Questions

1. Simplify the fraction $\dfrac{18}{24}$.

2. Simplify the fraction $\dfrac{12}{36}$.

3. Change $\dfrac{17}{6}$ to a <u>mixed number</u>.

4. Change $\dfrac{111}{9}$ to a <u>mixed number</u>.

5. Change $3\dfrac{1}{5}$ to an <u>improper fraction</u>.

6. Change $6\dfrac{3}{5}$ to an <u>improper fraction</u>.

7. Find the missing numerator when $\dfrac{3}{11} = \dfrac{?}{44}$.

8. Find the missing numerator when $\dfrac{3}{5} = \dfrac{?}{35}$.

9. Charles Barkley of the Phoenix Suns once scored 1560 points during 68 games played during the season. Express as a mixed number in simplified form how many points he averaged per game.

10. The University of California system accepted 14,664 out of 18,330 California Community College students who applied as transfer in fall 2003. What simplified fraction shows what portion of applications were accepted?

11. Find the lowest common denominator, then add the fractions $\dfrac{7}{15}, \dfrac{11}{21}$.

12. Find the lowest common denominator, then add the fractions $\dfrac{7}{10}, \dfrac{1}{4}$.

13. Combine $\dfrac{3}{8} + \dfrac{2}{8}$.

14. Combine $\dfrac{5}{14} - \dfrac{1}{14}$.

15. Combine $\dfrac{5}{36} + \dfrac{7}{9} - \dfrac{5}{12}$.

16. Combine $4\dfrac{1}{3} + 3\dfrac{2}{5}$.

17. Combine $\dfrac{7}{9} + \dfrac{5}{6}$.

18. Combine $2\dfrac{1}{7} + 3\dfrac{11}{14}$.

19. Multiply then simplify $\dfrac{36}{7} \times \dfrac{5}{9}$.

20. Multiply then simplify $\dfrac{17}{18} \times \dfrac{3}{5}$.

21. Divide then simplify $\dfrac{\frac{5}{8}}{1\frac{3}{4}}$.

22. Divide then simplify $\dfrac{\frac{2}{3}}{1\frac{1}{4}}$.

23. Multiply then simplify $6 \times 4\dfrac{2}{3}$.

24. Multiply then simplify $2\dfrac{1}{2} \times \dfrac{1}{10} \times \dfrac{3}{4}$.

25. Jennifer rode her mountain bike for $4\frac{1}{5}$ miles after work. Two-thirds of the distance was over a mountain bike trail. How long is the mountain bike trail?

26. Change 2.39 to a percent.

27. Change 7.4% to a decimal.

28. What is 8% of 65?

29. 36 is what percent of 24?

30. 49 is what percent of 28?

31. What percent of 340 is 17?

32. In 2002 it was found that the average consumer 65 years or older had an annual income of $26,533.67. It was found that the average consumer spent $3,247.54 per year for health care. Find exactly the percentage of income spent for health care, then redo the calculation using estimates to speed calculation.

33. Carl bought a 20 gallon aquarium. He put $17\frac{3}{4}$ gallons of water into the aquarium, but it looked too low so he added $1\frac{1}{4}$ gallons of water. Then he added plants and gravel, but the water was too high so he removed $2\frac{2}{3}$ gallons of water. How many gallons of water are now in the aquarium?

34. A country club maintains the putting greens with a grass height of $\frac{7}{8}$ inches. The grass on the fairways is maintained at $2\frac{1}{2}$ inches. How much must the mower blade be lowered if the person mowing the fairways uses the same machine to mow the greens?

35. A denim shirt requires $2\frac{3}{4}$ yards of material to make. If you have 142 yards of material, how many shirts can you make (assume there is no wasted material).

36. What percent by weight of carbon tetrachloride, CCl_4, is carbon, C? The atomic weights of these elements are (C, 12: Cl, 35).

37. Dave Bagley traveled 24,500 miles last year. He is a salesperson and 74% of his mileage is business related. He was planning to deduct 31 cents per business mile on his income tax return, but his accountant told him he can deduct 35 cents per business mile. How much would his deduction increase if he uses the new larger amount?

Solutions

1. Technique: **prime factor** the numerator and denominator.

$$\frac{18}{24} = \frac{3 \times \cancel{3} \times \cancel{2}}{4 \times \cancel{3} \times \cancel{2}} = \frac{3}{4}$$

2.

$$\frac{12}{36} = \frac{1 \times \cancel{12}}{3 \times \cancel{12}} = \frac{1}{3}$$

3. Six goes into seventeen 2 times with a remainder of $\frac{5}{6}$.

$$\frac{17}{6} = 2\frac{5}{6}$$

4. Nine goes into one hundred and eleven 12 times with a remainder of $\frac{3}{9} = \frac{1 \times \cancel{3}}{3 \times \cancel{3}} = \frac{1}{3}$.

$$\frac{111}{9} = 12\frac{1}{3}$$

5. Write the addition in the mixed number explicitly.

$$3\frac{1}{5} = 3 + \frac{1}{5}$$
$$= \frac{3 \times 5}{5} + \frac{1}{5} \qquad \textbf{common denominator}$$
$$= \frac{15}{5} + \frac{1}{5} \qquad \text{simplify}$$
$$= \frac{15 + 1}{5} \qquad \text{simplify}$$
$$= \frac{16}{5} \qquad \text{done!}$$

6.

$$6\frac{3}{5} = 6 + \frac{3}{5} = \frac{6 \times 5}{5} + \frac{3}{5} = \frac{30}{5} + \frac{3}{5} = \frac{30 + 3}{5} = \frac{33}{5}$$

7. The numerator must be 12. Multiply (both numerator and denominator!) by 4 to get a denominator of 44.

$$\frac{3}{11} = \frac{3 \times 4}{11 \times 4}$$
$$= \frac{12}{44} \qquad \text{simplify}$$

8. The numerator must be 21. Multiply (both numerator and denominator!) by 7 to get a denominator of 35.

$$\frac{3}{5} = \frac{3 \times 7}{5 \times 7}$$
$$= \frac{21}{35} \qquad \text{simplify}$$

9. Start prime factoring and cancel factors until nothing else factors and cancels. Your solution might look different from mine.

$$\frac{1560}{68} = \frac{\cancel{4} \times 390}{\cancel{4} \times 17} = \frac{390}{17}$$

Now that we have the the simplified improper fraction, we can convert to a mixed number.

Seventeen goes into three hundred and ninety 22 times with a remainder of $\frac{16}{17}$.

$$\frac{390}{17} = 22\frac{16}{17}.$$

10. Start prime factoring and cancel factors until nothing else factors and cancels.

$$\frac{14664}{18330} = \frac{\cancel{2} \times 7332}{\cancel{2} \times 9165}$$
$$= \frac{7332}{9165}$$
$$= \frac{\cancel{3} \times 2444}{\cancel{3} \times 3055}$$
$$= \frac{2 \times 1222}{5 \times 611}$$
$$= \frac{2 \times 2 \times \cancel{611}}{5 \times \cancel{611}}$$
$$= \frac{4}{5}$$

11. Technique: **prime factor** the numerators to determine the LCD (lowest common denominator).

$$15 = 3 \times 5$$
$$\underline{21 = 3 \times \qquad 7}$$
$$\Rightarrow \text{LCD} = 3 \times 5 \times 7 = 105$$

Multiply by appropriate numbers to get common denominator

$$
\begin{aligned}
\frac{7}{15} + \frac{11}{21} &= \frac{7 \times 7}{15 \times 7} + \frac{11 \times 5}{21 \times 5} \\
&= \frac{49}{105} + \frac{55}{105} && \text{simplify} \\
&= \frac{49 + 55}{105} \\
&= \frac{104}{105} && \text{simplify}
\end{aligned}
$$

12.

$$10 = 2 \times 5$$
$$\underline{4 = 2 \times \qquad 2}$$
$$\Rightarrow \text{LCD} = 2 \times 5 \times 2 = 20$$

$$
\begin{aligned}
\frac{7}{10} + \frac{1}{4} &= \frac{7 \times 2}{10 \times 2} + \frac{1 \times 5}{4 \times 5} \\
&= \frac{14}{20} + \frac{5}{20} \\
&= \frac{14 + 5}{20} \\
&= \frac{19}{20}
\end{aligned}
$$

13. The denominators are already the same.

$$\frac{3}{8} + \frac{2}{8} = \frac{3 + 2}{8} = \frac{5}{8}$$

14. The denominators are already the same.

$$\frac{5}{14} - \frac{1}{14} = \frac{5 - 1}{14} = \frac{4}{14} = \frac{2 \times \cancel{2}}{\cancel{2} \times 7} = \frac{2}{7}$$

15.

$$36 = 3 \times 3 \times 4$$
$$9 = 3 \times 3$$
$$\underline{12 = 3 \times \qquad 4}$$
$$\Rightarrow \text{LCD} = 3 \times 3 \times 4 = 36$$

$$\begin{aligned}
\frac{5}{36} + \frac{7}{9} - \frac{5}{12} &= \frac{5}{36} + \frac{7 \times \mathbf{4}}{9 \times \mathbf{4}} - \frac{5 \times \mathbf{3}}{12 \times \mathbf{3}} \\
&= \frac{5}{36} + \frac{28}{36} - \frac{15}{36} \\
&= \frac{5 + 28 - 15}{36} \\
&= \frac{18}{36} \\
&= \frac{1 \times \cancel{2} \times \cancel{9}}{2 \times \cancel{2} \times \cancel{9}} \\
&= \frac{1}{2}
\end{aligned}$$

Note: we usually would write $18 = 2 \times 9$ but since we are going to cancel everything in the numerator, it helps to write as $18 = 1 \times 2 \times 9$ so we can see that there will still be a 1 left in the numerator.

16. First, convert the mixed numbers to improper fractions since we know how to add improper fractions.

Now we can add.

$$\begin{aligned}
4\frac{1}{3} = 4 + \frac{1}{3} &= \frac{4 \times \mathbf{3}}{\mathbf{3}} + \frac{1}{3} = \frac{12}{3} + \frac{1}{3} = \frac{12 + 1}{3} \\
&= \frac{13}{3} \\
3\frac{2}{5} = 3 + \frac{2}{5} &= \frac{3 \times \mathbf{5}}{\mathbf{5}} + \frac{2}{5} = \frac{15}{5} + \frac{2}{5} = \frac{15 + 2}{5} \\
&= \frac{17}{5}
\end{aligned}$$

Now get a lowest common denominator.

$$3 = 3$$
$$\underline{5 = \qquad 5}$$
$$\Rightarrow \text{LCD} = 3 \times 5 = 15$$

$$\begin{aligned}
4\frac{1}{3} + 3\frac{2}{5} &= \frac{13}{3} + \frac{17}{5} \\
&= \frac{13 \times \mathbf{5}}{3 \times \mathbf{5}} + \frac{17 \times \mathbf{3}}{5 \times \mathbf{3}} \\
&= \frac{65}{15} + \frac{51}{15} \\
&= \frac{65 + 51}{15} \\
&= \frac{116}{15} \\
&= 7\frac{11}{15}
\end{aligned}$$

17.

$$9 = 3 \times 3 \times 3$$
$$6 = 3 \times 2$$
$$\overline{}$$
$$\Rightarrow \text{LCD} = 3 \times 3 \times 2 = 18$$

$$
\begin{aligned}
\frac{7}{9} + \frac{5}{6} &= \frac{7 \times \mathbf{2}}{9 \times \mathbf{2}} + \frac{5 \times \mathbf{3}}{6 \times \mathbf{3}} \\
&= \frac{14}{18} + \frac{15}{18} \\
&= \frac{14 + 15}{18} \\
&= \frac{29}{18}
\end{aligned}
$$

18. First, convert the mixed numbers to improper fractions since we know how to add improper fractions.

$$
\begin{aligned}
2\frac{1}{7} &= 2 + \frac{1}{7} = \frac{2 \times \mathbf{7}}{\mathbf{7}} + \frac{1}{7} = \frac{14}{7} + \frac{1}{7} = \frac{14 + 1}{7} \\
&= \frac{15}{7} \\
3\frac{11}{14} &= 3 + \frac{11}{14} = \frac{3 \times \mathbf{14}}{\mathbf{14}} + \frac{11}{14} = \frac{42}{14} + \frac{11}{14} \\
&= \frac{42 + 11}{14} = \frac{53}{14}
\end{aligned}
$$

Now get a lowest common denominator.

$$7 = 7$$
$$14 = 7 \quad 2$$
$$\overline{}$$
$$\Rightarrow \text{LCD} = 7 \times 2 = 14$$

Now we can add.

$$
\begin{aligned}
2\frac{1}{7} + 3\frac{11}{14} &= \frac{15}{7} + \frac{53}{14} \\
&= \frac{15 \times \mathbf{2}}{7 \times \mathbf{2}} + \frac{53}{14} \\
&= \frac{30}{14} + \frac{53}{14} \\
&= \frac{30 + 53}{14} \\
&= \frac{83}{14} \\
&= 5\frac{13}{14}
\end{aligned}
$$

19. Multiply numerator and denominator

$$\frac{36}{7} \times \frac{5}{9} = \frac{36 \times 5}{7 \times 9}$$
$$= \frac{\cancel{9} \times 4 \times 5}{7 \times \cancel{9}}$$
$$= \frac{20}{7}$$

factor to simplify

20.

$$\frac{17}{18} \times \frac{3}{5} = \frac{17 \times 3}{18 \times 5}$$
$$= \frac{17 \times \cancel{3}}{\cancel{3} \times 6 \times 5}$$
$$= \frac{17}{30}$$

21. Convert mixed numbers to improper fractions.

$$1\frac{3}{4} = 1 + \frac{3}{4} = \frac{4}{4} + \frac{3}{4} = \frac{4+3}{4} = \frac{7}{4}$$

$$\frac{\frac{5}{8}}{1\frac{3}{4}} = \frac{\left(\frac{5}{8}\right)}{\left(\frac{7}{4}\right)}$$
$$= \frac{5}{8} \times \frac{4}{7}$$
$$= \frac{5 \times 4}{8 \times 7}$$
$$= \frac{5 \times \cancel{4}}{2 \times \cancel{4} \times 7}$$
$$= \frac{5}{14}$$

22. Convert mixed numbers to improper fractions.

$$1\frac{1}{4} = 1 + \frac{1}{4} = \frac{4}{4} + \frac{1}{4} = \frac{4+1}{4} = \frac{5}{4}$$

$$\frac{\frac{2}{3}}{1\frac{1}{4}} = \frac{\left(\frac{2}{3}\right)}{\left(\frac{5}{4}\right)}$$
$$= \frac{2}{3} \times \frac{4}{5}$$
$$= \frac{2 \times 4}{3 \times 5}$$
$$= \frac{8}{15}$$

23.

$$4\frac{2}{3} = 4 + \frac{2}{3} = \frac{4 \times \mathbf{3}}{\mathbf{3}} + \frac{2}{3} = \frac{12}{3} + \frac{2}{3} = \frac{12 + 2}{3} = \frac{14}{3}$$

$$6 \times 4\frac{2}{3} = 6 \times \frac{14}{3}$$
$$= \frac{6 \times 14}{3}$$
$$= \frac{2 \times \cancel{3} \times 14}{\cancel{3}} = 28$$

24.

$$2\frac{1}{2} = 2 + \frac{1}{2} = \frac{2 \times \mathbf{2}}{\mathbf{2}} + \frac{1}{2} = \frac{4}{2} + \frac{1}{2} = \frac{4 + 1}{2} = \frac{5}{2}$$

$$2\frac{1}{2} \times \frac{1}{10} \times \frac{3}{4} = \frac{5}{2} \times \frac{1}{10} \times \frac{3}{4}$$
$$= \frac{5 \times 1 \times 3}{2 \times 10 \times 4}$$
$$= \frac{\cancel{5} \times 1 \times 3}{2 \times 2 \times \cancel{5} \times 4} = \frac{3}{16}$$

25. The mountain bike trail will have length $\frac{2}{3}$ of the distance traveled.

$$\frac{2}{3} \times 4\frac{1}{5} = \frac{2}{3} \times \frac{21}{5}$$
$$= \frac{2 \times 21}{3 \times 5}$$
$$= \frac{2 \times 7 \times \cancel{3}}{\cancel{3} \times 5}$$
$$= \frac{14}{5}$$

The mountain bike trail is $\frac{14}{5} = 2\frac{4}{5}$ miles long.

26.

$$2.39 = \frac{239}{100} = 239\%$$

27.

$$7.4\% = \frac{7.4}{100} = 0.074$$

28.

$$8\% \times 65 = 0.08 \times 65 = 5.2$$

29.
$$\frac{36}{24} = 1.5 = \frac{150}{100} = 150\%$$

30.
$$\frac{49}{28} = 1.75 = \frac{175}{100} = 175\%$$

31.
$$\frac{17}{340} = 0.05 = \frac{5}{100} = 5\%$$

32.
$$\text{Exact: } \frac{3,247.54}{26,533.67} = 0.122393 = \frac{12.2393}{100}$$
$$= 12.2393\%$$

$$\text{Estimate: } \frac{3,247.54}{26,533.67} \sim \frac{3,000}{30,000} = \frac{1}{10} = 10\%$$

33.
$$17\frac{3}{4} + 1\frac{1}{4} - 2\frac{2}{3} = \frac{71}{4} + \frac{5}{4} - \frac{8}{3}.$$

The LCD for these improper fractions is 12.
$$\frac{71}{4} + \frac{5}{4} - \frac{8}{3} = \frac{71 \times 3}{4 \times 3} + \frac{5 \times 3}{4 \times 3} - \frac{8 \times 4}{3 \times 4}$$
$$= \frac{213}{12} + \frac{15}{12} - \frac{32}{12}$$
$$= \frac{213 + 15 - 32}{12}$$
$$= \frac{196}{12}$$
$$= 16\frac{1}{3}$$

Since 12 divides into 196 16 times with a remainder of $\frac{4}{12} = \frac{1}{3}$, there are $16\frac{1}{3}$ gallons of water in the tank.

34.
$$2\frac{1}{2} - \frac{7}{8} = \frac{5}{2} - \frac{7}{8}$$
$$= \frac{5 \times 4}{2 \times 4} - \frac{7}{8}$$
$$= \frac{20}{8} - \frac{7}{8}$$
$$= \frac{20 - 7}{8}$$
$$= \frac{13}{8} = 1\frac{5}{8}$$

The blade must be lowered $1\frac{5}{8}$ inches.

35.

$$\frac{142}{\left(2\frac{3}{4}\right)} = \frac{142}{\left(\frac{11}{4}\right)} = \frac{142}{1} \times \frac{4}{11} = \frac{568}{11} \sim 51.6$$

You only have enough material to make 51 shirts. Notice in this problem it made sense to convert to a decimal at the end.

36.

$$\frac{68}{80} = 0.85 = 85\%.$$

37.

$$1.8\% \times 36,000 = 0.018 \times 36,000 = 648 \text{ were defective.}$$

38. Carbon tetrachloride is made up of 1 carbon and 4 chlorine.

The atomic weight of carbon tetrachloride is therefore $1 \times 12 + 4 \times 35 = 152$ atomic units.

The fraction of this weight that is carbon is $\dfrac{1 \times 12}{152} = \dfrac{3 \times \cancel{4}}{38 \times \cancel{4}} = \dfrac{3}{38}$.

Since $\dfrac{3}{38} \sim 0.07894 = 7.89\%$, carbon tetrachloride gets roughly 7.89% of its weight from carbon.

It made sense to convert to decimals at the end.

39. First, we need to determine how many miles are business related miles.

$$24,500 \text{ miles} \times 74\% \text{ business related}$$
$$= 24,500 \times 74\% \text{ business related miles}$$

Let's drop the units and figure this out:

$$24,500 \times 74\% = 24,500 \times 0.74 = 18,130.$$

So Dave has 18,130 business miles.

His deduction at 31 cents per mile (or \$0.31 per mile) would be:

$$18,130 \times \$0.31 = \$5620.30.$$

His deduction at 35 cents per mile (or \$0.35 per mile) would be:

$$18,130 \times \$0.35 = \$6345.50.$$

Dave's accountant increased his deduction by

$$\$6345.50 - \$5620.30 = \$725.20.$$