1 The Real Number Line

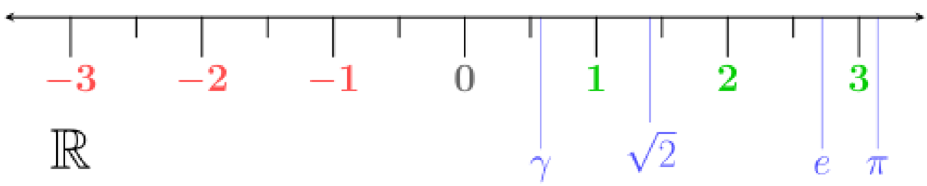
There are many sets of numbers, but important ones in math and life sciences are the following

- The **integers** $\mathbb{Z} = \{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$.
- The **positive integers**, sometimes called **natural numbers**, $\mathbb{N} = \{1, 2, 3, 4, \ldots\}$.
- **Rational numbers** $\mathbb{Q}$ are any number that can be expressed as a fraction $\frac{p}{q}$ where p and $q \neq 0$ are integers.

  Note that every integer is a rational number!
- An **irrational number** is a number that is not rational.

  Examples of irrational numbers are $\sqrt{2} \sim 1.41421\ldots$, $e \sim 2.71828\ldots$, $\pi \sim 3.14159265359\ldots$, and less familiar ones like Euler's constant $\gamma \sim 0.577215664901532\ldots$.

  The "$\ldots$" here represent that the number is a nonrepeating, nonterminating decimal.
- The **real numbers** $\mathbb{R}$ contain all the integer number, rational numbers, and irrational numbers. The real numbers are usually presented as a real number line, which extends forever to the left and right.



Note the real numbers are not bounded above or below. To indicate that the real number line extends forever in the positive direction, we use the terminology **infinity**, which is written as ∞, and in the negative direction the real number line extends to minus infinity, which is denoted $-\infty$.

The symbol ∞ is used to say that the real numbers extend without bound (for any real number, there is a larger real number and a smaller real number). Infinity is a somewhat tricky concept, and you will learn more about it in precalculus and calculus. For the moment, we just want to understand that the symbol ∞ means there is no bound.

Note a more formal way to write the real number line would be as a set of numbers. If x is a placeholder for a real number (i.e., x represents any real number), then we can say x is a real number in a variety of ways:

$x \in \mathbb{R}$	(read as "x is an element of the set of real numbers")
$-\infty < x < \infty$	(read as "x is between minus infinity and infinity")
$x \in (-\infty, \infty)$	(read as "x is in the interval from minus infinity and infinity")

The last notation is known as interval notation, which we will talk about in Section 4.

2 Arithmetic of Real Numbers

- **Addition Properties** for $\mathbb{R}$ (set of real numbers)
  - Addition is **commutative**: $a + b = b + a$
  - Addition of 0 leaves a number unchanged (0 is the *additive identity*): $a + 0 = a$
  - Addition is **associative**: $a + (b + c) = (a + b) + c$.
- **Multiplication Properties** for $\mathbb{R}$
  - Multiplication is commutative: $a \times b = b \times a$
  - Multiplication by 0 results in zero: $a \times 0 = 0$
  - Multiplication by 1 leaves a number unchanged (1 is the *multiplicative identity*): $a \times 1 = a$
  - Multiplication is associative: $a \times (b \times c) = (a \times b) \times c$.
  - Different symbols for multiplication (in math we rarely use the "$\times$" symbol)

    $$a \times (b + c) = a \cdot (b + c) = a(b + c)$$
- **Division Properties** for $\mathbb{R}$
  - In math, we will use a fraction notation for division, and rarely use the "$\div$" symbol:

    $$a \div b = \frac{a}{b} = a \times \frac{1}{b}$$
  - Division of a real number other than zero by 0 is undefined: $\frac{a}{0}$ is undefined if $a \neq 0$

    $$\frac{456}{0} = \text{ undefined}$$
  - Zero divided by any real number not equal to zero is zero: $\frac{0}{b} = 0$ if $b \neq 0$

    $$\frac{0}{456} = 0$$
  - Zero divided by zero is called an **indeterminant form** and requires more work to determine what the quantity simplifies to on a case-by-case basis (it could be anything). You will learn more about indeterminant forms in precalculus and calculus.

    $$\frac{0}{0} = \text{ indeterminant}$$
- **Integer Exponents**–pay close attention to the base (i.e., what is being raised to the power)!

  $$(-2)^4 = (-2)(-2)(-2)(-2) = 16$$

  $$-2^4 = -(2)(2)(2)(2) = -16$$

- **Absolute Value**

  The absolute value of a number can be thought of the distance to the origin on the number line. So

$$\begin{aligned} |6| &= 6 \\ |-6| &= 6 \\ |-126| &= 126 \\ |0| &= 0 \end{aligned}$$

Distance between two numbers: $d = |x_2 - x_1|$

Advice: Never write two "operations" without using bracket! What I mean is, never write $34 + -56$, since it obscures the fact that you are adding negative fifty-six. Other examples of poor notation include:

5×-7 should be written as $5 \times (-7) = -5(7)$.

$5 \div -7$ should be written as $5 \div (-7) = -\frac{5}{7}$.

$5 - -7$ should be written as $5 - (-7) = 5 + 7$.

Here is how we read the mathematics:

$34 - 56$ is read as "thirty-four minus fifty-six".

$34 + (-56)$ is read as "thirty four plus negative fifty-six".

<u>In this way, we can always think of subtraction as adding a negative number.</u> This may seem like a very subtle point, but it will pay big rewards later when we start dealing with variables.

Multiplying and Dividing with negative numbers

$(\text{positive}) \times (\text{positive}) = (\text{positive})$

$(\text{positive}) \times (\text{negative}) = (\text{negative})$

$(\text{negative}) \times (\text{negative}) = (\text{positive})$

$\dfrac{(\text{positive})}{(\text{positive})} = (\text{positive})$

$\dfrac{(\text{positive})}{(\text{negative})} = (\text{negative})$

$\dfrac{(\text{negative})}{(\text{positive})} = (\text{negative})$

$\dfrac{(\text{negative})}{(\text{negative})} = (\text{positive})$

2.1 Order of Operations

We have to have a way to simplify complicated expressions, and the order of operations is what tells us which operation takes priority.

Order of Operations

1. Do all operations inside parentheses.
2. Raise numbers to a power.
3. Multiply and Divide from left to right.
4. Add and subtract from left to right.

EXAMPLE Your favourite stock opened the day at \$37.20 per share, and closed the day at \$27.30 per share. If you owned 75 shares, describe your day at the market with respect to this particular stock.

SOLUTION: You are very sad, because your stock lost value that day: $\$37.20 - \$27.30 = -\$9.90$. Since you own 75 shares, in total you lost: $75 \times (-\$9.90) = \$742.50$.

You could write the solution to this problem all in one line: $75(\$37.20 - \$27.30) = 75(-\$9.90) = \$742.50$.

EXAMPLE The highest point in Africa is Mount Kilimanjaro, which is 5895 meters above sea level. The lowest point in Africa is Lake Assal, which is 156 meters below sea level. What is the difference in elevation between Mount Kilimanjaro and Lake Assal?

SOLUTION: This is a really good example of how negative numbers can arise–positive numbers can measure distance above sea level, and negative numbers measure distance below sea level. It's like you take a number line, and instead of it being horizontal it is vertical with the 0 at sea level.

The distance between the mountain and lake is $5895 - (-156) = 5895 + 156 = 6051$ meters.

EXAMPLE On June 22, 1943 the temperature in Spearfish South Dakota had a high of 44.6°F and a low of −4°F. What was the temperature change that day?

SOLUTION: Temperature change $= 44.6 - (-4) = 44.6 + 4 = 48.6$°F.

3 Arithmetic of Algebraic Expressions

- An **algebraic expression** contains numbers and variables.

  When working with an expression, we typically are trying to find **equivalent expressions**, and we indicate these equivalent expressions using equal signs.

  $$\begin{aligned} \text{original expression} &= \text{equivalent expression 1}, \\ &= \text{equivalent expression 2}, \\ &= \text{equivalent expression 3}, \\ &= \text{equivalent expression 4}, \text{ etc.} \end{aligned}$$

  The final expression usually has a property you wanted that the previous expressions did not.

- A **factor** are quantities that are multiplied together:

  xyz has factors x, y, and z.

  $x(x + 6)$ has factors x and $x + 6$.

- **Terms** are quantities that are added together:

  $a + b + c + d$ has terms a, b, c and d.

  $17x^3 + 8b + \dfrac{1}{c} + (-14)$ has terms $17x^3$, $8b$, $\dfrac{1}{c}$ and -14.

Sometimes you may see

$$x - 14 \text{ has terms } x \text{ and } 14,$$

which is the same as

$$x + (-14) \text{ has terms } x \text{ and } -14.$$

I prefer the latter since you will be less likely to make sign errors if you keep track of the sign by including it in the term. You can also ensure you keep a minus sign with the factor by writing

$$-18(x-1) = (-18)(x-1) = -18x + 18.$$

This will help you avoid the mistake that leads you to $-18x - 18$.

- **Distributive property** (notice that once you distribute, the parentheses are not needed):

$$a(b+c) = ab + ac.$$

- **Like terms** are terms with identical variables and exponents, but the numbers can be different. You must practice identifying like terms.

$$16x^3, -\frac{1}{5}x^3 \text{ and } x^3 \text{ are all like terms.}$$

$$x^2y^4, -24x^2y^4, \frac{1}{2}x^2y^4 \text{ are all like terms.}$$

EXAMPLE Distribute $(a-b)(a+b)$.

SOLUTION:

$$\begin{aligned}
\boldsymbol{(a-b)}(a+b) &= \boldsymbol{(a-b)}a + \boldsymbol{(a-b)}b, && \text{(distribute } a-b \text{ (in blue))} \\
&= (a-b)\boldsymbol{a} + (a-b)\boldsymbol{b}, && \text{(distribute } a \text{ and } b \text{ (in blue))} \\
&= a\boldsymbol{a} - b\boldsymbol{a} + a\boldsymbol{b} - b\boldsymbol{b}, && \text{(simplify, multiplication is commutative)} \\
&= a^2 - \cancel{ab} + \cancel{ab} - b^2, && \text{(collect like terms, in this case they cancel)} \\
&= a^2 - b^2.
\end{aligned}$$

Note this is the factoring rule **difference of squares**, $a^2 - b^2 = (a-b)(a+b)$.

3.1 Removing grouping symbols (simplifying parenthesis)

Use the distributive property and collect like terms.

$$\begin{aligned}
5(4x \boldsymbol{-3}(x-2)) &= 5(4x \boldsymbol{-3}x + \boldsymbol{(-3)}(-2)) && \text{(distribute the } -3 \text{ into the } (x-2)\text{)} \\
&= \boldsymbol{5}(x+6) && \text{(collect like terms } 4x \text{ and } -3x\text{)} \\
&= \boldsymbol{5}x + \boldsymbol{5} \times 6 && \text{(distribute the 5 into the } (x+6)\text{)} \\
&= 5x + 30 && \text{(simplify)}
\end{aligned}$$

When removing parentheses, many texts says quite emphatically that you should "Remember to remove the innermost parentheses first", as we did above. This is simply advice, not a mathematical rule like the order of operations.

The order of operations cannot be changed, but with parentheses you could work from outermost parentheses first if you use the distributive property correctly.

$$\begin{aligned}
\mathbf{5}(4x-3(x-2)) &= \mathbf{5\times}4x-\mathbf{5\times}3(x-2) && \text{(distribute the 5)}\\
&= 20x-\mathbf{15}(x-2) && \text{(simplify)}\\
&= 20x-\mathbf{15}x-\mathbf{(-15)\times}2 && \text{(distribute the } -15\text{)}\\
&= 5x+30 && \text{(collect like terms } 20x \text{ and } -15x\text{)}
\end{aligned}$$

Wikipedia had this to say about removing parentheses, and I think is much more beneficial than what the text says:

> If an expression involves parentheses, then do the arithmetic inside the innermost pair of parentheses first and work outward, or use the distributive law to remove parentheses.[a]
>
> [a]http://en.wikipedia.org/wiki/Order_of_operations retrieved on Aug 23, 2007.

In my opinion, the following is one of the most important concepts in mathematics, and one that we sometimes fail to teach to students:

> Your goal should be to understand what the rules of mathematics are that you cannot break, and what is simply advice to make problem solving go faster. There are many different paths to solution, and any path that uses mathematics correctly is a good one.

EXAMPLE Simplify $\left(\frac{1}{2}\right)^3+\frac{1}{4}-\left(\frac{1}{6}-\frac{1}{12}\right)-\frac{2}{3}\cdot\left(\frac{1}{4}\right)^2$.

SOLUTION: This is all about getting the order of operation right.

$$\begin{aligned}
\left(\frac{1}{2}\right)^3+\frac{1}{4}-\left(\frac{\mathbf{1}}{\mathbf{6}}-\frac{\mathbf{1}}{\mathbf{12}}\right)-\frac{2}{3}\cdot\left(\frac{1}{4}\right)^2 &= \left(\frac{1}{2}\right)^3+\frac{1}{4}-\left(\frac{\mathbf{2}}{\mathbf{12}}-\frac{\mathbf{1}}{\mathbf{12}}\right)-\frac{2}{3}\cdot\left(\frac{1}{4}\right)^2 && \text{(do parentheses)}\\
&= \left(\frac{\mathbf{1}}{\mathbf{2}}\right)^{\mathbf{3}}+\frac{1}{4}-\frac{\mathbf{1}}{\mathbf{12}}-\frac{2}{3}\cdot\left(\frac{\mathbf{1}}{\mathbf{4}}\right)^{\mathbf{2}} && \text{(do powers)}\\
&= \left(\frac{\mathbf{1}}{\mathbf{2}}\right)\left(\frac{\mathbf{1}}{\mathbf{2}}\right)\left(\frac{\mathbf{1}}{\mathbf{2}}\right)+\frac{1}{4}-\frac{1}{12}-\frac{2}{3}\cdot\left(\frac{\mathbf{1}}{\mathbf{4}}\right)\left(\frac{\mathbf{1}}{\mathbf{4}}\right)\\
&= \frac{\mathbf{1}}{\mathbf{8}}+\frac{1}{4}-\frac{1}{12}-\frac{2}{3}\cdot\left(\frac{\mathbf{1}}{\mathbf{16}}\right) && \text{(do multiplication from left to right)}\\
&= \frac{1}{8}+\frac{1}{4}-\frac{1}{12}-\frac{1}{24} && \text{(do addition/subtraction from left to right)}\\
\text{LCD is } 24\\
&= \frac{3}{24}+\frac{6}{24}-\frac{2}{24}-\frac{1}{24}=\frac{3+6-2-1}{24}=\frac{6}{24}=\frac{1}{4} && \text{(simplify)}
\end{aligned}$$

EXAMPLE Simplify $(-2)^3 - 4\left|\frac{1}{4} - \frac{1}{3}\right| - 6 \cdot \left(\frac{1}{4}\right)^2$.

$$\begin{aligned}
(-2)^3 - 4\left|\frac{1}{4} - \frac{1}{3}\right| - 6 \cdot \left(\frac{1}{4}\right)^2 &= (-2)^3 - 4\left|\frac{3}{12} - \frac{4}{12}\right| - 6 \cdot \left(\frac{1}{4}\right)^2 \text{ (do parentheses)} \\
&= (-2)^3 - 4\left|-\frac{1}{12}\right| - 6 \cdot \left(\frac{1}{4}\right)^2 \text{ (do absolute value (it's like a parentheses))} \\
&= (-2)^3 - 4\left(\frac{1}{12}\right) - 6 \cdot \left(\frac{1}{4}\right)^2 \text{ (do powers)} \\
&= (-2)(-2)(-2) - 4\left(\frac{1}{12}\right) - 6 \cdot \left(\frac{1}{4}\right)\left(\frac{1}{4}\right) \\
&= -8 - 4\left(\frac{1}{12}\right) - 6 \cdot \left(\frac{1}{16}\right) \text{ (do multiplication from left to right)} \\
&= -8 - \frac{1}{3} - \frac{3}{8} \text{ (do addition/subtraction from left to right)}
\end{aligned}$$

LCD is 24

$$= -\frac{192}{24} - \frac{8}{24} - \frac{9}{24} = \frac{-192 - 8 - 9}{24} = \frac{-209}{24} \quad \text{(simplify)}$$

NOTE: Because I am looking for an improper fraction as my answer, I never really do any division. This is why I say "do multiplication from left to right" rather than "do multiplication/division from left to right". If I want a decimal answer, I can do a division at the end to get it. This is typical of what you will do, since is more common to see quantities like $\frac{1}{4}$ rather than $1 \div 4$.

EXAMPLE Compare the following two simplifications, both which represent the same number:

Simplification 1:
$$\begin{aligned}
(3-7)^2 \div 8 + 3 &= (-4)^2 \div 8 + 3 && \text{(do the parentheses)} \\
&= (-4)(-4) \div 8 + 3 && \text{(do the powers)} \\
&= 16 \div 8 + 3 && \text{(do the multiplication/division from left to right)} \\
&= 2 + 3 && \text{(do the addition/subtraction from left to right)} \\
&= 5 && \text{(simplify)}
\end{aligned}$$

Simplification 2:
$$\begin{aligned}
\frac{(3-7)^2}{8} + 3 &= \frac{(-4)^2}{8} + 3 && \text{(do the parentheses)} \\
&= \frac{(-4)(-4)}{8} + 3 && \text{(do the powers)} \\
&= \frac{16}{8} + 3
\end{aligned}$$

LCD is 8

$$\begin{aligned}
&= \frac{16}{8} + \frac{24}{8} && \text{(do the the addition/subtraction from left to right)} \\
&= \frac{16 + 24}{8} && \text{(do the the addition/subtraction from left to right)} \\
&= \frac{40}{8} = 5 && \text{(simplify)}
\end{aligned}$$

Both techniques are correct. I am showing a lot of steps for those students who need it, if you can do some of the arithmetic in you head *correctly*, then you don't need to show as much work.

EXAMPLE Simplify $2a - (6b - 4(a - (b - 3a)))$.

$$\begin{aligned}
2a - [6b - 4(a - (b - 3a))] &= 2a - [6b + (-4)(a - (b - 3a))] && \text{(distribute factor } -4) \\
&= 2a - [6b + (-4)a - (-4)(b - 3a)] \\
&= 2a - [6b - 4a + 4(b - 3a)] && \text{(simplify)} \\
&= 2a - [6b - 4a + 4b - 4 \times 3a] && \text{(distribute the 4)} \\
&= 2a - [6b - 4a + 4b - 12a] && \text{(simplify)} \\
&= 2a - 6b + 4a - 4b + 12a && \text{(distributed the minus sign (which is } -1)) \\
&= 2a + 4a + 12a - 6b - 4b && \text{(put like terms together)} \\
&= 18a - 10b && \text{(collect like terms)}
\end{aligned}$$

What if we didn't work from innermost parentheses to outermost? We get the same answer, of course.

$$\begin{aligned}
2a - [6b - 4(a - (b - 3a))] &= 2a - 6b + 4(a - (b - 3a)) && \text{(distribute the } -1 \text{ (the minus sign))} \\
&= 2a - 6b + 4a - 4(b - 3a) && \text{(distribute the } +4) \\
&= 2a - 6b + 4a + (-4)(b - 3a) && \text{(distribute the } -4) \\
&= 2a - 6b + 4a + (-4)b - (-4) \times 3a \\
&= 2a - 6b + 4a - 4b + 12a && \text{(simplify)} \\
&= 2a + 4a + 12a - 6b - 4b \\
&= 18a - 10b && \text{(collect like terms)}
\end{aligned}$$

Working from innermost parentheses first is usually what to do since it makes the simplifying easier. In this case it really didn't.

3.2 Substituting into Variable Expressions

- When substituting into variable expressions, it can help to use parenthesis but not fill in what goes into them right away.
- Think of the variables as placeholders.
- This can help you avoid some sign errors that might occur if you aren't careful with your substitution.

EXAMPLE Evaluate $\dfrac{x-1}{1+x^2}$ when $x = -1$.

$$\begin{aligned}
\frac{x-1}{1+x^2} &= \frac{(\ \ ) - 1}{1 + (\ \ )^2} && \text{(put parenthesis where there were } x\text{'s)} \\
&= \frac{(-1) - 1}{1 + (-1)^2} && \text{(put the value for } x \text{ in the parentheses)} \\
&= \frac{-1 - 1}{1 + (-1)(-1)} && \text{(simplify, using rules of algebra)} \\
&= \frac{-2}{1+1} = \frac{-2}{2} = -1
\end{aligned}$$

EXAMPLE Density d, mass m and volume V are related by the formula $d = \dfrac{m}{V}$. When working with formulas where the variables have physical meaning, you should include units in your calculations.

What is the density of mercury if you measure that 24 milliliters of mercury has a mass of 326 grams?

$$
\begin{aligned}
d &= \frac{m}{v} && \text{(write down equation you will use)}\\
d &= \frac{(326 \text{ grams})}{(24 \text{ milliliters})} && \text{(substitute values, including units)}\\
d &= \frac{163 \times \cancel{2} \text{ grams}}{12 \times \cancel{2} \text{ milliliters}} && \text{(simplify, using rules of algebra)}\\
d &= \frac{163}{12}\frac{\text{grams}}{\text{milliliters}} && \text{(convert to decimal at the last step)}\\
d &= 13.5833\frac{\text{grams}}{\text{milliliters}}
\end{aligned}
$$

If you performed this measurement, you would also need to do an error analysis. You could convert to a decimal earlier if you do the error analysis. Rounding and error analysis is something you will study in your science classes.

Temperature conversion

$$T_C = \frac{5}{9}\left(T_F - 32\right)$$

where T_C is temperature in Celsius, T_F is temperature in Fahrenheit.

EXAMPLE What is $-40°$F in Celsius?

$$
\begin{aligned}
T_C &= \frac{5}{9}\left(T_F - 32\right)\\
T_C &= \frac{5}{9}\left(-40 - 32\right)\\
T_C &= \frac{5}{9}\left(-72\right)\\
T_C &= -\frac{5 \times 8 \times \cancel{9}}{\cancel{9}}\\
T_C &= -40
\end{aligned}
$$

So $-40°\text{F} = -40°\text{C}$. Note: We typically don't include units in this calculation since the $\frac{5}{9}$ really has units $\frac{5°\text{C}}{9°\text{F}}$ and it is a simple calculation so we just leave them out.

4 Inequalities and Intervals

Often we need to talk about part of the set of real numbers, not the entire set of real numbers. This is where inequalities and interval notation can help. An **inequality** is a relation between two expressions that are not equal.

When sketching an inequality on a number line you use an open circle if the endpoint is not included, and a filled in circle if the endpoint is included. Here's how you can remember this:

For $<$ (one thing)	draw $\circ$	one thing (draw the circle)
For $>$ (one thing)	draw $\circ$	one thing (draw the circle)
For $\geq$ (two things)	draw $\bullet$	two things (draw the circle and then shade it in)
For $\leq$ (two things)	draw $\bullet$	two things (draw the circle and then shade it in)

4.1 Set Notation and Interval Notation for Inequalities

$a \leq x \leq b$	is equivalent to	$x \in [a, b]$
$a < x < b$	is equivalent to	$x \in (a, b)$
$a \leq x < b$	is equivalent to	$x \in [a, b)$
$a < x \leq b$	is equivalent to	$x \in (a, b]$

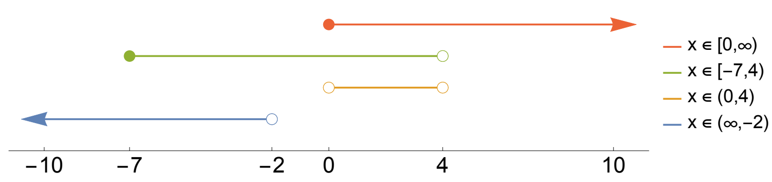
Note: the symbol $\in$ can be read as "is in the interval".

Note a more formal way to write these would be $\{x \mid a \leq x \leq b\}$ which we read as:
the set of numbers x such that x is greater than or equal to a and x is less than or equal to b.

If the intervals are not bounded above or below, we have

$x \geq a$	is equivalent to	$x \in [a, \infty)$	(closed interval)
$x \leq a$	is equivalent to	$x \in (-\infty, a]$	(closed interval)
$x > a$	is equivalent to	$x \in (a, \infty)$	(open interval)
$x < a$	is equivalent to	$x \in (-\infty, a)$	(open interval)
$x \in \mathbb{R}$	is equivalent to	$x \in (-\infty, \infty)$	(open interval)

4.2 Number Line Examples of Interval Notation



With inequalities, you often have to present the solution in different ways, although I find interval notation the easiest to use for complicated sets and number lines useful to visualize the sets.

For now, you should understand the different notations, and we will use these notations in future units.

5 Converting Between English and Algebraic Expressions

I sometimes get asked “When do you teach word problems in math?”, and the answer is that word problem show up almost everywhere. And in different applications, different mathematical techniques are used to solve the word problems. So in some sense, learning how to solve word problems is something that you will be continually doing in your math and science classes.

The first step in solving word problems is converting an English phrase to an algebraic expression.

You should take a moment to review these examples that show how to convert common phrases into algebraic expressions, as this will help you with word problems you will see later in your math and science classes.

In all these examples, your answers may vary.

1. Write an algebraic expression for the quantity “three more than half a number”.

   Let x be the unknown number.
   half a number is $\frac{1}{2}x$.
   more than is addition.
   three more than half a number is $3 + \frac{1}{2}x$.

2. Write an algebraic expression for the quantity “one-fifth of a number reduced by double the same number”.

   Let x be the unknown number.
   one-fifth of a number is $\frac{1}{5}x$.
   double the same number is $2x$.
   reduced by is subtraction.
   one-fifth of a number reduced by double the same number is $\frac{1}{5}x - 2x$.

3. The number of boxes of cookies sold by Sarah was 43 fewer than the number of boxes of cookies sold by Keiko. The number of boxes of cookies sold by Imelda was 53 more than the number sold by Keiko. Write algebraic expressions for these relations.

   Let x be the number of boxes of cookies sold by Keiko.
   The number of boxes sold by Sarah is $x - 43$.
   The number of boxes sold by Imelda is $x + 53$.

   Other solutions are possible, but they get more complicated to figure out. For example,

   Let y be the number of boxes of cookies sold by Sarah.
   The number of boxes sold by Keiko is $y + 43$ (she sold 43 more than Sarah).
   The number of boxes sold by Imelda is $y + 43 + 53 = y + 76$ (she sold 53 more than Keiko).

4. The first angle of a triangle is 16 degrees less than the second angle. The angle is double the second angle. Write algebraic expressions for these relations.

   Let x be the second angle.
   The first angle is $x - 16$.
   The third angle is $2x$.

5. Kentucky has about half the land area of Minnesota. The land area of Maine is approximately two-fifths the land area of Minnesota. Write algebraic expressions for these relations.

   Let A be the land area of Minnesota.
   The land area of Maine is $\frac{2}{5}A$.
   The land area of Kentucky is $\frac{1}{2}A$.

6. A census of a middle school found that the number of 7th graders was fifty more than the number of eighth graders. The number of sixth graders was three-fourths the number of eighth graders. Write algebraic expressions for these relations.

   Let w be the number of eighth graders.
   The number of seventh graders is $50 + x$.
   The number of sixth graders is $\frac{3}{4}x$.

7. In an archery tournament, the number of points awarded for an arrow in the gold circle (bull's eye) is six less than triple the points awarded for an arrow in the blue ring. Write algebraic expressions for these relations.

   Let x be the number of points for an arrow in the blue ring.
   Then the points for an arrow in the gold ring is $3x - 6$.

8. The number of pounds of fish caught by Jack was 813 pounds more than the amount of fish caught by Sally. The amount of fish caught by Ben was 623 pounds less than the amount caught by Sally. Write algebraic expressions for these relations.

   Let the number of pounds of fish caught by Sally be x.
   The number of pounds of fish caught by Jack is $x + 813$.
   The number of pounds of fish caught by Ben is $x - 623$.

   You can have different answers that are correct, but they are more complicated. Consider the following:

   Let the number of pounds of fish caught by Jack be y.
   Since Jack caught 813 pounds more than Sally, Sally caught 813 pounds less than Jack. The number of pounds of fish caught by Sally is $y - 813$.
   The number of pounds of fish caught by Ben is $y - 813 - 623 = y - 1436$.

   This is correct, but it required more translation of the English phrases than our first answer.

6 Reference

6.1 Useful Perimeter and Area formulas

parallelogram base b and perpendicular height a:	Perimeter = sum of all four sides Area = ab
rectangle width w and length l:	Perimeter = $2l + 2w$ Area = lw
square side length s:	Perimeter = $4s$ Area = s^2
triangle base b perpendicular height a:	Perimeter = sum of all four sides Area = $\frac{1}{2}ab$
trapezoid:	Perimeter = sum of all four sides Area = $\frac{1}{2}a(b_1 + b_2)$
circle radius r:	Perimeter = $2\pi r$ (commonly called the circumference) Area = πr^2

6.2 Triangle facts

1. The sum of the interior angles of a triangle is 180 degrees.
2. An **equilateral triangle** has three sides of equal lengths and three angles that measure 60 degrees.
3. An **isosceles triangle** has two sides of equal length. The two angles opposite the equal sides are also equal.
4. A **right triangle** has one angle that is 90 degrees.