Note: all the previous quadratics we have solved we were able to solve by *factoring*. If you can factor the quadratic, you can still solve it using those techniques. *Completing the square* is the technique you can use when you cannot factor the quadratic.

Questions

Include complex solutions in your answers.

1. Solve $(x+9)^2 = 21$.
2. Solve $(4x-3)^2 = 36$.
3. Solve $(5x-2)^2 - 25 = 0$.
4. Solve by completing the square $x^2 + 6x + 2 = 0$.
5. Solve by completing the square $x^2 - 14x = -48$.
6. Solve by completing the square $\frac{x^2}{3} - \frac{x}{3} = 3$.
7. Solve by completing the square $2y^2 - y = 15$.
8. Solve $x^2 - 2x = -7$.
9. Solve $3x^2 + 8x + 3 = 2$.
10. Write down the quadratic formula that solves $ax^2 + bx + c = 0$.
11. Solve $x^2 = \frac{2}{3}x$.
12. Solve $7x^2 + 4x - 3 = 0$.
13. Solve $4x^2 + 3x - 2 = 0$.
14. Solve $\frac{1}{4} + \frac{6}{y+2} = \frac{6}{y}$.
15. Solve $2x^2 + 15 = 0$.
16. Solve $5x^2 = -3$.
17. Use the discriminant to find what type of solution $9x^2 + 4 = 12x$ has.
18. Write a quadratic equation which has the solutions $1-4i$ and $1+4i$.
19. Solve $x^4 - 11x^2 + 18 = 0$.
20. Solve $3x^4 = 10x^2 + 8$.
21. Solve $x^6 - 3x^3 = 0$.
22. Solve $x^8 - 6x^4 = 0$.
23. Solve $x^4 - 81 = 0$.
24. Solve $x^{2/5} + x^{1/5} - 2 = 0$.
25. Solve $x^{-2} + 3x^{-1} = 0$.
26. Solve $S = 4\pi r^2$ for r.
27. Solve $A = \frac{1}{2}r^2\theta$ for r.
28. Solve $A = P(1+r)^2$ for r.
29. Find the length of the sides in the following right angle triangles:

30. The brace for a bookshelf has the shape of a right triangle. It's hypotenuse is 10 inches long and the two legs are equal in length. How long are the legs of the triangle?
31. Knox college is creating a new rectangular parking lot. The length is 0.07 miles longer than the width and the area of the parking lot is 0.026 square miles. Find the length and width of the parking lot.

32. The formulas $A = P(1+r)^2$ gives the amount A in dollars that will be obtained in 2 years if P dollars are invested at an annual compound interest rate of r. If you invest $P = \$1400$ and it grows to \$1514.24 in 2 years, what is the annual interest rate r?

33. Sketch $f(x) = 2x^2 + 2x - 4$.

34. Sketch $w(x) = x^2 - 6x + 8$.

35. Sketch $g(x) = x^2 - 2x - 8$.

36. Sketch $r(x) = -3x^2 + 6x - 4$.

37. Given the sketch below for

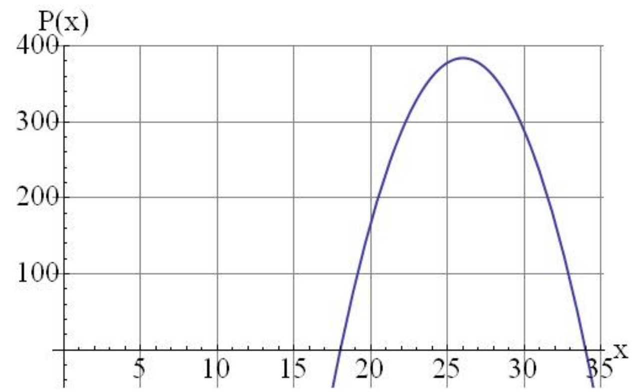
$$P(x) = -6x^2 + 312x - 3672,$$

which represents the profit P in dollars where x is the number of widgets manufactured by a company each day.

What is the maximum profit? How many widgets are produced each day to get the maximum profit?

How any widgets are made each day if the company has a daily profit of zero dollars?

How many widgets are made per day if the company has a profit of \$288 per day?



Solutions

1. Use the square root property, $w^2 = a \quad \Rightarrow \quad w = \pm\sqrt{a}$.

$$\begin{aligned}
(x+9)^2 &= 21 && \\
x + 9 &= \pm\sqrt{21} && \text{(square root property)} \\
x + \cancel{9} \mathbf{- \cancel{9}} &= \mathbf{-9} \pm \sqrt{21} && \text{(addition principle)} \\
x &= -9 \pm \sqrt{21} &&
\end{aligned}$$

2. Use the square root property.

$$\begin{aligned}
(4x-3)^2 &= 36 && \\
4x - 3 &= \pm\sqrt{36} && \text{(square root property)} \\
4x - \cancel{3} \mathbf{+ \cancel{3}} &= \mathbf{+3} \pm \sqrt{36} && \text{(addition principle)} \\
\frac{\cancel{4}x}{\cancel{\mathbf{4}}} &= \frac{3 \pm 6}{\mathbf{4}} && \text{(division principle)} \\
x &= \frac{3 \pm 6}{4} = \frac{3+6}{4} \text{ or } \frac{3-6}{4} && \text{(simplify)} \\
x &= \frac{3+6}{4} \text{ or } \frac{3-6}{4} && \\
x &= \frac{9}{4} \text{ or } -\frac{3}{4} &&
\end{aligned}$$

3. Use the square root property.

$$\begin{aligned}
(5x-2)^2 - \cancel{25} \mathbf{+ \cancel{25}} &= 0 \mathbf{+ 25} && \text{(addition principle)} \\
(5x-2)^2 &= 25 && \\
5x - 2 &= \pm\sqrt{25} && \text{(square root property)} \\
5x - \cancel{2} \mathbf{+ \cancel{2}} &= \mathbf{2} \pm 5 && \text{(simplify square root, and addition principle)} \\
\frac{\cancel{5}x}{\cancel{\mathbf{5}}} &= \frac{2 \pm 5}{\mathbf{5}} && \text{(division principle)} \\
x &= \frac{2 \pm 5}{5} && \text{(simplify)} \\
x &= \frac{2+5}{5} \text{ or } \frac{2-5}{5} && \\
x &= \frac{7}{5} \text{ or } -\frac{3}{5} &&
\end{aligned}$$

4. Complete the square, Step 1 is done, we already have a factor of 1 in front of x^2.

$$x^2 + 6x + 2 = 0 \qquad \text{(Step 2: identify } \left(\frac{6}{2}\right)^2 = 9\text{)}$$

$$x^2 + 6x + \ + 9 - 9 + 2 = 0 \qquad \text{(Step 3: blue terms add zero)}$$

$$\mathbf{x^2 + 6x + 9} - 9 + 2 = 0 \qquad \text{(Step 4: red terms are perfect square)}$$

$$\mathbf{(x+3)^2} - 7 = 0$$

$$(x+3)^2 - \cancel{7} + \cancel{7} = 0 + 7 \qquad \text{(addition principle)}$$

$$(x+3)^2 = 7$$

$$x + 3 = \pm\sqrt{7} \qquad \text{(square root property)}$$

$$x + \cancel{3} - \cancel{3} = \ -3 \pm \sqrt{7} \qquad \text{(addition principle)}$$

$$x = -3 \pm \sqrt{7}$$

5. Complete the square, Step 1 is done, we already have a factor of 1 in front of x^2.

$$x^2 - 14x = -48 \qquad \text{(Step 2: identify } \left(\frac{14}{2}\right)^2 = 49\text{)}$$

$$x^2 - 14x + 49 - 49 = -48 \qquad \text{(Step 3: blue terms add zero)}$$

$$\mathbf{x^2 - 14x + 49} - 49 = -48 \qquad \text{(Step 4: red terms are perfect square)}$$

$$\mathbf{(x-7)^2} - 49 = -48$$

$$(x-7)^2 - \cancel{49} + \cancel{49} = -48 + 49 \qquad \text{(addition principle)}$$

$$(x-7)^2 = 1$$

$$x - 7 = \pm\sqrt{1} \qquad \text{(square root property)}$$

$$x - \cancel{7} + \cancel{7} = \ +7 \pm 1 \qquad \text{(addition principle)}$$

$$x = 7 \pm 1$$

$$x = 8 \text{ or } 6$$

6.

$$\frac{x^2}{\cancel{3}} \times \cancel{3} - \frac{x}{\cancel{3}} \times \cancel{3} = 3 \times \mathbf{3} \qquad \text{(Step 1: multiplication principle to get 1 in front of } x^2\text{)}$$

$$x^2 - \mathbf{1}x = 9 \qquad \text{(Step 2: identify } \left(\frac{1}{2}\right)^2 = \frac{1}{4}\text{)}$$

$$x^2 - x + \frac{\mathbf{1}}{\mathbf{4}} - \frac{\mathbf{1}}{\mathbf{4}} = 9 \qquad \text{(Step 3: blue terms add zero)}$$

$$\boldsymbol{x^2 - x + \frac{1}{4}} - \frac{1}{4} = 9 \qquad \text{(Step 4: red terms are perfect square)}$$

$$\boldsymbol{\left(x - \frac{1}{2}\right)^2} - \frac{1}{4} = 9$$

$$\left(x - \frac{1}{2}\right)^2 - \cancel{\frac{1}{4}} + \cancel{\frac{1}{4}} = 9 + \frac{\mathbf{1}}{\mathbf{4}} \qquad \text{(addition principle)}$$

$$\left(x - \frac{1}{2}\right)^2 = \frac{37}{4}$$

$$x - \frac{1}{2} = \pm\sqrt{\frac{37}{4}} \qquad \text{(square root property)}$$

$$x - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} = + \frac{\mathbf{1}}{\mathbf{2}} \pm \frac{\sqrt{37}}{2} \qquad \text{(addition principle)}$$

$$x = \frac{1}{2} \pm \frac{\sqrt{37}}{2}$$

7.

$$\frac{2y^2 - y}{\mathbf{2}} = \frac{15}{\mathbf{2}}$$ (Step 1: division principle to get 1 in front of y^2)

$$y^2 - \frac{\mathbf{1}}{\mathbf{2}}y = \frac{15}{2}$$ (Step 2: identify $\left(\frac{1}{2}\cdot\frac{1}{2}\right)^2 = \frac{1}{16}$)

$$y^2 - \frac{1}{2}y + \frac{\mathbf{1}}{\mathbf{16}} - \frac{\mathbf{1}}{\mathbf{16}} = \frac{15}{2}$$ (Step 3: blue terms add zero)

$$\boldsymbol{y^2 - \frac{1}{2}y + \frac{1}{16}} - \frac{1}{16} = \frac{15}{2}$$ (Step 4: red terms are perfect square)

$$\boldsymbol{\left(y - \frac{1}{4}\right)^2} - \frac{1}{16} = \frac{15}{2}$$

$$\boldsymbol{\left(y - \frac{1}{4}\right)^2} - \cancel{\frac{1}{16}} + \cancel{\frac{\mathbf{1}}{\mathbf{16}}} = \frac{15}{2} + \frac{\mathbf{1}}{\mathbf{16}}$$ (addition principle)

$$\left(y - \frac{1}{4}\right)^2 = \frac{121}{16}$$

$$y - \frac{1}{4} = \pm\sqrt{\frac{121}{16}}$$ (square root property)

$$y - \frac{1}{4} = \pm\frac{11}{4}$$ (property of radicals)

$$y - \cancel{\frac{1}{4}} + \cancel{\frac{\mathbf{1}}{\mathbf{4}}} = + \frac{\mathbf{1}}{\mathbf{4}} \pm \frac{11}{4}$$ (addition property)

$$y = \frac{1}{4} + \frac{11}{4} \text{ or } \frac{1}{4} - \frac{11}{4}$$

$$y = 3 \text{ or } -\frac{5}{2}$$

8. Solve by completing the square. Step 1 is already completed.

$$x^2 - \mathbf{2}x = -7$$ (Step 2: identify $\left(\frac{2}{2}\right)^2 = 1$)

$$x^2 - 2x + \mathbf{1} - \mathbf{1} = -7$$ (Step 3: blue terms add zero)

$$\boldsymbol{x^2 - 2x + 1} - 1 = -7$$ (Step 4: red terms are perfect square)

$$\boldsymbol{(x-1)^2} - 1 = -7$$

$$(x-1)^2 - \cancel{1} + \cancel{1} = -7 + \mathbf{1}$$ (addition principle)

$$(x-1)^2 = -6$$

$$x - 1 = \pm\sqrt{-6}$$ (square root property)

$$x - 1 = \pm i\sqrt{6}$$ (property of complex numbers)

$$x - \cancel{1} + \cancel{1} = + \mathbf{1} \pm i\sqrt{6}$$ (addition principle)

$$x = 1 \pm i\sqrt{6}$$

9. Solve by completing the square.

$$\frac{3x^2+8x+3}{\mathbf{3}} = \frac{2}{\mathbf{3}} \qquad \text{(Step 1: division principle to get 1 in front of } x^2\text{)}$$

$$x^2 + \frac{\mathbf{8}}{\mathbf{3}}x + 1 = \frac{2}{3} \qquad \text{(Step 2: identify } \left(\frac{1}{2}\cdot\frac{8}{3}\right)^2 = \frac{16}{9}\text{)}$$

$$x^2 + \frac{8}{3}x + \frac{\mathbf{16}}{\mathbf{9}} - \frac{\mathbf{16}}{\mathbf{9}} + 1 = \frac{2}{3} \qquad \text{(Step 3: blue terms add zero)}$$

$$\boldsymbol{x^2 + \frac{8}{3}x + \frac{16}{9}} - \frac{16}{9} + 1 = \frac{2}{3} \qquad \text{(Step 4: red terms are perfect square)}$$

$$\boldsymbol{\left(x+\frac{4}{3}\right)^2} - \frac{7}{9} = \frac{2}{3}$$

$$\boldsymbol{\left(x+\frac{4}{3}\right)^2} - \cancel{\frac{7}{9}} + \cancel{\frac{\mathbf{7}}{\mathbf{9}}} = \frac{2}{3} + \frac{\mathbf{7}}{\mathbf{9}} \qquad \text{(addition principle)}$$

$$\left(x+\frac{4}{3}\right)^2 = \frac{13}{9}$$

$$x + \frac{4}{3} = \pm\sqrt{\frac{13}{9}} \qquad \text{(square root property)}$$

$$x + \frac{4}{3} = \pm\frac{\sqrt{13}}{3} \qquad \text{(property of radicals)}$$

$$x + \cancel{\frac{4}{3}} - \cancel{\frac{\mathbf{4}}{\mathbf{3}}} = -\frac{\mathbf{4}}{\mathbf{3}} \pm \frac{\sqrt{13}}{3} \qquad \text{(addition property)}$$

$$x = -\frac{4}{3} \pm \frac{\sqrt{13}}{3}$$

10. $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$.

11. Solve using the quadratic formula, where $a = 1$, $b = -2/3$, and $c = 0$:

$$x^2 - \frac{2}{3}x = 0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-2/3) \pm \sqrt{(-2/3)^2 - 4(1)(0)}}{2(1)}$$

$$x = \frac{2/3 \pm 2/3}{2}$$

$$x = 1/3 \pm 1/3$$

$$x = 1/3 + 1/3 \text{ or } 1/3 - 1/3$$

$$x = 2/3 \text{ or } 0$$

Since $c = 0$, this could have been solved by factoring:

$$x^2 - \frac{2}{3}x = 0$$

$$x\left(x - \frac{2}{3}\right) = 0$$

$$x = 0 \text{ or } x - \frac{2}{3} = 0$$

$$x = 0 \text{ or } x = \frac{2}{3}$$

12. Solve using quadratic formula, where $a = 7$, $b = 4$, and $c = -3$.

$$7x^2 + 4x - 3 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(7)(-3)}}{2(7)}$$
$$x = \frac{-4 \pm \sqrt{100}}{14}$$
$$x = \frac{-4 \pm 10}{14}$$
$$x = \frac{-4 + 10}{14} \text{ or } \frac{-4 - 10}{14}$$
$$x = \frac{3}{7} \text{ or } -1$$

This could also have been done using factoring by grouping.

13. Solve using quadratic formula, where $a = 4$, $b = 3$, and $c = -2$.

$$4x^2 + 3x - 2 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(4)(-2)}}{2(4)}$$
$$x = \frac{-3 \pm \sqrt{41}}{8}$$

Because the roots are complicated, the only way to do this one was using quadratic formula or completing the square.

14.

$$\frac{1}{\cancel{4}} \cdot \mathbf{\cancel{4}(y+2)(y)} + \frac{6}{\cancel{y+2}} \cdot \mathbf{4\cancel{(y+2)}(y)} = \frac{6}{\cancel{y}} \cdot \mathbf{4(y+2)\cancel{(y)}} \qquad \text{(multiply by LCD)}$$

$$(y+2)(y) + 6 \cdot 4y = 6 \cdot 4(y+2)$$

$$y^2 + 2y + 24y - 24y - 48 = 0 \qquad \text{(collect like terms)}$$

$$y^2 + 2y - 48 = 0 \qquad (a = 1,\ b = 2,\ \text{and}\ c = -48)$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-48)}}{2(1)}$$

$$y = \frac{-2 \pm \sqrt{196}}{2}$$

$$y = \frac{-2 \pm 14}{2}$$

$$y = -1 \pm 7$$

$$y = -1 + 7 \text{ or } -1 - 7$$

$$y = 6 \text{ or } -8$$

Recall than when you multiply by an LCD to solve an equation, you must check for extraneous solutions.

$$\text{Check } y = 6: \quad \frac{1}{4} + \frac{6}{(6)+2} = \frac{6}{(6)} \Rightarrow 1 = 1 \text{ True}$$

$$\text{Check } y = -8: \quad \frac{1}{4} + \frac{6}{(-8)+2} = \frac{6}{(-8)} \Rightarrow -\frac{3}{4} = -\frac{3}{4} \text{ True}$$

15. Solve using the square root property $w^2 = a \quad \Rightarrow \quad w = \pm\sqrt{a}$.

$$2x^2 + 15 = 0$$

$$2x^2 = -15$$

$$x^2 = -\frac{15}{2}$$

$$x = \pm\sqrt{-\frac{15}{2}}$$

$$x = \pm\sqrt{\frac{15}{2}}i$$

16. Solve using quadratic formula.

$$5x^2 + 3 = 0 \qquad (a = 5,\ b = 0,\ c = 3)$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(0) \pm \sqrt{(0)^2 - 4(5)(3)}}{2(5)}$$
$$x = \frac{\pm\sqrt{-60}}{10}$$
$$x = \pm\frac{\sqrt{60}}{10}i$$
$$x = \pm\frac{\sqrt{2^2 \cdot 15}}{10}i$$
$$x = \pm\frac{2\sqrt{15}}{10}i$$
$$x = \pm\frac{\sqrt{15}}{5}i$$

This could also have been solved using the square root property.

17. The discriminant is $b^2 - 4ac = (-12)^2 - 4(9)(4) = 0$. This means there will be one rational root.

18. If the quadratic has solution r, then it has a factor $(x - r)$.

Complex solutions will appear in complex conjugate pairs, so $1 + 4i$ and $1 - 4i$ are the solutions.

$$(x - (1 - 4i))\boldsymbol{(x - (1 + 4i))} = 0$$

Now, carefully multiply out (distribute) to get the quadratic. Use $i^2 = -1$ to simplify near the end.

$$(x)\boldsymbol{(x - (1 + 4i))} + (-(1 - 4i))\boldsymbol{(x - (1 + 4i))} = 0$$
$$(x)(x - 1 - 4i) + \boldsymbol{(-1 + 4i)}(x - 1 - 4i)) = 0$$
$$x^2 - x - 4xi + \boldsymbol{(-1 + 4i)}(x) - \boldsymbol{(-1 + 4i)}(1 + 4i) = 0$$
$$x^2 - x - 4xi - x + 4xi + (1 - 4i)(1 + 4i) = 0$$
$$x^2 - x - 4xi - x + 4xi + 1 + 4i - 4i - 16\boldsymbol{i^2} = 0$$
$$x^2 - 2x + 1 - 16\boldsymbol{(-1)} = 0$$
$$x^2 - 2x + 17 = 0$$

19. Let $u = x^2$. From this substitution, it follows that $x^2 = u$ and $x^4 = u^2$.

$$x^4 - 11x^2 + 18 = 0$$
$$u^2 - 11u + 18 = 0 \qquad \text{(factor: two numbers whose product is 18 and sum is } -11\text{: } -9, -2)$$
$$(u - 9)(u - 2) = 0$$
$$u - 9 = 0 \text{ or } u - 2 = 0$$
$$\boldsymbol{u} = 9 \text{ or } \boldsymbol{u} = 2 \qquad \text{(now we must back-substitute using } u = x^2 \text{ and solve for } x)$$
$$\boldsymbol{x^2} = 9 \text{ or } \boldsymbol{x^2} = 2$$
$$x = \pm 3 \text{ or } x = \pm\sqrt{2} \qquad \text{(four solutions: } x = +3, -3, +\sqrt{2}, -\sqrt{2}))$$

20. Let $u = x^2$. From this substitution, it follows that $x^2 = u$ and $x^4 = u^2$.

$$3x^4 - 10x^2 - 8 = 0$$
$$3u^2 - 10u - 8 = 0$$
$$u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{(quadratic Formula to solve for } u\text{)}$$
$$u = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(3)(-8)}}{2(3)}$$
$$u = \frac{10 \pm \sqrt{196}}{6}$$
$$u = \frac{10 \pm 14}{6}$$
$$u = \frac{10 + 14}{6} \text{ or } u = \frac{10 - 14}{6}$$
$$\boldsymbol{u} = 4 \text{ or } \boldsymbol{u} = -\frac{2}{3}$$
$$\boldsymbol{x^2} = 4 \text{ or } \boldsymbol{x^2} = -\frac{2}{3} \qquad \text{(now we must back-substitute)}$$
$$x = \pm 2 \text{ or } x = \pm\sqrt{-\frac{2}{3}} = \pm i\sqrt{\frac{2}{3}}$$

Four solutions: $x = -2, 2, -\sqrt{2/3}, \sqrt{2/3}$.

21. Let $u = x^3$. From this substitution, it follows that $x^3 = u$ and $x^6 = u^2$.

$$x^6 - 3x^3 = 0$$
$$u^2 - 3u = 0$$
$$u(u - 3) = 0\text{(actor to solve for } u\text{)}$$
$$u = 0 \text{ or } u - 3 = 0$$
$$\boldsymbol{u} = 0 \text{ or } \boldsymbol{u} = 3$$
$$\boldsymbol{x^3} = 0 \text{ or } \boldsymbol{x^3} = 3 \qquad \text{(now we must back-substitute)}$$
$$x = 0 \text{ or } x = \sqrt[3]{3}$$

22. Let $u = x^4$. From this substitution, it follows that $x^4 = u$ and $x^8 = u^2$.

$$x^8 - 6x^4 = 0$$
$$u^2 - 6u = 0$$
$$u(u - 6) = 0 \qquad \text{(factor to solve for } y\text{)}$$
$$u = 0 \text{ or } u - 6 = 0$$
$$\boldsymbol{u} = 0 \text{ or } \boldsymbol{u} = 6$$
$$\boldsymbol{x^4} = 0 \text{ or } \boldsymbol{x^4} = 6 \qquad \text{(now we must back-substitute)}$$
$$x = 0 \text{ or } x = \pm\sqrt[4]{6}$$

23. You can use any variable you like for the substitution.

Let $y = x^2$. From this substitution, it follows that $x^2 = y$.

$$\begin{aligned} x^4 - 81 &= 0 \\ y^2 - 81 &= 0 \\ y &= \pm 9 \\ \boldsymbol{y} = 9 \text{ or } \boldsymbol{y} &= -9 \\ \boldsymbol{x^2} = 9 \text{ or } \boldsymbol{x^2} &= -9 \qquad \text{(now we must back-substitute)} \\ x = \pm 3 \text{ or } x &= \pm\sqrt{-9} = \pm 3i \end{aligned}$$

24. You can use any variable you like for the substitution.

Let $w = x^{1/5}$. From this substitution, it follows that $x^{1/5} = w$ and $x^{2/5} = w^2$.

$$\begin{aligned} x^{2/5} + x^{1/5} - 2 &= 0 \\ w^2 + w - 2 &= 0 \\ (w+2)(w-1) &= 0 \qquad \text{(factor to solve for } w\text{)} \\ y + 2 = 0 \text{ or } w - 1 &= 0 \\ \boldsymbol{w} = -2 \text{ or } \boldsymbol{w} &= 1 \\ \boldsymbol{x^{1/5}} = -2 \text{ or } \boldsymbol{x^{1/5}} &= 1 \qquad \text{(now we must back-substitute)} \\ x = (-2)^5 \text{ or } x &= 1^5 \\ x = -32 \text{ or } x &= 1 \end{aligned}$$

25. You can use any variable you like for the substitution.

Let $w = x^{-1}$. From this substitution, it follows that $x^{-1} = w$ and $x^{-2} = w^2$.

$$\begin{aligned} x^{-2} + 3x^{-1} &= 0 \\ w^2 + 3w &= 0 \\ w(w+3) &= 0 \qquad \text{(factor to solve for } y\text{)} \\ w = 0 \text{ or } w + 3 &= 0 \\ \boldsymbol{w} = 0 \text{ or } \boldsymbol{w} &= -3 \\ \boldsymbol{x^{-1}} = 0 \text{ or } \boldsymbol{x^{-1}} &= -3 \qquad \text{(now we must back-substitute)} \\ x = \frac{1}{0} \text{ or } x &= \frac{1}{-3} = -\frac{1}{3} \end{aligned}$$

The quantity $\frac{1}{0}$ is not defined (division by zero). There is only one solution, $x = -1/3$.

26.

$$\begin{aligned}
\frac{S}{\mathbf{4\pi}} &= \frac{\cancel{4\pi}r^2}{\cancel{\mathbf{4\pi}}} && \text{(division principle)}\\
\frac{S}{4\pi} &= r^2\\
r^2 &= \frac{S}{4\pi}\\
r &= \pm\sqrt{\frac{S}{4\pi}} && \text{(square root property)}\\
r &= \pm\frac{1}{2}\sqrt{\frac{S}{\pi}}
\end{aligned}$$

With formulas, typically the variables have some meaning. If r is a length, we would eliminate the negative solution and only keep $r = \frac{1}{2}\sqrt{\frac{S}{\pi}}$.

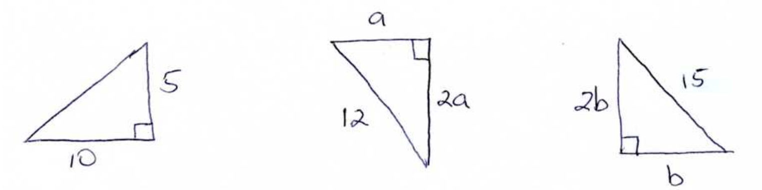
27.

$$\begin{aligned}
A \times \frac{\mathbf{2}}{\boldsymbol{\theta}} &= \frac{1}{\cancel{2}}r^2\cancel{\theta} \times \frac{\cancel{\mathbf{2}}}{\cancel{\boldsymbol{\theta}}} && \text{(multiplication principle)}\\
\frac{2A}{\theta} &= r^2\\
r^2 &= \frac{2A}{\theta}\\
r &= \pm\sqrt{\frac{2A}{\theta}} && \text{(square root property)}
\end{aligned}$$

28.

$$\begin{aligned}
\frac{A}{\boldsymbol{P}} &= \frac{\cancel{P}(1+r)^2}{\cancel{\boldsymbol{P}}} && \text{(division principle)}\\
\frac{A}{P} &= (1+r)^2\\
\pm\sqrt{\frac{A}{P}} &= 1+r && \text{(square root property)}\\
\mathbf{-1} \pm \sqrt{\frac{A}{P}} &= \cancel{\mathbf{-1}} + \cancel{1} + r && \text{(addition property)}\\
-1 \pm \sqrt{\frac{A}{P}} &= r\\
r &= -1 \pm \sqrt{\frac{A}{P}}
\end{aligned}$$

29. Choose positive square roots since we are looking for lengths which are greater than zero.



First:

$$a^2 + b^2 = c^2$$
$$a^2 + 25 = 100$$
$$a = \sqrt{75} = 5\sqrt{3}$$

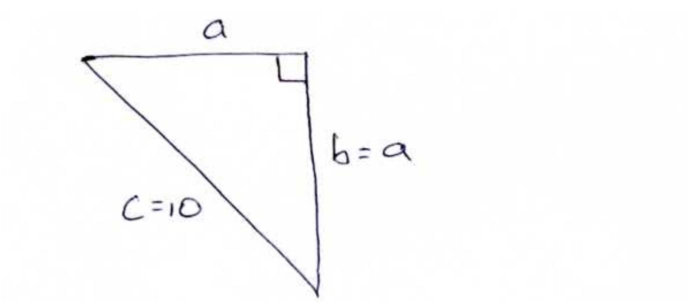
Second:

$$a^2 + b^2 = c^2$$
$$a^2 + (2a)^2 = (12)^2$$
$$5a^2 = 144 \quad \Rightarrow \quad a = \sqrt{\frac{144}{5}}; \quad b = 2a = 2\sqrt{\frac{144}{5}}$$

Third:

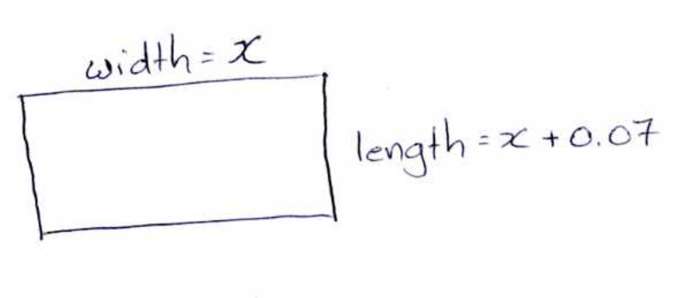
$$a^2 + b^2 = c^2$$
$$(2b)^2 + b^2 = 15^2$$
$$5b^2 = 225 \Rightarrow b = 3\sqrt{5}; a = 6\sqrt{5}$$

30. Sketch:



$$a^2 + b^2 = c^2$$
$$a^2 + a^2 = 10^2$$
$$a^2 = 50 \Rightarrow a = \sqrt{50} = 5\sqrt{2} \text{ inches.}$$

31. Sketch:



$$\begin{aligned}
\text{Area} &= (\text{length})(\text{width}) \\
0.026 &= (x+0.07)x \\
x^2 + 0.07x - 0.026 &= 0 \\
x &= \frac{-b \pm \sqrt{b^2-4ac}}{2a} \\
x &= \frac{-0.07 \pm \sqrt{(0.07)^2 - 4(1)(-0.026)}}{2(1)} \\
x &= 0.13
\end{aligned}$$

The lot is $x = 0.13$ miles wide and $x + 0.07 = 0.2$ miles long.

32.

$$\begin{aligned}
A &= P(1+r)^2 \\
1514.24 &= 1400(1+r)^2 \\
1.0816 &= (1+r)^2 \\
\pm\sqrt{1.0816} &= 1+r \\
\pm 1.04 &= 1+r \\
-1 \pm 1.04 &= r \\
r = 0.04 \text{ or } r &= -2.04
\end{aligned}$$

The interest rate is $0.04 = 4\%$ per year (the interest rate must be positive).

An alternate solution would be to expand $(1+r)^2 = 1 + 2r + r^2$ and then use the quadratic formula.

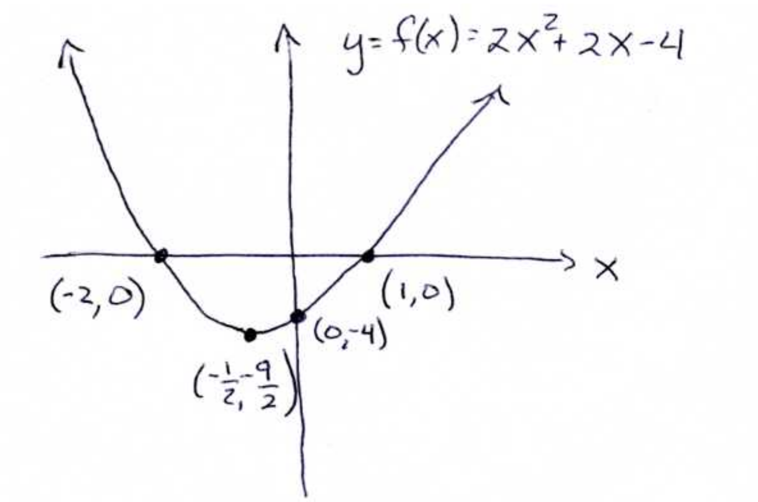
33. Sketch $f(x) = 2x^2 + 2x - 4$.

Since $a = 2 > 0$, quadratic opens up

$$\text{Vertex: } x = -\frac{b}{2a} = -\frac{2}{2(2)} = -\frac{1}{2}$$

$$y = f(-1/2) = 2(-1/2)^2 + 2(-1/2) - 4 = -\frac{9}{2}$$

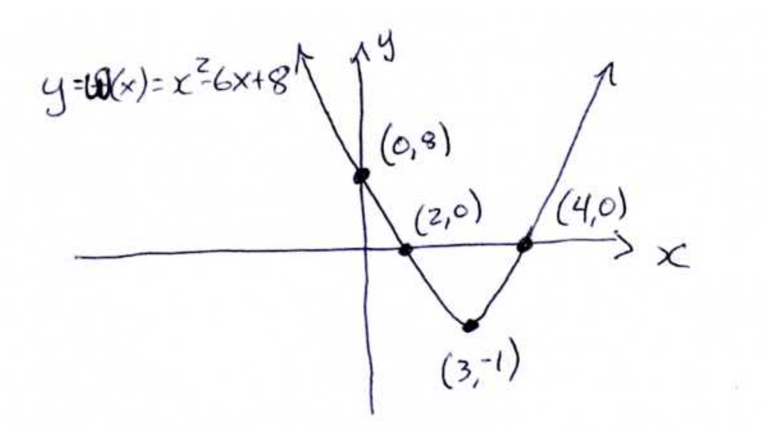
$$\begin{aligned} x\text{-intercepts: } & 2x^2 + 2x - 4 = 0 \\ & x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ & x = \frac{-2 \pm \sqrt{2^2 - 4(2)(-4)}}{2(2)} \\ & x = \frac{-2 \pm 6}{4} \\ & x = 1 \text{ or } x = -2 \\ y\text{-intercept: } & f(0) = 2(0)^2 + 2(0) - 4 = -4 \end{aligned}$$



34. Sketch $w(x) = x^2 - 6x + 8$.

Since $a = 1 > 0$, quadratic opens up

$$\begin{aligned}
\text{Vertex: } x &= -\frac{b}{2a} = -\frac{(-6)}{2(1)} = 3 \\
y &= f(3) = (3)^2 - 6(3) + 8 = -1 \\
\\
x\text{-intercepts: } x^2 - 6x + 8 &= 0 \\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
x &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(8)}}{2(1)} \\
x &= \frac{6 \pm 2}{2} \\
x &= 4 \text{ or } x = 2 \\
y\text{-intercept: } f(0) &= (0)^2 - 6(0) + 8 = +8
\end{aligned}$$

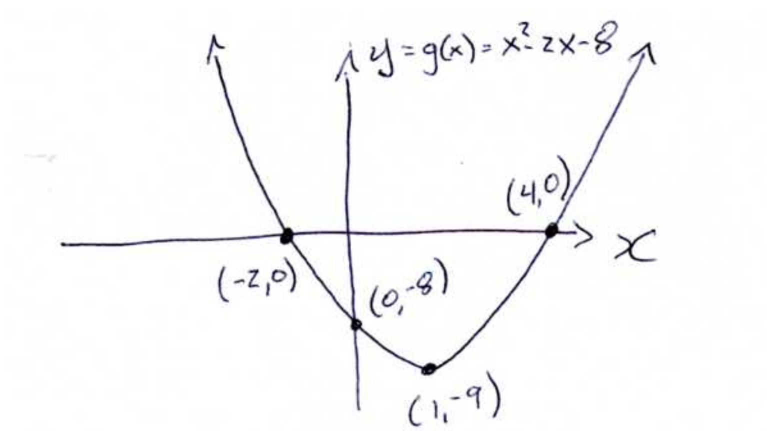


35. Sketch $g(x) = x^2 - 2x - 8$.

Since $a = 1 > 0$, quadratic opens up

$$\begin{aligned}
\text{Vertex: } x &= -\frac{b}{2a} = -\frac{(-2)}{2(1)} = -1 \\
y &= f(-1) = (-1)^2 - 2(-1) - 8 = -9
\end{aligned}$$

$$\begin{aligned}
x\text{-intercepts: } & x^2 - 2x - 8 = 0 \\
& x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
& x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-8)}}{2(1)} \\
& x = \frac{2 \pm 6}{2} \\
& x = 4 \text{ or } x = -2 \\
y\text{-intercept: } & f(0) = (0)^2 - 2(0) - 8 = -8
\end{aligned}$$



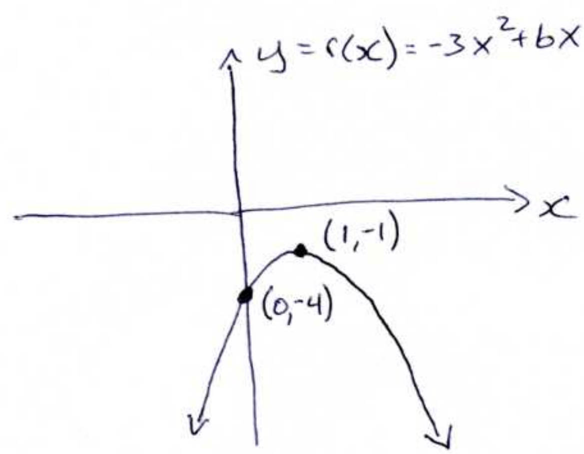
36. Sketch $r(x) = -3x^2 + 6x - 4$.

Since $a = -3 < 0$, quadratic opens down

$$\text{Vertex: } x = -\frac{b}{2a} = -\frac{6}{2(-3)} = 1$$
$$y = f(1) = -3(1)^2 + 6(1) - 4 = -1$$

$$x\text{-intercepts: } -3x^2 + 6x - 4 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-6 \pm \sqrt{6^2 - 4(-3)(-4)}}{2(-3)}$$
since $b^2 - 4ac = -12 < 0$, there are no x-intercepts

$$y\text{-intercept: } f(0) = -3(0)^2 + 6(0) - 4 = -4$$



37. We don't actually need the sketch to answer these questions.

Let's analyze $P(x) = -6x^2 + 312x - 3672$ to answer the questions.

Maximum profit occurs at the vertex.

$$\text{Vertex: } x = -\frac{b}{2a} = -\frac{312}{2(-6)} = 26 \qquad \text{(number of widgets to get max profit)}$$
$$y = f(26) = -6(26)^2 + 312(26) - 3672 = 384 \qquad \text{(maximum profit)}$$

Daily profit will be zero at the x-intercepts.

$$\begin{aligned}
x\text{-intercepts: } & -6x^2 + 312x - 3672 = 0 \\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
x &= \frac{-312 \pm \sqrt{(312)^2 - 4(-6)(-3672)}}{2(-6)} \\
x &= \frac{-312 \pm 96}{-12} \\
x &= 18 \text{ or } x = 34
\end{aligned}$$

Profit is zero if they produce 18 or 34 widgets.

For a profit of \$288, we must solve the equation $P(x) = 288$.

$$\begin{aligned}
& -6x^2 + 312x - 3672 = 288 \\
& -6x^2 + 312x - 3960 = 0 \\
& x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
& x = \frac{-312 \pm \sqrt{(312)^2 - 4(-6)(-3960)}}{2(-6)} \\
& x = \frac{-312 \pm 48}{-12} \\
& x = 22 \text{ or } x = 30
\end{aligned}$$

Daily profit is \$288 if they produce 22 or 30 widgets.