1 Rational Exponents and Radicals

1.1 Rules of Exponents

The rules for exponents are the same as what you saw earlier. Memorize these rules if you haven't already done so.

- $x^0 = 1$ if $x \neq 0$ (0^0 is indeterminant and is dealt with in calculus).
- Product Rule: $x^a \cdot x^b = x^{a+b}$.
- Quotient Rule: $\dfrac{x^a}{x^b} = x^{a-b}$.
- Power Rule: $(x^a)^b = x^{ab}$.
- Product Raised to Power Rule: $(xy)^a = x^a y^a$.
- Quotient Raised to a Power Rule: $\left(\dfrac{x}{y}\right)^a = \dfrac{x^a}{y^a}$ if $y \neq 0$.
- Negative Exponent: $x^{-a} = \dfrac{1}{x^a}$, if $x \neq 0$.

What is new in this section is the powers a and b in our rules are extended to rational numbers, so you will be working with quantities like $(8)^{1/3}$.

1.2 Radical Notation and Rules of Radicals

If x is a nonnegative real number, then $\sqrt{x} > 0$ is the **principal square root of x**. This is because $(\sqrt{x})^2 = x$.

Higher order roots are defined using **radical notation** as: $\sqrt[n]{x}$.

In words, to evaluate the expression $\sqrt[n]{x} = y$ means you are looking for a number y that when multiplied by itself n times gives you the quantity x.

$$\sqrt[4]{16} = 2 \text{ since } (2)(2)(2)(2) = 16.$$

Note that is is true that $(-2)(-2)(-2)(-2) = 16$ but we choose $+2$ since we want the principal root.

1.3 Rules of Radicals

Working with radicals is important, but looking at the rules may be a bit confusing. Here are examples to help make the rules more concrete. The rules are fairly straightforward when everything is positive, which is most likely what you will see in your science classes.

1. If x is a **positive real number**, then
   - $\sqrt[n]{x}$ is the nth root of x and $(\sqrt[n]{x})^n = x$,

     $\left(\sqrt[3]{17}\right)^3 = 17$

     $\left(\sqrt[3]{8}\right)^3 = 8$ since $\sqrt[3]{8} = 2$ since $(2)(2)(2) = 8$
   - if n is a positive integer, we can write $x^{1/n} = \sqrt[n]{x}$.

     $8^{1/4} = \sqrt[4]{8}$

     $625^{1/4} = \sqrt[4]{625} = 5$ since $(5)(5)(5)(5) = 625$
2. If x is a **negative real number**, then
   - $(\sqrt[n]{x})^n = x$ when n is an odd integer,

     $\left(\sqrt[3]{-6}\right)^3 = -6$

     $\left(\sqrt[3]{-8}\right)^3 = -8$ since $\sqrt[3]{-8} = -2$ since $(-2)(-2)(-2) = -8$
   - $(\sqrt[n]{x})^n$ is not a real number when n is an even integer.

     $\left(\sqrt[2]{-6}\right)^2$ is not a real number.

   This last result is because there is no real number that you can square and get a negative number.
3. For **all real numbers** x **(including negative values)**
   - $\sqrt[n]{x^n} = \left|x\right|$ when n is an even positive integer,

     $\sqrt[4]{(-16)^4} = \left|-16\right| = 16$ since you take fourth power first, you are removing the negative sign

     $\sqrt[2]{(-6)^2} = \sqrt[2]{36} = \sqrt[2]{6^2} = 6$
   - $\sqrt[n]{x^n} = x$ when n is an odd positive integer.

     $\sqrt[3]{(-19)^3} = -19$

     $\sqrt[3]{(-8)^3} = \sqrt[3]{(-8)(-8)(-8)} = -8$

Summary:

$$\left(\sqrt[n]{x}\right)^n = x \qquad \text{(when } x \text{ is positive)}$$
$$\left(\sqrt[n]{x}\right)^n = x \qquad \text{(when } x < 0 \text{ and } n \text{ odd)}$$
$$\left(\sqrt[n]{x}\right)^n \text{ is not real number.} \qquad \text{(when } x < 0 \text{ and } n \text{ even)}$$
$$\sqrt[n]{x} = x^{1/n} \qquad \text{(when } x \text{ is positive and } n \text{ is positive integer)}$$
$$\sqrt[n]{x^n} = |x| \qquad \text{(for all real } x \text{ and } n \text{ even)}$$
$$\sqrt[n]{x^n} = x \qquad \text{(for all real } x \text{ and } n \text{ odd)}$$

Product Rule for radicals: When a, b are nonnegative real numbers,

$$\sqrt[n]{a}\,\sqrt[n]{b} = \sqrt[n]{ab},$$

which is really just the exponent rule $a^m b^m = (ab)^m$ where $m = 1/n$.

Quotient Rule for radicals: When a, b are nonnegative real numbers (and $b \neq 0$),

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}.$$

Absolute Value: $|x| = \sqrt{x^2}$ which is just an earlier result with $n = 2$.

EXAMPLE Evaluate $\left(\dfrac{16}{81}\right)^{3/4}$.

Since the radical for this expression would be $\left(\sqrt[4]{\dfrac{16}{81}}\right)^3$, we should look for a way to write $16/81$ as $(\text{something})^4$.

$$\left(\frac{16}{81}\right)^{3/4} = \left(\left(\frac{2}{3}\right)^4\right)^{3/4} \qquad \text{(rewrite with number to power 4)}$$
$$= \left(\frac{2}{3}\right)^{4(3/4)} = \left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27} \qquad \text{(power rule of exponents)}$$

Notice that writing this as $\left(\dfrac{16}{81}\right)^{3/4} = \sqrt[4]{\left(\dfrac{16}{81}\right)^3} = \sqrt[4]{\dfrac{4096}{531441}}$ is mathematically true, it doesn't help us simplify.

EXAMPLE Simplify $\sqrt{8} + \sqrt{50} - 2\sqrt{72}$.

Since we are dealing with square roots, we simplify by looking for quantities that can be written as $(\text{something})^2$.

$$\begin{aligned}
\sqrt{\mathbf{8}} + \sqrt{\mathbf{50}} - 2\sqrt{\mathbf{72}} &= \sqrt{\mathbf{4 \cdot 2}} + \sqrt{\mathbf{25 \cdot 2}} - 2\sqrt{\mathbf{36 \cdot 2}} \\
&= \sqrt{2^2 \cdot 2} + \sqrt{5^2 \cdot 2} - 2\sqrt{6^2 \cdot 2} \\
&= \sqrt{2^2}\sqrt{2} + \sqrt{5^2}\sqrt{2} - 2\sqrt{6^2}\sqrt{2} \\
&= 2\sqrt{2} + 5\sqrt{2} - 2 \cdot 6\sqrt{2} \\
&= -5\sqrt{2}
\end{aligned}$$

EXAMPLE Common Factor $x^{1/2}$ from the expression $3x^2 - 2x^{3/2} + x^{1/2}$.

SOLUTION: I like to do common factoring with radicals by using the rules of exponents.

$$\begin{aligned}
3x^2 - 2x^{3/2} + x^{1/2} &= 3x^{1/2+3/2} - 2x^{1/2+2/2} + x^{1/2} && \text{(rewrite exponents with a power of 1/2 in each)}\\
&= 3x^{1/2}x^{3/2} - 2x^{1/2}x^{2/2} + x^{1/2} && \text{(rules of exponents)}\\
&= 3\boldsymbol{x^{1/2}}x^{3/2} - 2\boldsymbol{x^{1/2}}x^{2/2} + \boldsymbol{x^{1/2}} && \text{(identify common factor)}\\
&= \boldsymbol{x^{1/2}}\left(3x^{3/2} - 2x + 1\right) && \text{(common factor)}
\end{aligned}$$

EXAMPLE Common Factor $x^{3/2}$ from the expression $x^{9/2} - x^{3/2}$.

SOLUTION: I like to do common factoring with radicals by using the rules of exponents.

$$\begin{aligned}
x^{9/2} - x^{3/2} &= x^{3/2+6/2} - x^{3/2} && \text{(rewrite exponents with a power of 3/2 in each)}\\
&= x^{3/2}x^{6/2} - x^{3/2} && \text{(rules of exponents)}\\
&= \boldsymbol{x^{3/2}}x^{6/2} - \boldsymbol{x^{3/2}} && \text{(identify common factor)}\\
&= \boldsymbol{x^{3/2}}\left(x^3 - 1\right) && \text{(common factor)}
\end{aligned}$$

EXAMPLE Common Factor $x^{1/4}$ from the expression $x^{9/4} - x^{3/4} - 2x^{1/4}$.

$$\begin{aligned}
x^{9/4} - x^{3/4} - 2x^{1/4} &= x^{8/4+1/4} - x^{2/4+1/4} - 2x^{1/4} && \text{(rewrite exponents with a power of 1/4 in each)}\\
&= x^{8/4}x^{1/4} - x^{2/4}x^{1/4} - 2x^{1/4} && \text{(rules of exponents)}\\
&= x^2\boldsymbol{x^{1/4}} - x^{1/2}\boldsymbol{x^{1/4}} - 2\boldsymbol{x^{1/4}} && \text{(identify common factor)}\\
&= \boldsymbol{x^{1/4}}\left(x^2 - \sqrt{x} - 2\right) && \text{(common factor)}
\end{aligned}$$

1.4 Rationalizing

Rationalizing something means getting rid of any radicals.

To rationalize a numerator, you want to modify the expression so as to remove any radicals from the numerator.

To rationalize a denominator, you want to modify the expression so as to remove any radicals from the denominator.

The expression $a+\sqrt{b}$ has the **conjugate** expression $a-\sqrt{b}$, which can be useful when rationalizing a denominator or numerator.

For example, $2 - \sqrt{43 - x}$ and $2 + \sqrt{43 - x}$ are conjugate expressions.

To **rationalize the denominator**, we multiply both the numerator and denominator by the conjugate of the denominator.

To **rationalize the numerator**, we multiply both the numerator and denominator by the conjugate of the numerator.

EXAMPLE Rationalize the denominator in the expression $\dfrac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-2\sqrt{y}}$.

To rationalize the denominator, we multiply both the numerator and denominator by the conjugate of the denominator

$$\begin{aligned}\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-2\sqrt{y}} &= \frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-2\sqrt{y}}\cdot\frac{\boldsymbol{\sqrt{x}+2\sqrt{y}}}{\boldsymbol{\sqrt{x}+2\sqrt{y}}}\\ &= \frac{(\sqrt{x}+\sqrt{y})(\sqrt{x}+2\sqrt{y})}{(\sqrt{x}-2\sqrt{y})(\sqrt{x}+2\sqrt{y})}\\ &= \frac{x+3\sqrt{x}\sqrt{y}+2y}{x-4y}\end{aligned}$$

We could also rationalize the numerator by multiplying both the numerator and denominator by the conjugate of the numerator.

$$\begin{aligned}\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-2\sqrt{y}} &= \frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-2\sqrt{y}}\cdot\frac{\boldsymbol{\sqrt{x}-\sqrt{y}}}{\boldsymbol{\sqrt{x}-\sqrt{y}}}\\ &= \frac{(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})}{(\sqrt{x}-2\sqrt{y})(\sqrt{x}-\sqrt{y})}\\ &= \frac{x-y}{x-3\sqrt{x}\sqrt{y}+2y}\end{aligned}$$

Some textbooks will say "If an expression contains a square root in the denominator, it is not considered simplified." I consider this statement crap. The problem is, what do we mean by *simplified*? Sometimes we may want to get rid of radicals in the denominator, but sometimes we may want to get rid of radicals in the numerator. To say one is more simplified than the other is completely misguided, in my opinion.

Simplification: To simplify an expression may mean different things in different situations. I view something as simplified if it is in a form that makes the next thing you want to do with it easier. Do you want to sketch a function? Look for roots? Find a numeric value? Substitute it into something else? Depending on what you want to do, you may want slightly different forms.

Your goal is to become proficient with the algebraic techniques of simplification (rationalizing numerator, rationalizing denominator, finding a common denominator, factoring, etc), so you can easily do whatever simplification is required.

EXAMPLE Rationalize the denominator in the expression $\dfrac{x}{\sqrt[4]{x}}$ so the denominator is x.

SOLUTION: This one is a little bit different than the previous examples, since the denominator we wish to rationalize has only one term.

What we need to do here is multiply both the numerator and denominator of the expression by something that will make the denominator x (since this is a rationalized denominator).

To figure out what that something is, we can do the following:

$$\begin{aligned}
\sqrt[4]{x}w &= x && \text{(where } w \text{ is the something we want to find)}\\
w &= \frac{x}{\sqrt[4]{x}} && \text{(division principle)}\\
&= \frac{x}{x^{1/4}} && \text{(rules of radicals)}\\
&= x^{1-1/4} && \text{(rules of exponents)}\\
&= x^{3/4}
\end{aligned}$$

Notice how similar this is to what we did when doing polynomial long division when fractions arose!

Now we can create an equivalent expression:

$$\begin{aligned}
\frac{x}{\sqrt[4]{x}} &= \frac{x \cdot \boldsymbol{x^{3/4}}}{\sqrt[4]{x} \cdot \boldsymbol{x^{3/4}}}\\
&= \frac{x^{1+3/4}}{x^{1/4}x^{3/4}}\\
&= \frac{x^{7/4}}{x^{1/4+3/4}} = \frac{x^{7/4}}{x} && \text{(equivalent expression with rationalized denominator)}
\end{aligned}$$

EXAMPLE Rationalize the denominator in the expression $\dfrac{x}{\sqrt[4]{x}}$.

SOLUTION: If you don't require that the denominator must be x, just use the rules of exponents to simplify:

$$\frac{x}{\sqrt[4]{x}} = \frac{x}{x^{1/4}} = x^{1-1/4} = x^{3/4} = \frac{x^{3/4}}{1} \qquad \text{(denominator is rationalized to 1)}$$

EXAMPLE Find a common denominator for the expression $\dfrac{13}{x^{1/2}} - \dfrac{2}{x^{7/2}}$.

SOLUTION: The process of finding a common denominator is exactly the same as before, only the fractional powers means we have to be a bit careful when finding LCD.

The LCD would be $x^{7/2}$, since $x^{7/2} = x^{6/2+1/2} = x^{3+1/2} = x^3x^{1/2}$.

$$\begin{aligned}
\frac{13}{x^{1/2}} - \frac{2}{x^{7/2}} &= \frac{13 \cdot \boldsymbol{x^3}}{x^{1/2} \cdot \boldsymbol{x^3}} - \frac{2}{x^{7/2}} && \text{(give first term same denominator as second term)}\\
&= \frac{13x^3}{x^{7/2}} - \frac{2}{x^{7/2}} && \text{(rules of exponents)}\\
&= \frac{13x^3 - 2}{x^{7/2}} && \text{(since } a/d + b/d = (a+b)/d\text{)}
\end{aligned}$$

If you wanted to, you could rationalize the denominator now.

EXAMPLE Solve the equation $-2x^{9/2} + 7x^{7/2} - 3x^{5/2} = 0$.

SOLUTION: When you first see this question it looks very challenging, but let's think it through using all the tools we have learned so far.

First, we can't "isolate an x" to solve the equation like we did back in Unit 3 since the equation is not linear in x.

We have also seen that we can solve quadratic equations by factoring in Unit 7, but this equation is not quadratic in x. But the equation has three terms so it kind of looks like this type.

In this Unit, we have seen how to work with fractional exponents, so let's try to factor out the smallest fractional power of x and see what happens. In this case the smallest power of x would be $x^{5/2}$.

$$\begin{aligned} -2x^{9/2} + 7x^{7/2} - 3x^{5/2} &= 0 && \text{(original equation)} \\ -2x^{4/2+5/2} + 7x^{2/2+5/2} - 3x^{5/2} &= 0 && \text{(rules of exponents)} \\ -2x^2 \cdot \boldsymbol{x^{5/2}} + 7x \cdot \boldsymbol{x^{5/2}} - 3 \cdot \boldsymbol{x^{5/2}} &= 0 && \text{(identify common factor)} \\ \boldsymbol{x^{5/2}}\Big(-2x^2 + 7x - 3\Big) &= 0 && \text{(common factor)} \end{aligned}$$

Notice we now have a quadratic to factor, which we can do using the Grouping Method from Unit 7.

Two numbers who product is $(-2)(-3) = 6$ and sum is 7: 1,6.

$$\begin{aligned} -2x^2 + 7x - 3 &= -2x^2 + 6x + 1x - 3 \\ &= -2x(x-3) + 1(x-3) \\ &= (-2x+1)(x-3) \end{aligned}$$

Our equation now becomes

$$x^{5/2}(-2x+1)(x-3) = 0 \qquad \text{(factored form of original equation)}$$

which can be solved using the Zero Factor Property from Unit 7.

$$\begin{aligned} x^{5/2} = 0 &\quad \Rightarrow \quad x = 0 \\ (-2x+1) = 0 &\quad \Rightarrow \quad x = 1/2 \\ (x-3) = 0 &\quad \Rightarrow \quad x = 3 \end{aligned}$$

So there are three solutions to $-2x^{9/2} + 7x^{7/2} - 3x^{5/2} = 0$, $x = 0, 1/2, 3$.

2 The Square Root Function

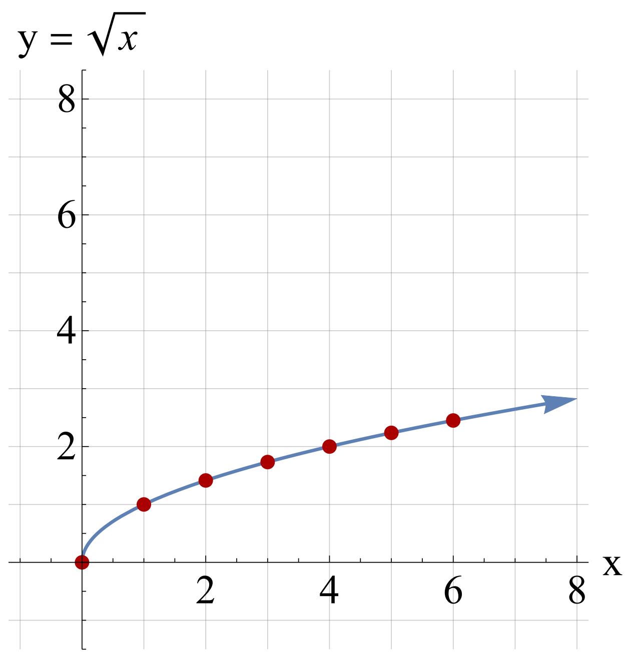
The **square root function** is defined as $f(x) = \sqrt{x}$.

Domain: The quantity under the square root must be positive or zero, so $x \geq 0$ or $x \in [0, \infty)$.

Range: $y \geq 0$ or $y \in [0, \infty)$.

$$
\begin{array}{lll}
f(\mathbf{-1}) = \sqrt{\mathbf{-1}} \text{ not a real number} & x = -1 \text{ is not in domain, no ordered pair} & \\
f(\mathbf{0}) = \sqrt{\mathbf{0}} = 0 & \Rightarrow & \text{ordered pair is } (0, 0) \\
f(\mathbf{1}) = \sqrt{\mathbf{1}} = 1 & \Rightarrow & \text{ordered pair is } (1, 1) \\
f(\mathbf{2}) = \sqrt{\mathbf{2}} & \Rightarrow & \text{ordered pair is } (2, \sqrt{2}) \\
f(\mathbf{3}) = \sqrt{\mathbf{3}} & \Rightarrow & \text{ordered pair is } (3, \sqrt{3}) \\
f(\mathbf{4}) = \sqrt{\mathbf{4}} = 2 & \Rightarrow & \text{ordered pair is } (4, 2) \\
f(\mathbf{5}) = \sqrt{\mathbf{5}} & \Rightarrow & \text{ordered pair is } (5, \sqrt{5}) \\
f(\mathbf{6}) = \sqrt{\mathbf{6}} & \Rightarrow & \text{ordered pair is } (6, \sqrt{6})
\end{array}
$$

Here is a plot of the points we have just found for the function. The line represents the other ordered pairs that we did not compute.



For more general square root functions, use the following ideas to get domain/range:

Domain: The quantity under the square root must be positive or zero.

Range: depends on the specific form of the square root function, and is usually found from looking at the sketch of the function.

EXAMPLE You are given the function $f(x) = \sqrt{x-2}$.

What is the value of this function at $x = 0$, $x = 2$, $x = 3$, $x = 6$, $x = 11$, and $x = 18$?

What is the domain of the function?

$$f(\boldsymbol{x}) = \sqrt{\boldsymbol{x} - 2}$$

$$f(\mathbf{0}) = \sqrt{\mathbf{0} - 2} = \sqrt{-2} \text{ not a real number} \qquad x = 0 \text{ is not in domain, no ordered pair}$$

$$f(\mathbf{2}) = \sqrt{\mathbf{2} - 2} = \sqrt{0} = 0 \quad \Rightarrow \quad \text{ordered pair is } (2, 0)$$

$$f(\mathbf{3}) = \sqrt{\mathbf{3} - 2} = \sqrt{1} = 1 \quad \Rightarrow \quad \text{ordered pair is } (3, 1)$$

$$f(\mathbf{6}) = \sqrt{\mathbf{6} - 2} = \sqrt{4} = 2 \quad \Rightarrow \quad \text{ordered pair is } (6, 2)$$

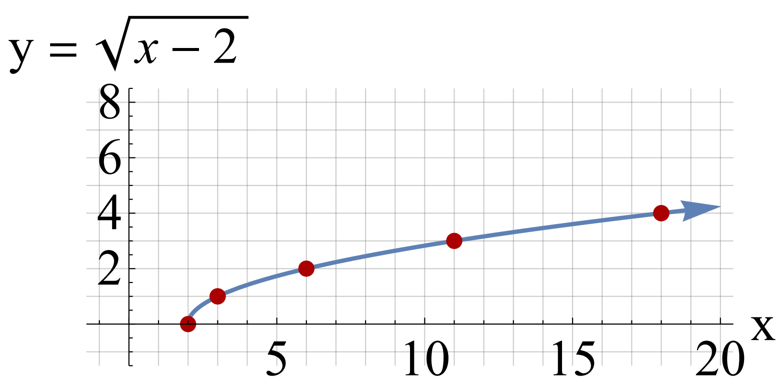
$$f(\mathbf{11}) = \sqrt{\mathbf{11} - 2} = \sqrt{9} = 3 \quad \Rightarrow \quad \text{ordered pair is } (11, 3)$$

$$f(\mathbf{18}) = \sqrt{\mathbf{18} - 2} = \sqrt{16} = 4 \quad \Rightarrow \quad \text{ordered pair is } (18, 4)$$

For the domain of the function, we note that square root function is real valued only if the quantity you are taking the square root of is nonnegative (sometimes shortened incorrectly to "you can't take the square root of a negative"), so we must have

$$x - 2 \geq 0$$
$$x \geq 2$$

which means the domain is $x \geq 2$. We will get a real number out of our function if we put real numbers x into the function that satisfy $x \geq 2$. Here is a plot of the function $f(x) = \sqrt{x-2}$ and the points we found.



EXAMPLE What is the domain of the function $f(x) = \sqrt{2 - 3x}$?

$$2 - 3x \geq 0 \qquad \text{(quantity under square root must be positive or zero)}$$

$$\cancel{2} - 3x \mathbf{-\cancel{2}} \geq 0 \mathbf{- 2} \qquad \text{(addition principle)}$$

$$\frac{\cancel{-3}x}{\cancel{\mathbf{-3}}} \leq \frac{-2}{\mathbf{-3}} \qquad \text{(division principle, change direction since dividing by negative)}$$

$$x \leq \frac{2}{3}$$

EXAMPLE What is the domain of the function $f(x) = \sqrt{45x - 7}$?

$$\begin{aligned} 45x - 7 &\geq 0 && \text{(quantity under square root must be positive or zero)} \\ 45x - \cancel{7} + \cancel{\mathbf{7}} &\geq 0 + \mathbf{7} && \text{(addition principle)} \\ \frac{\cancel{45}x}{\cancel{\mathbf{45}}} &\geq \frac{7}{\mathbf{45}} && \text{(division principle)} \\ x &\geq \frac{7}{45} \end{aligned}$$

EXAMPLE What is the domain of the function $f(x) = \sqrt{12 - \frac{1}{3}x}$?

$$\begin{aligned} 12 - \frac{1}{3}x &\geq 0 && \text{(quantity under square root must be positive or zero)} \\ \cancel{12} - \frac{1}{3}x - \cancel{\mathbf{12}} &\geq 0 - \mathbf{12} && \text{(addition principle)} \\ \cancel{-\frac{1}{3}}x \times \cancel{(\mathbf{-3})} &\leq (-12) \times (\mathbf{-3}) && \text{(division principle, change direction since dividing by negative)} \\ x &\leq 36 \end{aligned}$$

EXAMPLE What is the domain of the function $f(x) = \sqrt{-3x}$?

$$\begin{aligned} -3x &\geq 0 && \text{(quantity under square root must be positive or zero)} \\ \frac{\cancel{-3}x}{\cancel{\mathbf{-3}}} &\leq \frac{0}{\mathbf{-3}} && \text{(division principle, change direction since dividing by negative)} \\ x &\leq 0 \end{aligned}$$

EXAMPLE What is the domain of the function $f(x) = \sqrt{34x + 57}$?

$$\begin{aligned} 34x + 57 &\geq 0 && \text{(quantity under square root must be positive or zero)} \\ 34x + \cancel{57} - \cancel{\mathbf{57}} &\geq 0 - \mathbf{57} && \text{(addition principle)} \\ \frac{\cancel{34}x}{\cancel{\mathbf{34}}} &\geq \frac{-57}{\mathbf{34}} && \text{(division principle)} \\ x &\geq \frac{57}{34} \end{aligned}$$

EXAMPLE What is the domain of the function $f(x) = \sqrt{-3x - 7}$?

$$\begin{aligned} -3x - 7 &\geq 0 && \text{(quantity under square root must be positive or zero)} \\ -3x - \cancel{7} + \cancel{\mathbf{7}} &\geq 0 + \mathbf{7} && \text{(addition principle)} \\ \frac{\cancel{-3}x}{\cancel{\mathbf{-3}}} &\geq \frac{7}{\mathbf{-3}} && \text{(division principle, change direction since dividing by negative)} \\ x &\geq -\frac{7}{3} \end{aligned}$$

3 For Your Information: Applications involving the square root

Electric Current. We can approximate the amount of current in amps I drawn by an appliance in the home using the formula

$$I = \sqrt{\frac{P}{R}}$$

where P is the power measured in watts and R is the resistance measured in ohms.

Period of a Pendulum. The period of the pendulum is the amount of time T it takes to complete one full swing back and forth. If T is measured in seconds, and the length of the pendulum is L in feet, then the period is given by the formula

$$T = 2\pi\sqrt{\frac{L}{32}}.$$

A big part of precalculus is studying the behavior of functions. You will see a lot more about functions there (polynomial functions, rational functions, squaring functions, cubing functions, exponential functions, logarithmic functions, etc.).

Radicals and Factoring. You might have noticed that there is a relationship between radicals and factoring that works in some cases.

For example, what if we wanted to figure out how to simplify $y = \left(\frac{16}{81}\right)^{1/4}$? Here's something we could do.

$$\begin{aligned}
y &= \left(\frac{16}{81}\right)^{1/4} \\
y^4 &= \left[\left(\frac{16}{81}\right)^{1/4}\right]^4 && \text{(fourth power of both sides to remove fractional exponent)} \\
y^4 &= \frac{16}{81} && \text{(simplify)} \\
81y^4 &= 16 \\
81y^4 - 16 &= 0 && \text{(identify as a difference of squares)} \\
\left(9y^2\right)^2 - (4)^2 &= 0 \\
\left(9y^2 + 4\right)\left(9y^2 - 4\right) &= 0 && \text{(first factor is prime; second is a difference of squares)} \\
\left(9y^2 + 4\right)\left((3y)^2 - 2^2\right) &= 0 \\
\left(9y^2 + 4\right)(3y - 2)(3y + 2) &= 0
\end{aligned}$$

Now the zero factor property tells us either $9y^2 + 4 = 0$ (which is not possible for y a real number), or $3y - 2 = 0$ ($y = 2/3$), or $3y + 2 = 0$ ($y = -2/3$). We choose the positive solution since we want the principal root. So $y = 2/3$.

All the above is mathematically correct, but it is obviously easier to just do the following:

$$\left(\frac{16}{81}\right)^{1/4} = \left(\left(\frac{2}{3}\right)^4\right)^{1/4} = \frac{2}{3}.$$