Questions

1. Solve $12 + \sqrt{4x+5} = 7$.
2. Solve $y - \sqrt{y-3} = 5$.
3. Solve $\sqrt{2y-4} + 2 = y$.
4. Solve $\sqrt[3]{3-5x} = 2$.
5. Solve $\sqrt{8x+17} = \sqrt{2x+8} + 3$.
6. Solve $\sqrt{2x+9} - \sqrt{x+1} = 2$.
7. In geology, the water depth d near a mid-ocean spreading ridge depends on the square root of the distance x from the ridge axis according to the relation $d = d_0 + a\sqrt{x}$, where d_0 is the depth of the ridge axis and a is some constant. Solve the equation $d = d_0 + a\sqrt{x}$ for x.
8. Solve Graham's law of effusion (used in molecular chemistry) $\frac{\rho_1}{\rho_2} = \sqrt{\frac{M_2}{M_1}}$ for M_2, then solve for M_1. Describe how M_1 and M_2 vary with respect to the other variables.
9. Simplify $\sqrt{-169}$.
10. Simplify $\sqrt{-50}$.
11. Simplify $\sqrt{-48}$.
12. Simplify $-\frac{3}{2} + \sqrt{-81}$.
13. Simplify $\sqrt{-25}\sqrt{-9}$.
14. Simplify $\left(\frac{3}{4} - \frac{3}{4}i\right) + \left(\frac{9}{4} + \frac{5}{4}i\right)$.
15. Simplify $\left(\frac{1}{2} + i\right)^2$.
16. Simplify $(\sqrt{2}i)(\sqrt{6}i)$.
17. Simplify $\frac{2+i}{3-i}$.
18. Simplify $\frac{4+2i}{2-i}$.
19. If y varies directly with x, and $y = 15$ when $x = 40$, find y when $x = 64$.
20. The distance a spring stretches varies directly with the weight of the object hung from the spring (this is Hooke's Law). If a 10 lb weight stretches the spring 6 inches, how far will a 35 lb weight stretch the spring?
21. When an object is dropped, the distance it falls in feet varies directly with the square of the duration of the fall in seconds. An apple that falls from a tree falls 1 ft in $\frac{1}{4}$ second. How far will it fall in 1 second? How far will it fall in 2 seconds?
22. The weight that can be safely supported by a 2 by 6 inch beam varies inversely with its length. An engineer finds that a beam 8 ft long will support 900 lbs. Find the weight that can be safely supported by a beam that is 18 ft long.
23. The strength of a rectangular beam varies jointly with its width and the square of its thickness. If a beam 5 inches wide and 2 inches thick supports 400 lbs, how much can a beam of the same material that is 4 inches wide and 3.5 inches thick support?
24. The *kinetic energy* of an object is the energy the object has due to its motion, and is directly proportional to the mass and directly proportional to the square of velocity. If an object of mass 10kg moving at a velocity of 8 m/s has kinetic energy 320 kg m^2/s^2 = 320 Newtons = 320 N, determine the formula for kinetic energy. Pay attention to what is happening with the units.
25. Reduce $|x-6| = 16$.
26. Reduce $|2x-5| = 13$.
27. Reduce $\left|\frac{1}{2} - \frac{3}{4}x\right| + 1 = 3$.
28. Reduce $\left|4 - \frac{5}{2}x\right| = 12$.
29. Reduce $|x+6| = |2x-3|$.
30. Reduce $|1.5x-2| = |x-0.5|$.

31. Reduce $\left|\frac{2}{5}x + 1\right| = |1 - x|$.

32. Reduce $|x| \leq 8$.

33. Reduce $|x| < 6$.

34. Reduce $|2x - 5| \leq 7$.

35. Reduce $\left|\frac{3}{5}(1 - 7x)\right| < 6$.

36. Reduce $|2 - 9x| > 20$.

Solutions

1.

$$\begin{aligned}
\cancel{12} + \sqrt{4x+5} \mathbf{- \cancel{12}} &= 7 \mathbf{- 12} && \text{(addition principle to isolate square root)}\\
(\sqrt{4x+5})^{\mathbf{2}} &= (-5)^{\mathbf{2}} && \text{(square both sides of the equation)}\\
4x + 5 &= 25\\
4x &= 25 - 5\\
4x &= 20\\
x &= 5
\end{aligned}$$

Check for Extraneous Solutions:

$$x = 5: \qquad 12 + \sqrt{4(5)+5} = 12 + \sqrt{25} = 12 + 5 = 17 \neq 7$$

So $x = 5$ is extraneous, and the original equation has no solution.

2.

$$\begin{aligned}
y - \cancel{\sqrt{y-3}} \mathbf{- 5 + \cancel{\sqrt{y-3}}} &= \cancel{5} \mathbf{- \cancel{5} + \sqrt{y-3}} && \text{(addition principle to isolate square root)}\\
(y-5)^{\mathbf{2}} &= (\sqrt{y-3})^{\mathbf{2}} && \text{(square both sides of the equation)}\\
y^2 - 10y + 25 &= y - 3 && \text{(collect like terms)}\\
y^2 - 11y + 28 &= 0 && \text{(factor: sum is } -11 \text{ product is 28: } -7, -4)\\
(y-7)(y-4) &= 0\\
y - 7 = 0 \text{ or } y - 4 &= 0 && \text{(zero factor property)}\\
y = 7 \text{ or } y &= 4
\end{aligned}$$

Check for Extraneous Solutions:

$$y = 7: \qquad (7) - \sqrt{(7)-3} = 7 - \sqrt{4} = 7 - 2 = 5$$
$$y = 4: \qquad (4) - \sqrt{(4)-3} = 4 - \sqrt{1} = 3 \neq 5$$

So $y = 7$ is the only solution to the original equation.

3.

$$\sqrt{2y-4}+\cancel{2}\mathbf{-\cancel{2}} = y \mathbf{- 2} \quad \text{(addition principle to isolate square root)}$$
$$(\sqrt{2y-4})^{\mathbf{2}} = (y-2)^{\mathbf{2}} \quad \text{(square both sides of the equation)}$$
$$2y-4 = (y-2)^2$$
$$2(y-2) = (y-2)^2 \quad \text{(common factor)}$$
$$2\boldsymbol{(y-2)} - \boldsymbol{(y-2)}^2 = 0$$
$$\boldsymbol{(y-2)}(2-(y-2)) = 0$$
$$(y-2)(2-y+2) = 0$$
$$(y-2)(4-y) = 0$$
$$y-2=0 \text{ or } 4-y=0 \quad \text{(zero factor property)}$$
$$y=2 \text{ or } y=4$$

Check for Extraneous Solutions:

$$y=2: \qquad \sqrt{2(2)-4}+2=2$$
$$y=4: \qquad \sqrt{2(4)-4}+2=\sqrt{4}+2=4$$

So both $y=2$ and $y=4$ are solutions.

4. Since we have a cube root, we cube both sides of the equation here.

$$(\sqrt[3]{3-5x})^{\mathbf{3}} = (2)^{\mathbf{3}}$$
$$3-5x=8$$
$$-5x=5 \quad \Rightarrow \quad x=-1$$

Check for Extraneous Solutions:

$$x=-1: \qquad \sqrt[3]{3-5(-1)}=\sqrt[3]{8}=2$$

So $x=-1$ is a solution.

5.

$$(\sqrt{8x+17})^{\mathbf{2}} = (\sqrt{2x+8}+3)^{\mathbf{2}} \quad \text{(square both sides of the equation)}$$
$$8x+17 = 2x+8+9+6\sqrt{2x+8} \quad \text{(collect like terms)}$$
$$\frac{\cancel{6}x}{\cancel{6}} = \frac{\cancel{6}\sqrt{2x+8}}{\cancel{6}} \quad \text{(division principle)}$$
$$x = \sqrt{2x+8}$$
$$(x)^{\mathbf{2}} = (\sqrt{2x+8})^{\mathbf{2}} \quad \text{(square both sides of the equation)}$$
$$x^2 = 2x+8$$
$$x^2-2x-8=0 \quad \text{(factor: numbers sum } -2 \text{ product } -8\text{: } -4, 2)$$
$$(x-4)(x+2)=0$$
$$x-4=0 \text{ or } x+2=0 \quad \text{(zero factor property)}$$
$$x=4 \text{ or } x=-2$$

Check for Extraneous Solutions:

$$x = 4: \quad \sqrt{8(4)+17} = \sqrt{2(4)+8}+3 \Rightarrow \sqrt{49} = \sqrt{16}+3 \Rightarrow 7 = 7 \text{ True}$$
$$x = -2: \quad \sqrt{8(-2)+17} = \sqrt{2(-2)+8}+3 \Rightarrow \sqrt{1} = \sqrt{4}+3 \Rightarrow 1 = 5 \text{ False}$$

So $x = 4$ is the only solution.

6.

$$\sqrt{2x+9} - \cancel{\sqrt{x+1}} + \cancel{\sqrt{x+1}} = 2 + \sqrt{x+1} \quad \text{(addition principle to isolate square root)}$$
$$(\sqrt{2x+9})^2 = (2+\sqrt{x+1})^2 \quad \text{(square both sides of the equation)}$$
$$2x+9 = 4+4\sqrt{x+1}+(x+1)$$
$$2x+9-5-x = 5+x+4\sqrt{x+1}-\cancel{5}-\cancel{x} \quad \text{(collect like terms, addition principle)}$$
$$x+4 = 4\sqrt{x+1}$$
$$(x+4)^2 = (4\sqrt{x+1})^2 \quad \text{(square both sides of the equation)}$$
$$x^2+8x+16 = 16(x+1)$$
$$x^2+8x+16 = 16x+16 \quad \text{(collect like terms)}$$
$$x^2-8x = 0 \quad \text{(common factor)}$$
$$x(x-8) = 0$$
$$x = 0 \text{ or } x-8 = 0 \quad \text{(zero factor property)}$$
$$x = 0 \text{ or } x = 8$$

Check for Extraneous Solutions:

$$x = 0: \quad \sqrt{2(0)+9} - \sqrt{(0)+1} = 3-1 = 2 \text{ True}$$
$$x = 8: \quad \sqrt{2(8)+9} - \sqrt{(8)+1} = 5-3 = 2 \text{ True}$$

So both $x = 0$ and $x = 8$ are solutions.

7.

$$d - d_0 = \cancel{d_0} + a\sqrt{x} - \cancel{d_0} \quad \text{(addition principle to isolate square root)}$$
$$\frac{d-d_0}{a} = \frac{\cancel{a}\sqrt{x}}{\cancel{a}} \quad \text{(division principle)}$$
$$\frac{d-d_0}{a} = \sqrt{x}$$
$$\left(\frac{d-d_0}{a}\right)^2 = (\sqrt{x})^2 \quad \text{(square both sides of the equation)}$$
$$\left(\frac{d-d_0}{a}\right)^2 = x$$

8.

$$\left(\frac{\rho_1}{\rho_2}\right)^{\mathbf{2}} = \left(\sqrt{\frac{M_2}{M_1}}\right)^{\mathbf{2}} \quad \text{(square both sides of the equation)}$$

$$\left(\frac{\rho_1}{\rho_2}\right)^2 \times \boldsymbol{M_1} = \frac{M_2}{\cancel{M_1}} \times \cancel{\boldsymbol{M_1}} \quad \text{(multiplication principle)}$$

$$\left(\frac{\rho_1}{\rho_2}\right)^2 M_1 = M_2$$

$$M_2 = \frac{M_1\rho_1^2}{\rho_2^2} \quad \text{(solve for } M_2\text{)}$$

$$M_2 \times \frac{\boldsymbol{\rho_2^2}}{\boldsymbol{\rho_1^2}} = \frac{M_1\cancel{\rho_1^2}}{\cancel{\rho_2^2}} \times \frac{\cancel{\boldsymbol{\rho_2^2}}}{\cancel{\boldsymbol{\rho_1^2}}} \quad \text{(multiplication and division principles)}$$

$$\frac{M_2\rho_2^2}{\rho_1^2} = M_1$$

$$M_1 = \frac{M_2\rho_2^2}{\rho_1^2} \quad \text{(solve for } M_1\text{)}$$

Using $M_2 = \frac{M_1\rho_1^2}{\rho_2^2}$, M_2 jointly varies

- directly with respect to M_1
- directly with respect to the square of ρ_1
- inversely with respect to the square of ρ_2

Using $M_1 = \frac{M_2\rho_2^2}{\rho_1^2}$, M_1 jointly varies

- directly with respect to M_2
- directly with respect to the square of ρ_2
- inversely with respect to the square of ρ_1

9. $\sqrt{-169} = \sqrt{-1 \cdot 169} = \sqrt{-1}\sqrt{169} = i \cdot 13 = 13i.$

10. $\sqrt{-50} = \sqrt{-1 \cdot 5^2 \cdot 2} = \sqrt{-1}\sqrt{5^2}\sqrt{2} = 5\sqrt{2}i.$

11. $\sqrt{-48} = \sqrt{-1 \cdot 4^2 \cdot 3} = \sqrt{-1}\sqrt{4^2}\sqrt{3} = 4\sqrt{3}i.$

12. $-\frac{3}{2} + \sqrt{-81} = -\frac{3}{2} + \sqrt{-1 \cdot 9^2} = -\frac{3}{2} + \sqrt{-1}\sqrt{9^2} = -\frac{3}{2} + 9i.$

13. $\sqrt{-25}\sqrt{-9} = (5i)(3i) = 15i^2 = 15(-1) = -15.$

14. $\left(\frac{3}{4} - \frac{3}{4}i\right) + \left(\frac{9}{4} + \frac{5}{4}i\right) = \left(\frac{3}{4} + \frac{9}{4}\right) + \left(-\frac{3}{4} + \frac{5}{4}\right)i = \left(\frac{3+9}{4}\right) + \left(\frac{-3+5}{4}\right)i = (3) + \left(\frac{1}{2}\right)i = 3 + \frac{i}{2}.$

15. $\left(\frac{1}{2} + i\right)^2 = \left(\frac{1}{2}\right)^2 + i^2 + 2\frac{1}{2}i = \frac{1}{4} - 1 + i = -\frac{3}{4} + i.$

16. $(\sqrt{2}i)(\sqrt{6}i) = \sqrt{2 \cdot 6}i^2 = \sqrt{2^2 \cdot 3}(-1) = \sqrt{2^2}\sqrt{3}(-1) = -2\sqrt{3}$.

17. Use the complex conjugate of denominator to divide two complex numbers.

$$\begin{aligned}\frac{2+i}{3-i} &= \frac{(2+i)\boldsymbol{(3+i)}}{(3-i)\boldsymbol{(3+i)}} \\ &= \frac{6+5i+\boldsymbol{i^2}}{9-\boldsymbol{i^2}} && \text{(distribute)} \\ &= \frac{6+5i+\boldsymbol{(-1)}}{9-\boldsymbol{(-1)}} && \text{(replace } i^2 \text{ with } -1\text{)} \\ &= \frac{5+5i}{10} \\ &= \frac{1+i}{2}\end{aligned}$$

18.

$$\begin{aligned}\frac{4+2i}{2-i} &= \frac{(4+2i)\boldsymbol{(2+i)}}{(2-i)\boldsymbol{(2+i)}} \\ &= \frac{8+4i+4i+2\boldsymbol{i^2}}{4-\boldsymbol{i^2}} && \text{(distribute)} \\ &= \frac{8+8i+2\boldsymbol{(-1)}}{4-\boldsymbol{(-1)}} && \text{(replace } i^2 \text{ with } -1\text{)} \\ &= \frac{6+8i}{5}\end{aligned}$$

19. Since y varies directly with x: $y = kx$.
Use given information to determine k: $15 = k(40) \Rightarrow k = \frac{3}{8}$.
The relationship between x and y now that we know k: $y = \frac{3}{8}x$.
Find y when $x = 64$: $y = \frac{3}{8}(64) = 24$.

20. Let d be the distance stretched, and w be the weight hung from the spring: $d = kw$.
Use given information to determine k: $6 \text{ inches} = k(10 \text{ lbs}) \Rightarrow k = \frac{3}{5}\frac{\text{inches}}{\text{lb}}$.
The relationship is given by: $d = \frac{3}{5}w$.
Find d when $w = 35$ lbs: $d = \frac{3}{5}(35) = 21$ inches.

21. Let d be the distance fallen in ft, and t be the duration of the fall in seconds: $d = kt^2$.
Use given information to determine k: $1 \text{ ft} = k(\frac{1}{4} \text{ sec})^2 \Rightarrow k = 16\frac{\text{ft}}{\text{sec}^2}$.
The relationship is given by: $d = 16t^2$.
Find d when $t = 1$ s: $d = 16(1)^2 = 16$ ft.
Find d when $t = 2$ s: $d = 16(2)^2 = 64$ ft.

22. Let w be the weight in lbs safely supported, and l be the length in ft of the beam: $w = \frac{k}{l}$.
Use given information to determine k: $900 \text{ lbs} = \frac{k}{8 \text{ ft}} \Rightarrow k = 7200 \text{ lbs} \cdot \text{ft}$.
The relationship is given by: $w = \frac{7200}{l}$.
Find w when $l = 18$ ft: $w = \frac{7200}{18} = 400$ lbs.

23. Let s be the weight in lbs safely supported, w be the width in inches of the beam, and t the thickness in inches of the beam: $s = kwt^2$.

    Use given information to determine k: 400 lbs $= k$(5 inches)(2 inches)$^2 \Rightarrow k = 20\frac{\text{lbs}}{\text{inches}^3}$.

    The relationship is given by: $s = 20wt^2$.

    Find s when $w = 4$ inches and $t = 3.5$ inches: $w = 20(4)(3.5)^2 = 980$ lbs.

    Notice that if we simply tip this beam over so the width is $w = 3.5$ inches and the thickness is $t = 4$ inches it becomes stronger: $w = 20(3.5)(4)^2 = 1120$ lbs.

    It's obviously important in construction to understand in which direction the strength is required, and place your beam accordingly.

24. Let T be the kinetic energy in Newtons (N), m be the mass in kg, and v the velocity in m/s: $T = kmv^2$.

    Use given information to determine k: 320 N $= k$(10 kg)(8 m/s)$^2 \Rightarrow k = \frac{1}{2}\frac{\text{N}}{kg\cdot(\text{m/s})^2} = \frac{1}{2}$.

    Note: k has no units! It is dimensionless.

    The formula for kinetic energy is given by: $T = \frac{1}{2}mv^2$.

25. $|x - 6| = 16 \quad \Rightarrow$

$$\begin{aligned} x - 6 &= 16 &\quad \text{or} \quad& x - 6 = -16 \\ x &= 22 &\quad \text{or} \quad& x = -10 \end{aligned}$$

26. $|2x - 5| = 13 \quad \Rightarrow$

$$\begin{aligned} 2x - 5 &= 13 &\quad \text{or} \quad& 2x - 5 = -13 \\ 2x &= 18 &\quad \text{or} \quad& 2x = -8 \\ x &= 9 &\quad \text{or} \quad& x = -4 \end{aligned}$$

27. $\left|\frac{1}{2} - \frac{3}{4}x\right| + 1 = 3 \quad \Rightarrow \quad \left|\frac{1}{2} - \frac{3}{4}x\right| = 2 \quad \Rightarrow$

$$\begin{aligned} \frac{1}{2} - \frac{3}{4}x &= 2 &\quad \text{or} \quad& \frac{1}{2} - \frac{3}{4}x = -2 \\ 2 - 3x &= 8 &\quad \text{or} \quad& 2 - 3x = -8 \\ -3x &= 6 &\quad \text{or} \quad& -3x = -10 \\ x &= -2 &\quad \text{or} \quad& x = \frac{10}{3} \end{aligned}$$

28. $\left|4 - \frac{5}{2}x\right| = 12 \quad \Rightarrow$

$$\begin{aligned} 4 - \frac{5}{2}x &= 12 &\quad \text{or} \quad& 4 - \frac{5}{2}x = -12 \\ -\frac{5}{2}x &= 8 &\quad \text{or} \quad& -\frac{5}{2}x = -16 \\ x &= -\frac{16}{5} &\quad \text{or} \quad& x = \frac{32}{5} \end{aligned}$$

29. $|x+6| = |2x-3| \quad \Rightarrow$

$$\begin{aligned} x+6 &= 2x-3 && \text{or} && x+6 = -(2x-3) \\ -x &= -9 && \text{or} && x+6 = -2x+3 \\ x &= 9 && \text{or} && 3x = -3 \\ x &= 9 && \text{or} && x = -1 \end{aligned}$$

30. $|1.5x-2| = |x-0.5| \quad \Rightarrow$

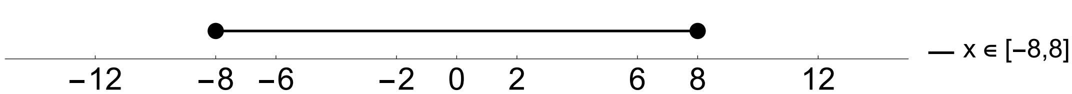
$$\begin{aligned} 1.5x-2 &= x-0.5 && \text{or} && 1.5x-2 = -(x-0.5) \\ 0.5x &= 1.5 && \text{or} && 1.5x-2 = -x+0.5 \\ x &= 3 && \text{or} && 2.5x = 2.5 \\ x &= 3 && \text{or} && x = 1 \end{aligned}$$

31. $\left|\frac{2}{5}x+1\right| = |1-x| \quad \Rightarrow$

$$\begin{aligned} \frac{2}{5}x+1 &= 1-x && \text{or} && \frac{2}{5}x+1 = -(1-x) \\ \frac{2}{5}x &= -x && \text{or} && \frac{2}{5}x+1 = -1+x \\ x &= 0 && \text{or} && -\frac{3}{5}x = -2 \\ x &= 0 && \text{or} && x = \frac{10}{3} \end{aligned}$$

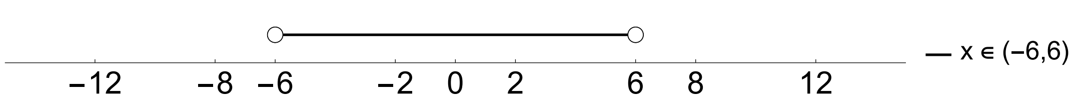
32. Reduce $|x| \le 8$.

Interval notation: $-8 \le x \le 8$
Set notation: $x \in [-8, 8]$



33. Reduce $|x| < 6$.

Interval notation: $-6 < x < 6$
Set notation: $x \in (-6, 6)$



34. Reduce $|2x-5| \le 7$.

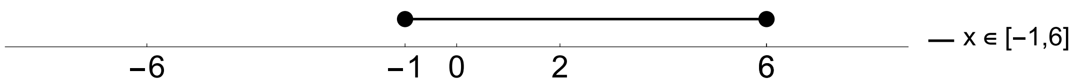
$$-7+5 \le 2x-\cancel{5}+\cancel{5} \le 7+5 \qquad \text{(addition principle))}$$

$$\frac{-2}{2} \le \frac{\cancel{2}x}{\cancel{2}} \le \frac{12}{2} \qquad \text{(division principle))}$$

$$-1 \le x \le 6$$

Interval notation: $-1 \le x \le 6$

Set notation: $x \in [-1, 6]$



35. Reduce $\left|\frac{3}{5}(1-7x)\right| < 6$. In this problem we have to remember to change direction of inequality when multiplying by negative!

$$-6 \times \frac{5}{3} < \frac{\cancel{3}}{\cancel{5}}(1-7x) \times \frac{\cancel{5}}{\cancel{3}} < 6 \times \frac{5}{3} \qquad \text{(multiplication principle))}$$

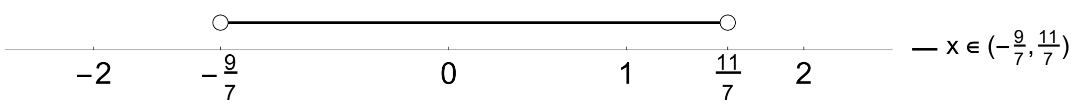
$$-10-1 < \cancel{1}-7x-\cancel{1} < 10-1 \qquad \text{(addition principle)}$$

$$\frac{-11}{-7} > \frac{\cancel{-7}x}{\cancel{-7}} > \frac{9}{-7} \qquad \text{(divide by negative, change direction of inequality)}$$

$$\frac{11}{7} > x > -\frac{9}{7}$$

Interval notation: $-\frac{9}{7} < x < \frac{11}{7}$

Set notation: $x \in \left(-\frac{9}{7}, \frac{11}{7}\right)$



36. Reduce $|2-9x| > 20$. In this problem we have to remember to change direction of inequality when multiplying by negative!

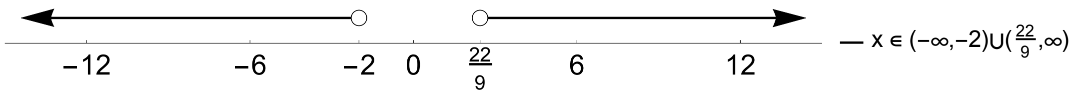
$$2-9x < -20 \quad \text{or} \quad 2-9x > 20$$

$$-9x < -22 \quad \text{or} \quad -9x > 18$$

$$x > \frac{22}{9} \quad \text{or} \quad x < -2$$

Interval notation: $x > \frac{22}{9}$ or $x < -2$

Set notation: $x \in (-\infty, -2) \cup \left(\frac{22}{9}, \infty\right)$


−12
−6
−2
0
22/9
6
12
x ∈ (−∞,−2)∪(22/9,∞)