

Questions

- Write down the rules of exponents.
- Simplify $\left(\frac{3xy^{-2}}{y^3}\right)^{-2}$.
- Simplify $\left(\frac{5x^{-2}y}{x^4}\right)^{-2}$.
- Simplify $2a^{-1/6}b^{3/4}$ so there are no negative exponents.
- Simplify $-5y^{-2/3}$ so there are no negative exponents.
- Simplify $(27)^{2/3}$.
- Simplify $(-27)^{5/3}$.
- Simplify $(-64)^{2/3}$.
- Simplify $(x^{-1/3}y^{2/3})(x^{1/3}y^{1/4})$.
- Factor the common factor $2a$ in $10a^{5/4} - 4a^{8/5}$.
- Factor the common factor $2a$ in $6a^{4/3} - 8a^{3/2}$.
- Factor the common factor $3x$ in $21x^{13/8} - 12x^{4/3}$.
- Simplify $-\sqrt{\frac{1}{9}}$.
- Simplify $\sqrt{0.04}$.
- Find the value of the function $f(x) = \sqrt{10x + 5}$ at $x = 0$, $x = 1$, $x = 2$, and $x = 3$. What is the domain of the function $f(x)$?
- Find the value of the function $f(x) = \sqrt{1.5x - 4}$ at $x = 4$, $x = 6$, $x = 8$, and $x = 14$. What is the domain of the function $f(x)$?
- Simplify $\sqrt[3]{-\frac{8}{27}}$.
- Replace radicals with rational exponents in $\sqrt[5]{2x}$.
- Replace radicals with rational exponents in $\sqrt[4]{3y}$.
- Replace radicals with rational exponents in $\sqrt[7]{(a+b)^3}$.
- Simplify $\sqrt[3]{-125x^{30}}$.
- Simplify $\sqrt[3]{-27a^6}$.
- Simplify $\sqrt[4]{a^{12}b^4}$.
- Simplify $\sqrt[4]{a^4b^{20}}$.
- Simplify $\sqrt{54}$.
- Simplify $\sqrt{44}$.
- Simplify $\sqrt{90}$.
- Simplify $5\sqrt{75} + \sqrt{48}$.
- Simplify $\sqrt{45} + \sqrt{80} - 3\sqrt{20}$.
- Simplify $-\sqrt{12} + 2\sqrt{48} - \sqrt{75}$.
- Simplify $5\sqrt{27x} - 4\sqrt{75x}$.
- Simplify $\sqrt{75a^3} + a\sqrt{12a}$.
- Simplify $\sqrt[3]{128} - 4\sqrt[3]{16}$.
- Simplify $(3\sqrt{3} + \sqrt{5})(\sqrt{3} - 2\sqrt{5})$.
- Simplify $(\sqrt{7} + 4\sqrt{5x})(2\sqrt{7} + 3\sqrt{5x})$.
- Simplify $(3\sqrt{5} + \sqrt{3})(\sqrt{2} + 2\sqrt{5})$.
- Simplify $\sqrt{\frac{49}{25}}$.
- Simplify $\sqrt{\frac{16}{36}}$.
- Simplify $\sqrt{\frac{12x}{49y^6}}$.
- Rationalize denominator in $\frac{3x}{\sqrt{10} - \sqrt{2}}$.
- Rationalize denominator in $\frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}}$.
- Rationalize numerator in $\frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}}$.
- Rationalize denominator in $\frac{\sqrt{3x} - 2\sqrt{y}}{\sqrt{3x} + \sqrt{y}}$.
- Rationalize numerator in $\frac{\sqrt{3x} - 2\sqrt{y}}{\sqrt{3x} + \sqrt{y}}$.
- Rationalize numerator in $\frac{\sqrt{3} + 2\sqrt{7}}{8}$.

Solutions

1. The rules of exponents are:

- $x^0 = 1$ if $x \neq 0$ (0^0 is indeterminate and is dealt with in calculus).
- Product Rule: $x^a \cdot x^b = x^{a+b}$.
- Quotient Rule: $\frac{x^a}{x^b} = x^{a-b}$.
- Power Rule: $(x^a)^b = x^{ab}$.
- Product Raised to Power Rule: $(xy)^a = x^a y^a$.
- Quotient Raised to a Power Rule: $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$ if $y \neq 0$.
- Negative Exponent: $x^{-n} = \frac{1}{x^n}$, if $x \neq 0$.

2.

$$\begin{aligned} \left(\frac{3xy^{-2}}{y^3}\right)^{-2} &= \frac{(3)^{-2}(x)^{-2}(y^{-2})^{-2}}{(y^3)^{-2}} && \text{(Product Raised to Power Rule)} \\ &= \frac{y^4}{3^2 x^2 (y^{-6})} && \text{(simplify, using Power Rule and Negative Exponent Rule)} \\ &= \frac{y^4 y^6}{9x^2} && \text{(simplify, using Negative Exponent Rule)} \\ &= \frac{y^{4+6}}{9x^2} && \text{(simplify, using Product Rule)} \\ &= \frac{y^{10}}{9x^2} && \text{(simplify)} \end{aligned}$$

3.

$$\begin{aligned} \left(\frac{5x^{-2}y}{x^4}\right)^{-2} &= \frac{5^{-2}(x^{-2})^{-2}y^{-2}}{(x^4)^{-2}} && \text{(Product Raised to Power Rule)} \\ &= \frac{x^4 x^8}{5^2 y^2} && \text{(simplify, using Power Rule and Negative Exponent Rule)} \\ &= \frac{x^{12}}{25y^2} && \text{(simplify, using Product Rule)} \end{aligned}$$

4. $2a^{-1/6}b^{3/4} = \frac{2b^{3/4}}{a^{1/6}}$.

5. $-5y^{-2/3} = \frac{-5}{y^{2/3}}$.

6. $(27)^{2/3} = (3^3)^{2/3} = (3)^{3 \times 2/3} = (3)^2 = 9$.

$$7. (-27)^{5/3} = ((-3)^3)^{5/3} = (-3)^{3 \times 5/3} = (-3)^5 = -243.$$

$$8. (-64)^{2/3} = ((-4)^3)^{2/3} = (-4)^{3 \times 2/3} = (-4)^2 = 16.$$

$$9. (x^{-1/3}y^{2/3})(x^{1/3}y^{1/4}) = x^{-1/3+1/3}y^{2/3+1/4} = x^0y^{8/12+3/12} = y^{11/12}$$

$$10. 10a^{5/4} - 4a^{8/5} = 2a \cdot 5a^{1/4} - 2a \cdot 2a^{3/5} = 2a(5a^{1/4} - 2a^{3/5}).$$

$$11. 6a^{4/3} - 8a^{3/2} = 2a \cdot 3a^{1/3} - 2a \cdot 4a^{1/2} = 2a(3a^{1/3} - 4a^{1/2}).$$

$$12. 21x^{13/8} - 12x^{4/3} = 3x \cdot 7x^{5/8} - 3x \cdot 4x^{1/3} = 3x(7x^{5/8} - 4x^{1/3}).$$

$$13. -\sqrt{\frac{1}{9}} = -\sqrt{\left(\frac{1}{3}\right)^2} = -\frac{1}{3}.$$

$$14. \sqrt{0.04} = \sqrt{(0.2)^2} = 0.2.$$

15. Use a calculator to approximate some of the square roots.

$$f(x) = \sqrt{10x + 5}$$

$$f(0) = \sqrt{10(0) + 5} = \sqrt{5} \sim 2.2$$

$$f(1) = \sqrt{10(1) + 5} = \sqrt{15} \sim 3.9$$

$$f(2) = \sqrt{10(2) + 5} = \sqrt{25} = 5$$

$$f(3) = \sqrt{10(3) + 5} = \sqrt{35} \sim 5.9$$

For the domain, we know that we can only get a real number out of a square root if we put in a number greater than or equal to zero, so for this expression the domain is

$$10x + 5 \geq 0$$

$$10x \geq -5$$

$$x \geq -\frac{5}{10}$$

$$x \geq -\frac{1}{2}$$

The domain is $x \geq -1/2$ or $x \in [-1/2, \infty)$.

16. Use a calculator to approximate the square roots.

$$f(x) = \sqrt{1.5x - 4}$$

$$f(4) = \sqrt{1.5(4) - 4} = \sqrt{2} \sim 1.4$$

$$f(6) = \sqrt{1.5(6) - 4} = \sqrt{5} \sim 2.2$$

$$f(8) = \sqrt{1.5(8) - 4} = \sqrt{8} \sim 2.8$$

$$f(14) = \sqrt{1.5(14) - 4} = \sqrt{17} \sim 4.1$$

For the domain, we know that we can only get a real number out of a square root if we put in a number greater than or equal to zero, so for this expression the domain is

$$\begin{aligned}
 1.5x - 4 &\geq 0 \\
 1.5x &\geq 4 \\
 x &\geq \frac{4}{1.5} \\
 x &\geq \frac{4}{3/2} \\
 x &\geq \frac{8}{3}
 \end{aligned}$$

The domain is $x \geq 8/3$ or $x \in [8/3, \infty)$.

17.

$$\begin{aligned}
 \sqrt[3]{-\frac{8}{27}} &= \left(-\frac{8}{27}\right)^{1/3} && \text{(radical to rational notation)} \\
 &= \left(-\frac{2^3}{3^3}\right)^{1/3} && \text{(identify cubed numbers)} \\
 &= \left(-\frac{2^3}{3^3}\right)^{1/3} \\
 &= \left(\left(-\frac{2}{3}\right)^3\right)^{1/3} && \text{(Quotient Raised to Power Rule)} \\
 &= -\frac{2}{3} && \text{(Power Rule)}
 \end{aligned}$$

18. $\sqrt[5]{2x} = (2x)^{1/5}$.

19. $\sqrt[4]{3y} = (3y)^{1/4}$.

20. $\sqrt[7]{(a+b)^3} = ((a+b)^3)^{1/7} = (a+b)^{3/7}$.

21.

$$\begin{aligned}
 \sqrt[3]{-125x^{30}} &= (-125x^{30})^{1/3} && \text{(radical to rational notation)} \\
 &= (-125)^{1/3}(x^{30})^{1/3} && \text{(Product Raised to Power Rule)} \\
 &= ((-5)^3)^{1/3}x^{10} && \text{(identify cubed numbers)} \\
 &= (-5)x^{10} && \text{(Power Rule)} \\
 &= -5x^{10}
 \end{aligned}$$

22.

$$\begin{aligned}\sqrt[3]{-27a^6} &= (-27a^6)^{1/3} && \text{(radical to rational notation)} \\ &= (-\mathbf{27})^{1/3} (a^6)^{1/3} && \text{(Product Raised to Power Rule)} \\ &= ((-\mathbf{3})^3)^{1/3} a^2 && \text{(identify cubed numbers)} \\ &= (-3)a^2 && \text{(Power Rule)} \\ &= -3a^2\end{aligned}$$

23.

$$\begin{aligned}\sqrt[4]{a^{12}b^4} &= (a^{12}b^4)^{1/4} && \text{(radical to rational notation)} \\ &= (\mathbf{a^{12}})^{1/4} (b^4)^{1/4} && \text{(Product Raised to Power Rule)} \\ &= ((\mathbf{a^3})^4)^{1/4} |b| && \text{(note: Need to use rule that } (x^n)^{1/n} = |x| \text{ if } n \text{ is even)} \\ &= |a^3||b| \\ &= |a^3b|\end{aligned}$$

24.

$$\begin{aligned}\sqrt[4]{a^4b^{20}} &= (a^4b^{20})^{1/4} && \text{(radical to rational notation)} \\ &= (a^4)^{1/4}(b^{20})^{1/4} && \text{(Product Raised to Power Rule)} \\ &= |a|((b^5)^4)^{1/4} && \text{(Power Rule)} \\ &= |a||b^5| && \text{(note: Need to use rule that } (x^n)^{1/n} = |x| \text{ if } n \text{ is even)} \\ &= |ab^5|\end{aligned}$$

25. $\sqrt{54} = \sqrt{9 \cdot 6} = \sqrt{9}\sqrt{6} = 3\sqrt{6}.$

26. $\sqrt{44} = \sqrt{4 \cdot 11} = \sqrt{4}\sqrt{11} = 2\sqrt{11}.$

27. $\sqrt{90} = \sqrt{9 \cdot 10} = \sqrt{9}\sqrt{10} = 3\sqrt{10}.$

28.

$$\begin{aligned}5\sqrt{75} + \sqrt{48} &= 5\sqrt{25 \cdot 3} + \sqrt{16 \cdot 3} && \text{(factor, identified squared numbers)} \\ &= 5\sqrt{25}\sqrt{3} + \sqrt{16}\sqrt{3} \\ &= 5 \cdot 5\sqrt{3} + 4\sqrt{3} \\ &= 25\sqrt{3} + 4\sqrt{3} && \text{(combine like terms)} \\ &= 29\sqrt{3}\end{aligned}$$

29.

$$\begin{aligned}\sqrt{45} + \sqrt{80} - 3\sqrt{20} &= \sqrt{9 \cdot 5} + \sqrt{16 \cdot 5} - 3\sqrt{4 \cdot 5} && \text{(factor, identified squared numbers)} \\ &= \sqrt{9}\sqrt{5} + \sqrt{16}\sqrt{5} - 3\sqrt{4}\sqrt{5} \\ &= 3\sqrt{5} + 4\sqrt{5} - 6\sqrt{5} && \text{(combine like terms)} \\ &= \sqrt{5}\end{aligned}$$

30.

$$\begin{aligned}
 -\sqrt{12} + 2\sqrt{48} - \sqrt{75} &= -\sqrt{4 \cdot 3} + 2\sqrt{16 \cdot 3} - \sqrt{25 \cdot 3} && \text{(factor, identified squared numbers)} \\
 &= -\sqrt{4}\sqrt{3} + 2\sqrt{16}\sqrt{3} - \sqrt{25}\sqrt{3} \\
 &= -2\sqrt{3} + 8\sqrt{3} - 5\sqrt{3} && \text{(combine like terms)} \\
 &= \sqrt{3}
 \end{aligned}$$

31.

$$\begin{aligned}
 5\sqrt{27x} - 4\sqrt{75x} &= 5\sqrt{9 \cdot 3x} - 4\sqrt{25 \cdot 3x} && \text{(factor, identified squared numbers)} \\
 &= 5\sqrt{9}\sqrt{3x} - 4\sqrt{25}\sqrt{3x} \\
 &= 15\sqrt{3x} - 20\sqrt{3x} && \text{(combine like terms)} \\
 &= -5\sqrt{3x}
 \end{aligned}$$

32.

$$\begin{aligned}
 \sqrt{75a^3} + a\sqrt{12a} &= \sqrt{25a^2 \cdot 3a} + a\sqrt{4 \cdot 3a} && \text{(factor, identified squared numbers)} \\
 &= \sqrt{25a^2}\sqrt{3a} + a\sqrt{4}\sqrt{3a} \\
 &= 5a\sqrt{3a} + 2a\sqrt{3a} && \text{(combine like terms)} \\
 &= 7a\sqrt{3a}
 \end{aligned}$$

33. Since this involves a cube root, look for numbers that are cubed, like $8 = 2^3$ and $64 = 4^3$.

$$\begin{aligned}
 \sqrt[3]{128} - 4\sqrt[3]{16} &= \sqrt[3]{64 \cdot 2} - 4\sqrt[3]{8 \cdot 2} && \text{(factor, identified cubed numbers)} \\
 &= \sqrt[3]{64}\sqrt[3]{2} - 4\sqrt[3]{8}\sqrt[3]{2} \\
 &= \sqrt[3]{4^3}\sqrt[3]{2} - 4\sqrt[3]{2^3}\sqrt[3]{2} \\
 &= 4\sqrt[3]{2} - 4(2)\sqrt[3]{2} \\
 &= 4\sqrt[3]{2} - 8\sqrt[3]{2} && \text{(combine like terms)} \\
 &= -4\sqrt[3]{2}
 \end{aligned}$$

34. Distribute, Distribute!

$$\begin{aligned}
 (3\sqrt{3} + \sqrt{5})(\sqrt{3} - 2\sqrt{5}) &= (3\sqrt{3})(\sqrt{3} - 2\sqrt{5}) + (\sqrt{5})(\sqrt{3} - 2\sqrt{5}) \\
 &= (3\sqrt{3})(\sqrt{3} - 2\sqrt{5}) + (\sqrt{5})(\sqrt{3} - 2\sqrt{5}) \\
 &= (3\sqrt{3})(\sqrt{3}) - (3\sqrt{3})(2\sqrt{5}) + (\sqrt{5})(\sqrt{3}) - (\sqrt{5})(2\sqrt{5}) \\
 &= 3(\sqrt{3})^2 - 6\sqrt{3}\sqrt{5} + \sqrt{5}\sqrt{3} - 2(\sqrt{5})^2 \\
 &= 3(3) - 6\sqrt{3 \cdot 5} + \sqrt{5 \cdot 3} - 2(5) \\
 &= -1 - 5\sqrt{15}
 \end{aligned}$$

35. Distribute, Distribute!

$$\begin{aligned}
 (\sqrt{7} + 4\sqrt{5x})(2\sqrt{7} + 3\sqrt{5x}) &= (\sqrt{7})(2\sqrt{7} + 3\sqrt{5x}) + (4\sqrt{5x})(2\sqrt{7} + 3\sqrt{5x}) \\
 &= (\sqrt{7})(2\sqrt{7} + 3\sqrt{5x}) + (4\sqrt{5x})(2\sqrt{7} + 3\sqrt{5x}) \\
 &= (\sqrt{7})(2\sqrt{7}) + (\sqrt{7})(3\sqrt{5x}) + (4\sqrt{5x})(2\sqrt{7}) + (4\sqrt{5x})(3\sqrt{5x}) \\
 &= 2(7) + 3\sqrt{7 \cdot 5x} + 8\sqrt{5x \cdot 7} + 12(5x) \\
 &= 14 + 11\sqrt{35x} + 60x
 \end{aligned}$$

36. Distribute, Distribute!

$$\begin{aligned}
 (3\sqrt{5} + \sqrt{3})(\sqrt{2} + 2\sqrt{5}) &= (3\sqrt{5})(\sqrt{2} + 2\sqrt{5}) + (\sqrt{3})(\sqrt{2} + 2\sqrt{5}) \\
 &= (3\sqrt{5})(\sqrt{2} + 2\sqrt{5}) + (\sqrt{3})(\sqrt{2} + 2\sqrt{5}) \\
 &= (3\sqrt{5})(\sqrt{2}) + (3\sqrt{5})(2\sqrt{5}) + (\sqrt{3})(\sqrt{2}) + (\sqrt{3})(2\sqrt{5}) \\
 &= 3\sqrt{5 \cdot 2} + (6)(5) + \sqrt{3 \cdot 2} + 2\sqrt{3 \cdot 5} \\
 &= 3\sqrt{10} + 30 + \sqrt{6} + 2\sqrt{15}
 \end{aligned}$$

$$37. \sqrt{\frac{49}{25}} = \frac{7}{5}.$$

$$38. \sqrt{\frac{16}{36}} = \frac{4}{6} = \frac{2}{3}.$$

$$39. \sqrt{\frac{12x}{49y^6}} = \sqrt{\frac{2^2 \cdot 3x}{(7y^3)^2}} = \frac{\sqrt{2^2} \cdot \sqrt{3x}}{\sqrt{(7y^3)^2}} = \frac{2\sqrt{3x}}{7y^3}.$$

40. Note the sign change when you rationalize! This is so the cross terms will cancel when you distribute.

$$\begin{aligned}
 \frac{3x}{\sqrt{10} - \sqrt{2}} &= \frac{(3x)(\sqrt{10} + \sqrt{2})}{(\sqrt{10} - \sqrt{2})(\sqrt{10} + \sqrt{2})} \\
 &= \frac{3x(\sqrt{10} + \sqrt{2})}{10 - 2} \\
 &= \frac{3x(\sqrt{10} + \sqrt{2})}{8}
 \end{aligned}$$

41.

$$\begin{aligned}
 \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} &= \frac{(\sqrt{5} + \sqrt{3})(\sqrt{5} + \sqrt{3})}{(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})} \\
 &= \frac{5 + 2\sqrt{15} + 3}{5 - 3} \\
 &= \frac{8 + 2\sqrt{15}}{2} \\
 &= 4 + \sqrt{15}
 \end{aligned}$$

42.

$$\begin{aligned} \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} &= \frac{(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3})}{(\sqrt{5} - \sqrt{3})(\sqrt{5} - \sqrt{3})} \\ &= \frac{5 - 3}{5 - 2\sqrt{15} + 3} \\ &= \frac{2}{8 - 2\sqrt{15}} \\ &= \frac{1}{4 - \sqrt{15}} \end{aligned}$$

43.

$$\begin{aligned} \frac{\sqrt{3x} - 2\sqrt{y}}{\sqrt{3x} + \sqrt{y}} &= \frac{(\sqrt{3x} - 2\sqrt{y})(\sqrt{3x} - \sqrt{y})}{(\sqrt{3x} + \sqrt{y})(\sqrt{3x} - \sqrt{y})} \\ &= \frac{3x - 2\sqrt{3x}\sqrt{y} - \sqrt{3x}\sqrt{y} + 2(y)}{3x - y} \\ &= \frac{3x - 3\sqrt{3xy} + 2y}{3x - y} \end{aligned}$$

44.

$$\begin{aligned} \frac{\sqrt{3x} - 2\sqrt{y}}{\sqrt{3x} + \sqrt{y}} &= \frac{(\sqrt{3x} - 2\sqrt{y})(\sqrt{3x} + 2\sqrt{y})}{(\sqrt{3x} + \sqrt{y})(\sqrt{3x} + 2\sqrt{y})} \\ &= \frac{3x - 4y}{3x + 2\sqrt{3x}\sqrt{y} + \sqrt{3x}\sqrt{y} + 2y} \\ &= \frac{3x - 4y}{3x + 3\sqrt{3xy} + 2y} \end{aligned}$$

45.

$$\begin{aligned} \frac{\sqrt{3} + 2\sqrt{7}}{8} &= \frac{(\sqrt{3} + 2\sqrt{7})(\sqrt{3} - 2\sqrt{7})}{8(\sqrt{3} - 2\sqrt{7})} \\ &= \frac{3 - 4(7)}{8(\sqrt{3} - 2\sqrt{7})} \\ &= \frac{-25}{8(\sqrt{3} - 2\sqrt{7})} \end{aligned}$$