I am including some word problems just so you can see what types of questions might lead to using the concepts from this Unit.

Questions

1. Solve for x if $-12 + x - 3 = 15 - 18 + 9$.
2. Solve for x if $-19 + x - 7 = 20 - 42 + 10$.
3. Is $x = 5$ the solution to $-7 + x = 2$? If not, find the correct solution.
4. Is $x = -8$ the solution to $-39 = x - 47$? If not, find the correct solution.
5. Find the value of x that satisfies $\frac{2}{3} + x = \frac{1}{6} + \frac{1}{4}$.
6. Find the value of x that satisfies $\frac{5}{12} - \frac{5}{6} = x - \frac{3}{2}$.
7. The Perception Toy Company wishes to cut costs and decides that improving quality control will help. It can do this by reducing the amount of rejected frisbees to 3%. In a recent week, a quality inspector found that 8 out of 413 frisbees were rejected. Did they meet the goal?
8. A 90 meter wide radar picture is taken of a swamp in Australia. The radar detects a rock outcrop that is 90 feet above sea level, and a vein of opal (a semi-precious stone) that is 27 feet below sea level. How far is the top of the rock from the location of the opal?
9. Trevor pays his monthly computer lease bill for \$49.99 but forgets to look at his checking account balance before doing so. When he gets his account statement at the local ATM, his balance reads −\$35.07. How much was in the account before he wrote the check?
10. Solve for x when $\frac{1}{9}x = 4$.
11. Solve for x when $-35 = 21x$.
12. Solve for x when $-3.9x = -15.6$.
13. Find the value x that satisfies $2x - 7x = 20$.
14. Find the value of x that satisfies $-6x - 3x = -7$.
15. Find the value of x that satisfies $\frac{3}{5}x = 39$.
16. We have said that if $a = b$ and $c \neq 0$, then $ac = bc$. Why is it important that $c \neq 0$? What would happen if we tried to solve an equation by multiplying both sides by zero?
17. We have said that if $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$. Why is it important that $c \neq 0$? What would happen if we tried to solve an equation by dividing both sides by zero?
18. In an average year, worldwide, there are 20 earthquakes of magnitude 7 on the Richter scale. If next year is predicted to be an exceptional year, and the number of earthquakes of magnitude 7 is expected to increase by 35%, about how many earthquakes of magnitude 7 can be expected?
19. Solve for x when $8x = 48 + 2x$.
20. Solve for x when $5x = 22 + 3x$.
21. Solve for x when $-6x = -27 + 3x$.
22. Solve for x when $72 - 4x = -12x$.
23. Solve for x when $9x - 5 = 7x + 43$.
24. Solve for x when $6(3x + 2) - 8 = -2$.
25. Solve for x when $7x - 3(5 - x) = 10$.
26. Solve for x when $5(x - 3) + 5 = 3(x + 2)$.
27. Solve for x when $3(2z - 4) - 4(z + 5) = 6$.
28. Solve for x when $\frac{2}{3}x = \frac{1}{15}x + \frac{3}{5}$.
29. Solve for x when $\frac{x}{2} + \frac{x}{5} = \frac{7}{10}$.
30. Solve for x when $20 - \frac{1}{3}x = \frac{1}{2}x$.
31. Is 4 a solution to $\frac{1}{2}(y - 2) + 2 = \frac{3}{8}(3y - 4)$?

32. Solve for x when $0.3x - 0.2(3 - 5x) = -0.5(x - 6)$.

33. Solve for x when $\frac{4}{5}x - \frac{2}{3} = \frac{3x+1}{2}$.

34. Solve for x when $\frac{4}{7}x + \frac{1}{3} = \frac{3x-2}{14}$.

35. Solve for x when $-1 + 5(x - 2) = 12x + 3 - 7x$.

36. Solve for x when $9(x + 3) - 6 = 24 - 2x - 3 + 11x$.

37. Suppose that the variable x must satisfy all of these conditions: $x \leq 2$, $x > -3$, $x < \frac{5}{2}$, $x \geq -\frac{5}{2}$. Graph on a number line the region that satisfies all these conditions.

38. Solve and graph on a number line the x values that satisfy $-4 + 5x < -3x + 8$.

39. Solve and graph on a number line the x values that satisfy $\frac{5x}{6} - 5 > \frac{x}{6} - 9$.

40. To pass a course with a B grade, a student must have an average of 80 or greater. A student's grade on three tests are 75, 83, and 86. Solve the inequality $\frac{75 + 83 + 86 + x}{4} \geq 80$ to find out what score the student must get on the final test to get a B average or better.

41. A computer hard drive is typically rated to work for temperatures between 30C and 50C. Write this condition as an inequality using both interval notation and set notation. Then, using the fact that $T_C = \frac{5}{9}(T_F - 32)$, find the temperature range the drive will work under in degrees Fahrenheit.

42. Solve the literal equation $2x + 2w = A$ for w.

43. Solve the literal equation $A = \frac{1}{2}bh$ for h.

44. Solve the literal equation $PV = nRT$ for T.

45. Solve the literal equation $A = 2x^2 + 4xh$ for h.

46. Solve the literal equation $W = \frac{RTs}{D(1 - vp)}$ for D.

47. Solve the literal equation $W = \frac{RTs}{D(1 - vp)}$ for T.

48. Solve the literal equation $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$ for V_1.

49. A number is doubled and then increased by seven. The result is ninety-three. What is the original number?

50. Six less than five times a number is the same as seven times the number. What is the number?

51. Brad is a waiter, and he gets paid \$5.75 per hour, and he can keep his tips. He knows his tips average \$8.80 per table. If he worked an eight-hour shift and took home \$169.20, how many tables did he serve?

52. On May 18, 1990 the fastest speed of any national railroad was achieved by the French high speed train *Train á Grande Vitess* as it traveled over a distance from Cortalain to Tours, France. A commentator said that this speed was so fast that if it continued at that rate, the train would travel 6404 miles in 20 hours. How fast did the train travel on that date?

53. Two trains leave a train station at the same time. One train travels east at 50 mph. The other train travels west at 55mph. In how many hours will the two trains be 315 miles apart?

54. In warmer climates, approximate temperature predictions can be made by counting the number of chirps a cricket makes during a minute. The temperature (in Fahrenheit) decreased by 40 is equivalent to one-forth of the number of cricket chirps in a minute.

    (a) Write an equation for this relationship.
    (b) Approximately how many chirps per minute should be recorded if the temperature is 90 F?
    (c) If a person recorded 48 cricket chirps in a minute, what would the temperature be?

Solutions

1.

$$\begin{aligned} -12 + x - 3 &= 15 - 18 + 9 \\ x - 15 &= 6 && \text{(arithmetic to simplify each side)} \\ x - 15 + 15 &= 6 + 15 && \text{(addition principle)} \\ x &= 21 \end{aligned}$$

2.

$$\begin{aligned} -19 + x - 7 &= 20 - 42 + 10 \\ x - 26 &= -12 \\ x - 26 + 26 &= -12 + 26 \\ x &= 14 \end{aligned}$$

3. Check if $x = 5$ is a solution by substitution:

$$\begin{aligned} -7 + (5) &= 2 \\ -2 &= 2 \end{aligned}$$ False, so $x = 5$ is not a solution.

Get correct solution:

$$\begin{aligned} -7 + x &= 2 \\ -7 + x + 7 &= 2 + 7 \\ x &= 9 \end{aligned}$$

4. Check if $x = -8$ is a solution by substitution:

$$\begin{aligned} -39 &= (-8) - 47 \\ -39 &= -55 \end{aligned}$$ False, so $x = -8$ is not a solution.

Get correct solution:

$$\begin{aligned} -39 &= x - 47 \\ -39 + 47 &= x - 47 + 47 \\ 8 &= x \end{aligned}$$

5.

$$\begin{aligned}
\frac{2}{3}+x &= \frac{1}{6}+\frac{1}{4}\\
\frac{2}{3}+x &= \frac{2}{12}+\frac{3}{12}\\
\frac{2}{3}+x &= \frac{5}{12}\\
\frac{2}{3}+x-\frac{2}{3} &= \frac{5}{12}-\frac{2}{3}\\
x &= \frac{5}{12}-\frac{8}{12}\\
x &= \frac{-3}{12}=-\frac{1}{4}
\end{aligned}$$

6.

$$\begin{aligned}
\frac{5}{12}-\frac{5}{6} &= x-\frac{3}{2}\\
\frac{5}{12}-\frac{10}{12} &= x-\frac{3}{2}\\
-\frac{5}{12} &= x-\frac{3}{2}\\
-\frac{5}{12}+\frac{3}{2} &= x-\frac{3}{2}+\frac{3}{2}\\
-\frac{5}{12}+\frac{18}{12} &= x\\
\frac{13}{12} &= x
\end{aligned}$$

7. $\frac{8}{413} = 0.019 = 1.9\% < 3\%$, so they met their goal.

8. The total distance is $90-(-27) = 117$ feet.

9. Let x be the amount of money in the account before Trev pays his bill.

$$\begin{aligned}
x-\$49.99 &= -\$35.07\\
x-\$49.99+\$49.99 &= -\$35.07+\$49.99\\
x &= \$14.92
\end{aligned}$$

10.

$$\begin{aligned}
9\cdot\frac{1}{9}x &= 4\cdot 9\\
x &= 36
\end{aligned}$$

11.

$$\begin{aligned} -35 &= 21x \\ \frac{1}{21}\cdot(-35) &= \frac{1}{21}\cdot 21x \\ -\frac{\cancel{7}\cdot 5}{\cancel{7}\cdot 3} &= x \\ -\frac{5}{3} &= x \end{aligned}$$

12.

$$\begin{aligned} \frac{1}{-3.9}\cdot(-3.9x) &= \frac{1}{-3.9}\cdot(-15.6) \\ x &= \frac{15.6}{3.9} = 4 \end{aligned}$$

13.

$$\begin{aligned} 2x - 7x &= 20 && \text{(collect like terms)} \\ -5x &= 20 \\ \frac{\cancel{-5}x}{\cancel{-5}} &= \frac{20}{-5} && \text{(division principal)} \\ x &= -4 && \text{(cancel common factors)} \end{aligned}$$

14.

$$\begin{aligned} -6x - 3x &= -7 \\ -9x &= -7 \\ \frac{1}{-9}\cdot(-9x) &= \frac{1}{-9}\cdot(-7) \\ x &= \frac{7}{9} \end{aligned}$$

15.

$$\begin{aligned} \frac{3}{5}x &= 39 \\ \frac{5}{3}\cdot\left(\frac{3}{5}x\right) &= \frac{5}{3}\cdot 39 \\ x &= \frac{39\cdot 5}{3} = \frac{\cancel{3}\cdot 13\cdot 5}{\cancel{3}} = 65 \end{aligned}$$

16. When we solve an equation, we are performing algebraic steps to obtain equivalent equations until we arrive at $x =$ something. When we multiply an equation by zero, we <u>do not</u> have an equivalent equation, since the new equation is immediately satisfied.

Consider the following

$$\begin{aligned} 3 &= 4 \text{ is false} \\ 0 \cdot 3 &= 0 \cdot 4 \text{ multiply by zero} \\ 0 &= 0 \text{ simplify, and we get a true statement!} \end{aligned}$$

The first and last statements are not equivalent.

17. As soon as we divide something by zero, we get an undefined quantity and have to stop.

18. We need a number that is 35% larger than 20.

    35% of 20 is $0.35 \cdot 20 = 7$.

    Expect $20 + 7 = 27$ earthquakes of magnitude 7 of more next year.

19.

$$\begin{aligned} 8x &= 48 + 2x \\ 8x - 2x &= 48 + 2x - 2x \\ 6x &= 48 \\ \frac{1}{6} \cdot 6x &= \frac{1}{6} \cdot 48 \\ x &= 8 \end{aligned}$$

20.

$$\begin{aligned} 5x &= 22 + 3x \\ 5x - 3x &= 22 + 3x - 3x \\ 2x &= 22 \\ \frac{1}{2} \cdot 2x &= \frac{1}{2} \cdot 22 \\ x &= 11 \end{aligned}$$

21.

$$\begin{aligned} -6x &= -27 + 3x \\ -6x - 3x &= -27 + 3x - 3x \\ -9x &= -27 \\ \frac{1}{-9} \cdot (-9x) &= \frac{1}{-9} \cdot (-27) \\ x &= 3 \end{aligned}$$

22.

$$\begin{aligned} 72 - 4x &= -12x \\ 72 - 4x + 4x &= -12x + 4x \\ 72 &= -8x \\ \frac{1}{-8} \cdot (72) &= \frac{1}{-8} \cdot (-8x) \\ -9 &= x \end{aligned}$$

23.

$$\begin{aligned} 9x - 5 &= 7x + 43 \\ 9x - 5 + 5 - 7x &= 7x + 43 + 5 - 7x \\ 2x &= 48 \\ \frac{1}{2} \cdot (2x) &= \frac{1}{2} \cdot (48) \\ x &= 24 \end{aligned}$$

24.

$$\begin{aligned} 6(3x + 2) - 8 &= -2 \\ 18x + 12 - 8 &= -2 && \text{(distribute)} \\ 18x + 4 &= -2 && \text{(simplify)} \\ 18x + 4 - 4 &= -2 - 4 && \text{(addition principle)} \\ 18x &= -6 && \text{(simplify)} \\ \frac{1}{18} \cdot (18x) &= \frac{1}{18} \cdot (-6) && \text{(multiplication principle)} \\ x &= -\frac{1}{3} && \text{(simplify)} \end{aligned}$$

25.

$$\begin{aligned} 7x - 3(5 - x) &= 10 \\ 7x - 15 + 3x &= 10 \\ 10x - 15 &= 10 \\ 10x - 15 + 15 &= 10 + 15 \\ 10x &= 25 \\ \frac{1}{10} \cdot (10x) &= \frac{1}{10} \cdot (25) \\ x &= \frac{5}{2} \end{aligned}$$

26.

$$\begin{aligned}
5(x-3)+5 &= 3(x+2)\\
5x-15+5 &= 3x+6\\
5x-10 &= 3x+6\\
5x-10-3x+10 &= 3x+6-3x+10\\
2x &= 16\\
\frac{1}{2}\cdot 2x &= \frac{1}{2}\cdot 16\\
x &= 8
\end{aligned}$$

27.

$$\begin{aligned}
3(2z-4)-4(z+5) &= 6\\
6z-12-4z-20 &= 6\\
2z-32 &= 6\\
2z-32+32 &= 6+32\\
2z &= 38\\
\frac{1}{2}\cdot 2z &= \frac{1}{2}\cdot 38\\
z &= 19
\end{aligned}$$

28. The LCD (lowest common denominator) is 15, so multiply the equation by 15 to remove the fractions.

$$\begin{aligned}
\frac{2}{3}x &= \frac{1}{15}x+\frac{3}{5}\\
15\cdot\left(\frac{2}{3}x\right) &= 15\cdot\left(\frac{1}{15}x+\frac{3}{5}\right)\\
10x &= 15\cdot\frac{1}{15}x+15\cdot\frac{3}{5} && \text{(distribute)}\\
10x &= x+9 && \text{(simplify)}\\
10x-x &= x+9-x && \text{(addition principle)}\\
9x &= 9 && \text{(combine like terms)}\\
\frac{1}{9}\cdot 9x &= \frac{1}{9}\cdot 9 && \text{(multiplication principle)}\\
x &= 1 && \text{(simplify)}
\end{aligned}$$

29. LCD is 10.

$$\begin{aligned}
\frac{x}{2}+\frac{x}{5} &= \frac{7}{10}\\
10\cdot\left(\frac{x}{2}+\frac{x}{5}\right) &= 10\cdot\frac{7}{10}\\
10\cdot\frac{x}{2}+10\cdot\frac{x}{5} &= 7\\
5x+2x &= 7\\
7x &= 7\\
\frac{1}{7}\cdot 7x &= \frac{1}{7}\cdot 7\\
x &= 1
\end{aligned}$$

30. LCD is 6.

$$\begin{aligned}
20-\frac{1}{3}x &= \frac{1}{2}x\\
6\cdot\left(20-\frac{1}{3}x\right) &= 6\cdot\frac{1}{2}x\\
6\cdot 20-6\cdot\frac{1}{3}x &= 3x\\
120-2x &= 3x\\
120-2x+2x &= 3x+2x\\
120 &= 5x\\
\frac{1}{5}\cdot 120 &= \frac{1}{5}\cdot 5x\\
24 &= x
\end{aligned}$$

31. You could substitute $y=4$ to check, but I am going to solve it instead. LCD is 8.

$$\begin{aligned}
\frac{1}{2}(y-2)+2 &= \frac{3}{8}(3y-4)\\
8\cdot\left(\frac{1}{2}(y-2)+2\right) &= 8\cdot\frac{3}{8}(3y-4)\\
8\cdot\frac{1}{2}(y-2)+8\cdot 2 &= 3(3y-4)\\
4(y-2)+16 &= 9y-12\\
4y-8+16 &= 9y-12\\
4y+8 &= 9y-12\\
4y+8-9y-8 &= 9y-12-9y-8\\
-5y &= -20\\
\frac{1}{-5}\cdot(-5y) &= \frac{1}{-5}\cdot(-20)\\
y &= 4
\end{aligned}$$

32.

$$\begin{aligned}
0.3x - 0.2(3 - 5x) &= -0.5(x - 6) \\
0.3x - 0.6 + x &= -0.5x + 3 \\
1.3x - 0.6 &= -0.5x + 3 \\
1.3x - 0.6 + 0.5x + 0.6 &= -0.5x + 3 + 0.5x + 0.6 \\
1.8x &= 3.6 \\
\frac{1}{1.8} \cdot 1.8x &= \frac{1}{1.8} \cdot 3.6 \\
x &= 2
\end{aligned}$$

33. LCD is 30.

$$\begin{aligned}
\frac{4}{5}x - \frac{2}{3} &= \frac{3x+1}{2} \\
30 \cdot \left(\frac{4}{5}x - \frac{2}{3}\right) &= 30 \cdot \frac{3x+1}{2} \\
30 \cdot \frac{4}{5}x - 30 \cdot \frac{2}{3} &= 30 \cdot \frac{1}{2} \cdot (3x+1)
\end{aligned}$$

Note in above I wrote $\frac{3x+1}{2}$ as $\frac{1}{2} \cdot (3x+1)$. Doing this helps reduce errors!

$$\begin{aligned}
24x - 20 &= 15 \cdot (3x+1) \\
24x - 20 &= 45x + 15 \\
24x - 20 - 45x + 20 &= 45x + 15 - 45x + 20 \\
-21x &= 35 \\
\frac{1}{-21} \cdot (-21x) &= \frac{1}{-21} \cdot 35 \\
x &= -\frac{35}{21} = -\frac{5}{3}
\end{aligned}$$

34. LCD is 42.

$$\begin{aligned}
\frac{4}{7}x+\frac{1}{3} &= \frac{3x-2}{14}\\
\frac{4}{7}x+\frac{1}{3} &= \frac{1}{14}(3x-2)\\
42\cdot\left(\frac{4}{7}x+\frac{1}{3}\right) &= 42\cdot\frac{1}{14}(3x-2)\\
42\cdot\frac{4}{7}x+42\cdot\frac{1}{3} &= 3(3x-2)\\
24x+14 &= 3(3x-2)\\
24x+14 &= 9x-6\\
24x+14-9x-14 &= 9x-6-9x-14\\
15x &= -20\\
\frac{1}{15}\cdot 15x &= \frac{1}{15}\cdot(-20)\\
x &= -\frac{20}{15}=-\frac{4}{3}
\end{aligned}$$

35.

$$\begin{aligned}
-1+5(x-2) &= 12x+3-7x\\
-1+5x-10 &= 5x+3\\
5x-9-5x &= 5x+3-5x\\
-9 &= 3
\end{aligned}$$

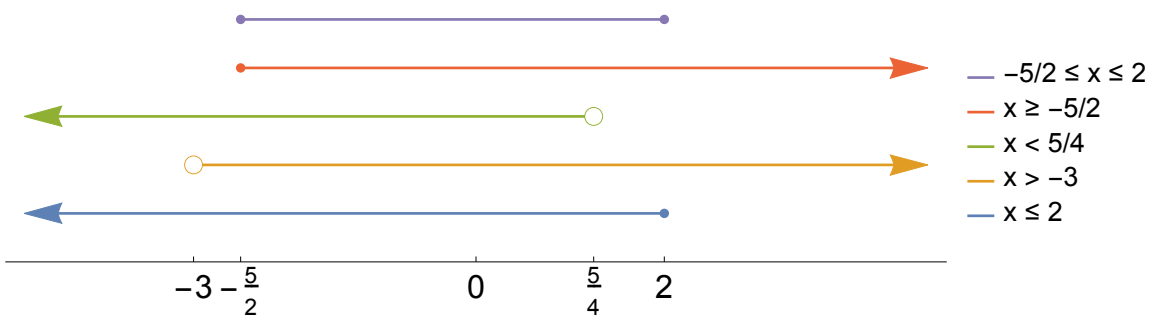
We have to interpret what we have found. Since -9 never equals 3, the equation is never true no matter what value of x we put in. This means the equation has no solution.

36.

$$\begin{aligned}
9(x+3)-6 &= 24-2x-3+11x\\
9x+27-6 &= 21+9x\\
9x+21 &= 21+9x\\
9x+21-9x &= 21+9x-9x\\
21 &= 21
\end{aligned}$$

We have to interpret what we have found. Since 21 is always equal to 21, the equation is true for any value of x that we try. Therefore, there are an infinite number of solutions.

37. The best way to solve this is to stack the graphs of the inequalities and identify the region of overlap. From the sketch below we see that all the inequalities are satisfied if $-\frac{5}{2}\le x\le 2$.



38.

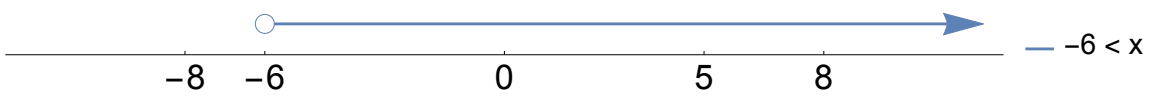
$$
\begin{aligned}
-4+5x &< -3x+8 && \text{(addition principle)}\\
-4+5x+4 &< -3x+8+4 && \text{(collect like terms)}\\
5x+3x &< -3x+12+3x && \text{(addition principle)}\\
8x &< 12 && \text{(collect like terms)}\\
\frac{8x}{8} &< \frac{12}{8} && \text{(division principle)}\\
x &< \frac{12}{8}\\
x &< \frac{3}{2}
\end{aligned}
$$

39.

$$
\begin{aligned}
\frac{5x}{6}-5 &> \frac{x}{6}-9\\
\frac{5x}{6}-5+\mathbf{9}-\frac{\mathbf{5x}}{\mathbf{6}} &> \frac{x}{6}-9+\mathbf{9}-\frac{\mathbf{5x}}{\mathbf{6}} && \text{(addition principle)}\\
4 &> \frac{x-5x}{6}\\
4 &> \frac{-2x}{3} && \text{(collect like terms)}\\
\frac{\mathbf{-3}}{\mathbf{2}}\cdot 4 &< \frac{\mathbf{-3}}{\mathbf{2}}\cdot\frac{-2x}{3} && \text{(multiplication principle)}
\end{aligned}
$$

Since we are multiplying by a negative number, change direction of inequality.

$$-6 < x$$



40.

$$\begin{aligned}\frac{75+83+86+x}{4} &\geq 80\\ 244+x &\geq 320\\ x &\geq 320-244\\ x &\geq 76\end{aligned}$$

41. Interval notation: $30 \leq T_C \leq 50$.

Set notation: $T_C \in [30, 50]$.

Note: the symbol $\in$ can be read as "is in the interval".

$$\begin{aligned}30 \leq T_C &\leq 50\\ 30 \leq \frac{5}{9}(T_F-32) &\leq 50\\ \frac{9}{5}\cdot 30 \leq \frac{9}{5}\cdot\frac{5}{9}(T_F-32) &\leq \frac{9}{5}\cdot 50\\ 54 \leq T_F-32 &\leq 90\\ 54+32 \leq T_F-32+32 &\leq 90+32\\ 86 \leq T_F &\leq 122\end{aligned}$$

It is safe to operate your hard drive between temperatures of 86F and 122F.

42.

$$\begin{aligned}\cancel{2x}+2w\ \mathbf{\cancel{-2x}} &= A\ \mathbf{-2x} && \text{(addition principle)}\\ 2w &= A-2x && \text{(cancel)}\\ \frac{\cancel{2}w}{\cancel{2}} &= \frac{A-2x}{2} && \text{(division principle)}\\ w &= \frac{A-2x}{2} && \text{(cancel common factor)}\end{aligned}$$

43.

$$\begin{aligned}\mathbf{2}A &= \mathbf{\cancel{2}}\cdot\frac{1}{\cancel{2}}bh && \text{(multiplication principle)}\\ 2A &= bh && \text{(cancel common factor)}\\ \frac{2A}{\mathbf{b}} &= \frac{\cancel{b}h}{\mathbf{\cancel{b}}} && \text{(division principle)}\\ \frac{2A}{b} &= h && \text{(cancel common factor)}\end{aligned}$$

44.

$$\begin{aligned}\frac{PV}{\mathbf{nR}} &= \frac{\cancel{nR}T}{\mathbf{\cancel{nR}}} && \text{(division principle)}\\ \frac{PV}{nR} &= T && \text{(cancel common factors)}\end{aligned}$$

45.

$$\begin{aligned}
A - 2x^2 &= 2x^2 + 4xh - 2x^2 && \text{(addition principle)}\\
A - 2x^2 &= 4xh && \text{(cancel)}\\
\frac{A - 2x^2}{4x} &= \frac{4xh}{4x} && \text{(division principle)}\\
\frac{A - 2x^2}{4x} &= h && \text{(cancel common factors)}
\end{aligned}$$

46.

$$\begin{aligned}
DW &= D \cdot \frac{RTs}{D(1 - vp)} && \text{(multiplication principle)}\\
DW &= \frac{RTs}{(1 - vp)} && \text{(cancel common factors)}\\
\frac{DW}{W} &= \frac{RTs}{W(1 - vp)} && \text{(division property)}\\
D &= \frac{RTs}{W(1 - vp)} && \text{(cancel common factor)}
\end{aligned}$$

47.

$$\begin{aligned}
D(1 - vp)W &= D(1 - vp) \cdot \frac{RTs}{D(1 - vp)} && \text{(multiplication principle)}\\
DW(1 - vp) &= RTs && \text{(cancel common factors, use commutativity on LHS)}\\
\frac{DW(1 - vp)}{Rs} &= \frac{RTs}{Rs} && \text{(division principle)}\\
\frac{DW(1 - vp)}{Rs} &= T && \text{(cancel common factors)}
\end{aligned}$$

48.

$$\begin{aligned}
T_1 \cdot \frac{P_1V_1}{T_1} &= T_1 \cdot \frac{P_2V_2}{T_2} && \text{(multiplication principle)}\\
P_1V_1 &= \frac{P_2V_2T_1}{T_2} && \text{(cancel common factors)}\\
\frac{P_1V_1}{P_1} &= \frac{P_2V_2T_1}{P_1T_2} && \text{(division principle)}\\
V_1 &= \frac{P_2V_2T_1}{P_1T_2} && \text{(cancel common factors)}
\end{aligned}$$

49. We are looking for a number, let's call it x.

The number is doubled ($2x$) and then increased by seven ($2x+7$). The result is ninety-three ($2x+7=93$).

$$\begin{aligned} 2x+7 &= 93 \\ 2x &= 86 \\ x &= 43 \end{aligned}$$

The number is 43.

Is seven more than two times forty-three equal to ninety-three?

$$2(43)+7 = 86+7 = 93\checkmark$$

50. We are looking for a number, let's call it x.

Six less than five times a number: $5x-6$

Seven times the same number: $7x$

These things are the same: $5x-6=7x$.

$$\begin{aligned} 5x-6 &= 7x \\ -2x &= 6 \\ x &= -3 \end{aligned}$$

The number is -3. Check:

$$5(-3)-6 \stackrel{?}{=} 7(-3) \Rightarrow -21 \stackrel{?}{=} -21\checkmark$$

51. **Gather Facts.**

Brad is paid \$5.75 per hour.
His tips average \$8.80 per table.
He worked 8 hours.
He took home \$169.20.

Assign variables.

We need to know the number of tables Brad waited on, so let that be x.

Write an equation.

The amount of money he earns from tips is $\$8.80x$.
The amount of money he earned in salary in 8 hours is $\$5.75 \times 8 = \$46$.
This money must add up to \$169.20: $8.80x+46=169.20$.

Solve and state the answer. Now, we can solve the equation:

$$\begin{aligned} 8.80x + \cancel{46} \ \textbf{−}\cancel{\textbf{46}} &= 169.20 \ \textbf{−46} \\ \frac{\cancel{8.80}x}{\cancel{\textbf{8.80}}} &= \frac{123.20}{\textbf{8.80}} \\ x &= 14 \end{aligned}$$

Brad waited on 14 tables.

Check. If he waited on 14 tables he would have earned $\$5.75 \times 8 + 14 \times \$8.80 = \$169.20\checkmark$.

52. **Gather Facts.**

Train would have traveled 6404 miles in 20 hours.
The distance is 6404 miles.
The time is 20 hours.

Assign variables.

We need to know the speed of the train, so let that be x.

Write an equation.

Speed is given by the formula $x = \frac{\text{distance}}{\text{time}}$.

Solve and state the answer.

$$x = \frac{\text{distance}}{\text{time}}$$
$$x = \frac{6404 \text{ miles}}{20 \text{ hours}}$$
$$x = 320.2 \frac{\text{miles}}{\text{hours}}$$

The speed of the train was 320.2 mph.

Check. In 20 hours, a train traveling at 320.2 miles per hour would travel a distance of $20 \times 320.2 = 6404$ miles. ✓

53. **Gather Facts.**

In 1 hours, EAST bound Train would have traveled 50×1 miles.
In 2 hours, EAST bound Train would have traveled 50×2 miles.
In t hours, EAST bound Train would have traveled $50t$ miles.
In t hours, WEST bound Train would have traveled $55t$ miles.

Assign variables.

We need to know the time it takes for something to happen, so let that be t (hours).

Write an equation.

The distance between the two trains at time t is $50t + 55t$ miles.
We want the distance to be 315 miles.
Set the two expressions for the distance equal to get an equation: $50t + 55t = 315$.

Solve and state the answer.

$$50t + 55t = 315 \qquad \text{(collect like terms)}$$
$$\frac{\cancel{105}t}{\cancel{105}} = \frac{315}{105} \qquad \text{(division principle)}$$
$$t = 3$$

After 3 hours the trains will be 315 miles apart.

Check. After 3 hours, the EAST bound train will be $50(3) = 150$ miles from the station, and the WEST bound train will be $55(3) = 165$ miles from the station, so they will be $150 + 165 = 315$ miles apart. ✓

54. **Assign variables.**

Let the number of cricket chirps in a minute be be x.
Let the temperature be T (in degrees Fahrenheit).

Write an equation.

The temperature decreased by forty is $T - 40$.
One-forth the number of chirps in a minute is $\frac{1}{4}x$.
These are equal: $T - 40 = \frac{1}{4}x$.

Solve and state the answer.

(a) The relationship between the number of cricket chirps per minute and the temperature is

$$T - 40 = \frac{1}{4}x$$

(b) If the temperature is $T = 90$ F, the number of chirps per minute will satisfy

$$\begin{aligned} 90 - 40 &= \frac{1}{4}x && \text{(substitute } T = 90 \text{ into equation)} \\ 50 \times 4 &= \frac{1}{\cancel{4}}x \times \cancel{4} && \text{(multiplication principle)} \\ 200 &= x \end{aligned}$$

There should be 200 chirps per minute when $T = 90$ F.

(c) If the number of chirps is 148, the temperature will satisfy

$$\begin{aligned} T - 40 &= \frac{1}{4}(148) && \text{(substitute } x = 148 \text{ into equation)} \\ T - \cancel{40} + \cancel{40} &= 37 + 40 && \text{(addition principle)} \\ T &= 77 \end{aligned}$$

The temperature is 77 F when the number of chirps per minute is 148.