

2. a i For the model $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + E$:

$H_0: \beta_1 = \beta_2 = 0$ vs. $H_A: \beta_1 \neq 0$ and/or $\beta_2 \neq 0$

$F = 6.24$ df: 2, 50

$P = 0.0038$

At $\alpha = 0.05$, we would reject H_0 and conclude that $\beta_1 \neq 0$ and/or $\beta_2 \neq 0$; either X_1 , or X_2 , or both are linearly associated with Y .

ii For the model $Y = \beta_0 + \beta_1 X_1 + E$:

$H_0: \beta_1 = 0$ vs. $H_A: \beta_1 \neq 0$

$$F = \frac{\text{Regression SS}(X_1)}{\text{Residual MS}(X_1)} = \frac{1535.8570}{12255.3128/51} = 6.39$$

Note: the regression sum of squares for this variables-added-in-order test is taken from the 'Type I SS' section of the SAS output. The residual MS given in Chapter 8, problem 2.

df: 1, 51

$0.01 < P < 0.025$

At $\alpha = 0.05$, we would reject H_0 and conclude that $\beta_1 \neq 0$. X_1 is linearly associated with Y .

iii For the model $Y = \beta_0 + \beta_2 X_2 + E$:

$H_0: \beta_2 = 0$ vs. $H_A: \beta_2 \neq 0$

$$F = \frac{\text{SSY-Residual SS}(X_2)}{\text{Residual MS}(X_2)} = \frac{13791.1698 - 13633.3225}{13633.3225/51} = 0.59$$

df: 1, 51

P -value: $P = Pr[F > 0.59]$ where $F \sim F_{1,51}$

$P > 0.25$

At $\alpha = 0.05$, we would not reject H_0 and conclude that $\beta_2 = 0$. X_2 is not linearly associated with Y .

The overall F -test and partial F -tests for each variable indicate that X_1 is significantly aids in predicting Y while X_2 does not; therefore, we would choose the model containing only X_1 . By contrast, in chapter 8, problem 2(b), the model containing both predictors was selected as the "best" model, based on the R^2 values. R^2 values should not be the sole criteria used in selecting models; they can be artificially inflated by adding variables that do not significantly aid in predicting the outcome.

b The two variables-added-in-order tests are:

i $H_0: \beta_1 = 0$ vs. $H_A: \beta_1 \neq 0$ in the model $Y = \beta_0 + \beta_1 X_1 + E$.

From part (a) above: $F = 6.39$; df: 1, 51; $0.01 < P < 0.025$.

At $\alpha = 0.05$, we reject H_0 and conclude that $\beta_1 \neq 0$.

ii $H_0: \beta_2 = 0$ vs. $H_A: \beta_2 \neq 0$ in the model $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + E$.

$F(X_2|X_1) = 5.52$ df: 1, 50

$P = 0.0228$

At $\alpha = 0.05$, we would reject H_0 and conclude that $\beta_2 \neq 0$ in the model $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + E$.