e We define X_i such that $X_i = 1$, for treatment i ; 0, otherwise, where $i = 1, 2, 3, 4$. Then the appropriate regression model is $Y = \beta_0 + \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 + \alpha_4 X_4 + E$ where the regression coefficients are as follows:
$\beta_0 = \mu_5$, $\alpha_1 = \mu_1 - \mu_5$, $\alpha_2 = \mu_2 - \mu_5$, $\alpha_3 = \mu_3 - \mu_5$, $\alpha_4 = \mu_4 - \mu_5$
For $X_i = -1$, for treatment 5 ; 1, for treatment i, (i=1,2,3,4) ; 0, otherwise

The regression coefficients are:
$\beta_0 = \mu$, $\alpha_1 = \mu_1 - \mu$, $\alpha_2 = \mu_2 - \mu$, $\alpha_3 = \mu_3 - \mu$, $\alpha_4 = \mu_4 - \mu$, and also
$-(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4) = \mu_5 - \mu$

f Calculating by hand: We rank the sample means in descending order of magnitude:
$\overline{Y}_1 > \overline{Y}_5 > \overline{Y}_4 > \overline{Y}_2 > \overline{Y}_3$
So the order of comparisons to be made is 1 vs 3, 1 vs 2, 1 vs 4, 1 vs 5, 5 vs 3, 5 vs 2, 5 vs 4, 4 vs 3, 4 vs 2, 2 vs 3.

MSE=2.34 (from the ANOVA table in (b)), and

$n_i = 6 = n^*$ ($i = 1, 2, 3, 4, 5$); $n = \sum_{i=1}^{5} n_i = 30$

$k = 5$, $\alpha = 0.05$

<u>Scheffé 's method:</u>
$S^2 = (k-1) F_{k-1, n-k, 1-\alpha} = 4 * F_{4, 25, 0.95} = 4 * 2.75 = 11$
$S = 3.317$
Thus the half width, w_S, by Scheffé's method is

$$w_s = S\sqrt{\text{MSE}\left(\frac{1}{6}+\frac{1}{6}\right)} = 3.317\sqrt{2.34\left(\frac{1}{3}\right)} = 2.929$$

<u>Tukey's method:</u>

$$T = \frac{1}{\sqrt{n^*}} q_{k,n-k,1-\alpha} = \frac{1}{\sqrt{6}} q_{5,25,0.95} = \left(\frac{1}{\sqrt{6}}\right)(4.158) = 1.697$$

Thus the half width, w_T, by Tukey's method is:
$w_T = T\sqrt{MSE} = (1.697)\sqrt{2.34} = 2.596$

<u>Bonferroni's method:</u>

There are $C_2^5 = 10$ pairwise comparisons; so $\alpha^* = \frac{\alpha}{10} = 0.005$.

The half width by Bonferroni's method is $w_B = t_{25,0.9975}\sqrt{\text{MSE}\left(\frac{1}{6}+\frac{1}{6}\right)} = 2.827$