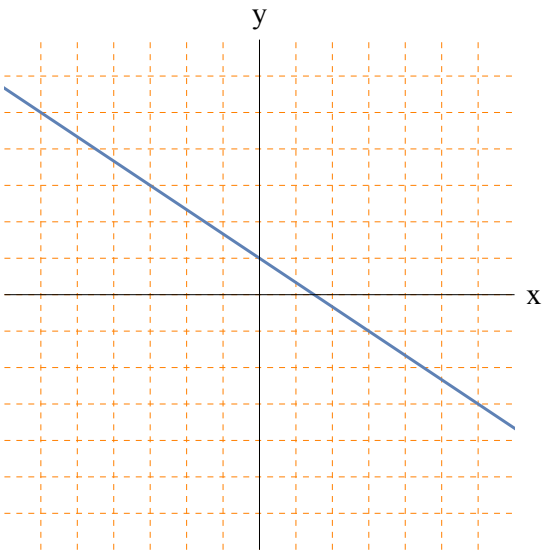
Questions

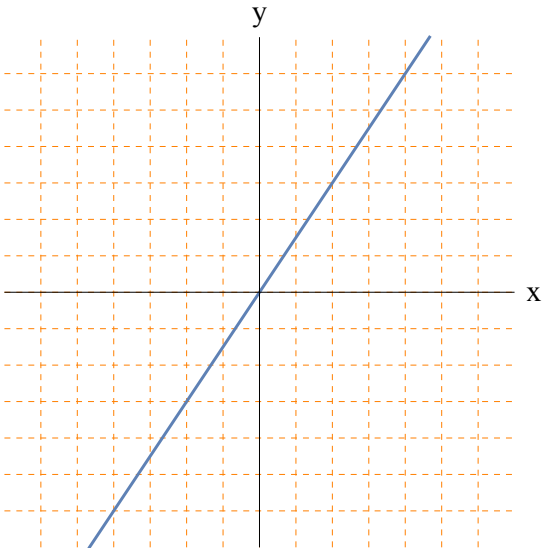
1. Find the midpoint and distance between the points $(1/3, 4/5)$ and $(7/6, 2/3)$.
2. Sketch $y = -2x + 1$. Find the value of y when $x = 0$, $x = -2$, and $x = 1$.
3. Sketch $y = 2x - 5$. Find the value of y when $x = 0$, $x = 2$, and $x = 4$.
4. Sketch $y = 3x + 2$. Find the value of y when $x = -1$, $x = 0$, and $x = 1$.
5. Sketch $4x + 3y = 12$.
6. Sketch $3x + 2y = 6$.
7. Sketch $y = 6 - 2x$.
8. Sketch $x - 6 = 2y$.
9. Sketch $y - 2 = 3y$.
10. Sketch $2x + 9 = 5x$.
11. Sketch $2x + 5y - 2 = -12$.
12. Find the slope of the straight line that passes through the points $(4, 1)$ and $(6, 7)$.
13. Find the slope of the straight line that passes through the points $(11, 2)$ and $(5, 14)$.
14. Find the slope of the straight line that passes through the points $(-6, -5)$ and $(2, -7)$.
15. Write the equation for a straight line in slope-intercept form with slope $m = \frac{2}{3}$ and y-intercept $(0, 5)$.
16. Write the equation for a straight line in slope-intercept form with slope $m = 5$ and y-intercept $(0, -6)$.
17. Write the equation for a straight line in slope-intercept form with slope $m = \frac{2}{3}$ and y-intercept $(0, 1/2)$.
18. Sketch the straight line $y = mx + b$ where $m = \frac{1}{3}$ and $b = -2$.
19. Sketch the straight line $y = mx + b$ where $m = -\frac{3}{2}$ and $b = 4$.
20. Sketch the straight line $y = 3x$.
21. A line has a slope of $\frac{11}{4}$. What is the slope of a line parallel to it? What is the slope of a line perpendicular to it?
22. A line has equation $y = \frac{3}{5}x - 5$. What is the slope of a line parallel to it? What is the slope of a line perpendicular to it?
23. During the years from 1980 to 2005 the total income for the U.S. federal budget can be approximated by the equation $y = 14(4x + 35)$, where x is the number of years since 1980 and y is the amount of money in billions of dollars (source: U.S. Office of Management and Budget).

    Write the equation in slope-intercept form. Find the slope and y-intercept. What is the meaning of the slope in this situation?
24. Find the equation of the line that passes through the point $(5, -3)$ and has slope $m = -\frac{2}{5}$.
25. Find the equation of the line that passes through the points $\left(1, \frac{5}{6}\right)$ and $\left(3, \frac{3}{2}\right)$.
26. Find the equation of the line that passes through the points $(2, 0)$ and $\left(\frac{3}{2}, \frac{1}{2}\right)$.
27. Find the equation of the line that passes through the point $(4, 3)$ and has slope $m = -2$.
28. Find the equation of the line that passes through the points $(1, -8)$ and $(2, -14)$.

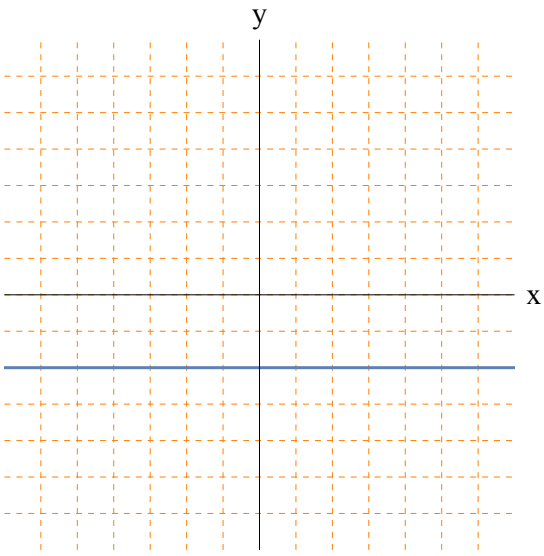
29. Write the equation of the line given below.



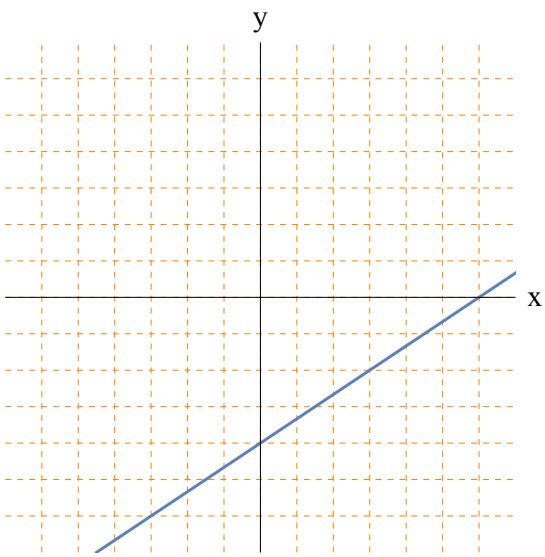
30. Write the equation of the line given below.



31. Write the equation of the line given below.



32. Write the equation of the line given below.



33. Sketch the linear inequality $3x \leq 4y + 2$.

34. Sketch the linear inequality $\frac{1}{5}y + \frac{2}{3}x \leq 1$.

35. Sketch the linear inequality $4x + 2y > 3$.

36. Sketch the linear inequality $4x - \frac{5}{7}y \geq 6$.

37. Sketch the linear inequality $x + y > 0$.

38. Sketch the linear inequality $x - y \leq 0$.

Solutions

1. Find the midpoint and distance between the points $(1/3, 4/5)$ and $(7/6, 2/3)$.

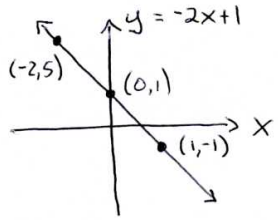
   Let $(x_1, y_1) = (1/3, 4/5)$ and $(x_2, y_2) = (7/6, 2/3)$. You will get the same results if you have these reversed.

$$\begin{aligned}
\text{midpoint} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) && \text{(midpoint formula)}\\
&= \left(\frac{1/3 + 7/6}{2}, \frac{4/5 + 2/3}{2}\right) && \text{(substitute in the values)}\\
&= \left(\frac{2/6 + 7/6}{2}, \frac{12/15 + 10/15}{2}\right) && \text{(common denominators)}\\
&= \left(\frac{9/6}{2}, \frac{22/15}{2}\right) && \text{(add fractions)}\\
&= \left(\frac{9}{6} \cdot \frac{1}{2}, \frac{22}{15} \cdot \frac{1}{2}\right) && \text{(change division to multiplication)}\\
&= \left(\frac{9}{12}, \frac{11}{15}\right) && \text{(multiplication of fractions)}
\end{aligned}$$

$$\begin{aligned}
\text{distance} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{(distance formula)}\\
&= \sqrt{(7/6 - 1/3)^2 + (2/3 - 4/5)^2} && \text{(substitute in values)}\\
&= \sqrt{(7/6 - 2/6)^2 + (10/15 - 12/15)^2} && \text{(common denominators)}\\
&= \sqrt{(5/6)^2 + (-2/15)^2} && \text{(subtract fractions)}\\
&= \sqrt{12/36 + 4/225} && \text{(square)}\\
&= \sqrt{0.333333 + 0.0177778} && \text{(calculator to simplify)}\\
&= 0.592546
\end{aligned}$$

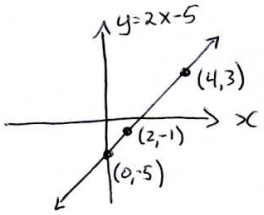
2. $y = -2x + 1$

   When $x = 0 \Rightarrow y = -2(0) + 1 = 1$, so the ordered pair is $(0, 1)$.
   When $x = -2 \Rightarrow y = -2(-2) + 1 = 5$, so the ordered pair is $(-2, 5)$.
   When $x = 1 \Rightarrow y = -2(1) + 1 = -1$, so the ordered pair is $(1, -1)$.



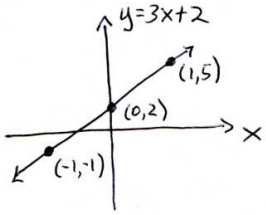
3. $y = 2x - 5$

   When $x = 0 \Rightarrow y = 2(0) - 5 = -5$, so the ordered pair is $(0, -5)$.
   When $x = 2 \Rightarrow y = 2(2) - 5 = -1$, so the ordered pair is $(2, -1)$.
   When $x = 4 \Rightarrow y = 2(4) - 5 = 3$, so the ordered pair is $(4, 3)$.



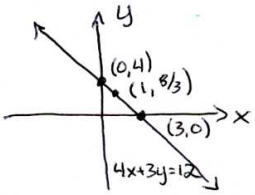
4. $y = 3x + 2$

When $x = -1 \Rightarrow y = 3(-1) + 2 = -1$, so the ordered pair is $(-1, -1)$.
When $x = 0 \Rightarrow y = 3(0) + 2 = 2$, so the ordered pair is $(0, 2)$.
When $x = 1 \Rightarrow y = 3(1) + 2 = 5$, so the ordered pair is $(1, 5)$.



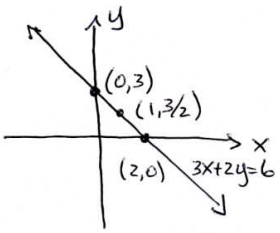
5. $4x + 3y = 12$

When $x = 0 \Rightarrow 4(0) + 3y = 12 \Rightarrow y = 4$ so the ordered pair is $(0, 4)$.
When $y = 0 \Rightarrow 4x + 3(0) = 12 \Rightarrow x = 3$ so the ordered pair is $(3, 0)$.
When $x = 1 \Rightarrow 4(1) + 3y = 12 \Rightarrow y = 8/3$ so the ordered pair is $(1, 8/3)$.



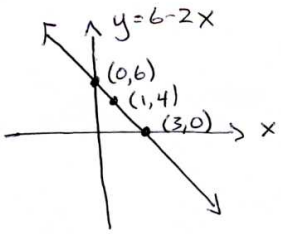
6. $3x + 2y = 6$

When $x = 0 \Rightarrow 3(0) + 2y = 6 \Rightarrow y = 3$ so the ordered pair is $(0, 3)$.
When $y = 0 \Rightarrow 3x + 2(0) = 6 \Rightarrow x = 2$ so the ordered pair is $(2, 0)$.
When $x = 1 \Rightarrow 3(1) + 2y = 6 \Rightarrow y = 3/2$ so the ordered pair is $(1, 3/2)$.



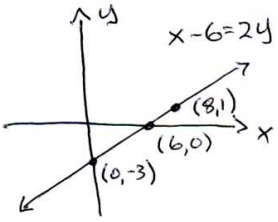
7. $y = 6 - 2x$

When $x = 0 \Rightarrow y = 6 - 2(0) \Rightarrow y = 6$ so the ordered pair is $(0, 6)$.
When $y = 0 \Rightarrow (0) = 6 - 2x \Rightarrow x = 3$ so the ordered pair is $(3, 0)$.
When $x = 1 \Rightarrow y = 6 - 2(1) \Rightarrow y = 4$ so the ordered pair is $(1, 4)$.

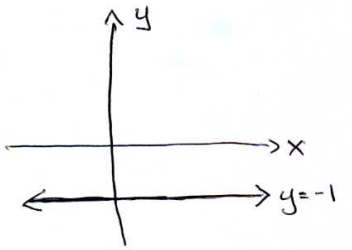


8. $x - 6 = 2y$

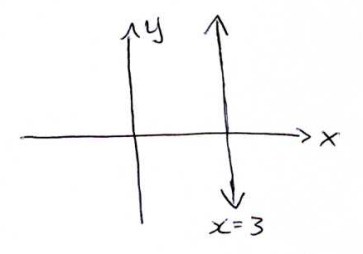
When $x = 0 \Rightarrow (0) - 6 = 2y \Rightarrow y = -3$ so the ordered pair is $(0, -3)$.
When $y = 0 \Rightarrow x - 6 = 2(0) \Rightarrow x = 6$ so the ordered pair is $(6, 0)$.
When $x = 8 \Rightarrow (8) - 6 = 2y \Rightarrow y = 1$ so the ordered pair is $(8, 1)$.



9. $y - 2 = 3y$. There is no x in the equation. Simplification shows this is a horizontal line, $y = -1$.

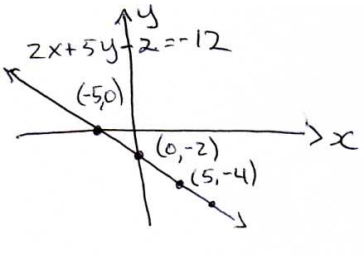


10. $2x + 9 = 5x$. There is no y in the equation. Simplification shows this is a vertical line, $x = 3$.



11. $2x + 5y - 2 = -12$
$\Rightarrow 2x + 5y = -10$

When $x = 0 \Rightarrow 2(0) + 5y = -10 \Rightarrow y = -2$ so the ordered pair is $(0, -2)$.
When $y = 0 \Rightarrow 2x + 5(0) = -10 \Rightarrow x = -5$ so the ordered pair is $(-5, 0)$.
When $x = 5 \Rightarrow 2(5) + 5y = -10 \Rightarrow y = -4$ so the ordered pair is $(5, -4)$.



12. slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{1-7}{4-6} = \dfrac{-6}{-2} = 3.$

13. slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{2-14}{11-5} = \dfrac{-12}{6} = -2.$

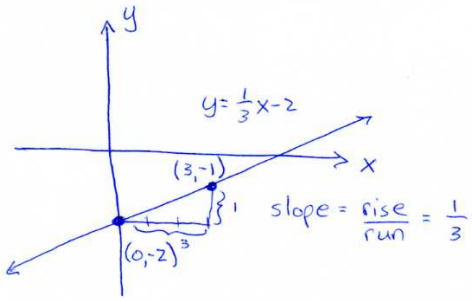
14. slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{-5-(-7)}{-6-2} = \dfrac{2}{-8} = -\dfrac{1}{4}.$

15. $y = \dfrac{2}{3}x + 5.$

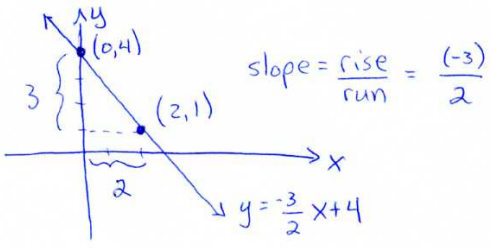
16. $y = 5x - 6.$

17. $y = \dfrac{2}{3}x + \dfrac{1}{2}.$

18. $y = \dfrac{1}{3}x - 2.$

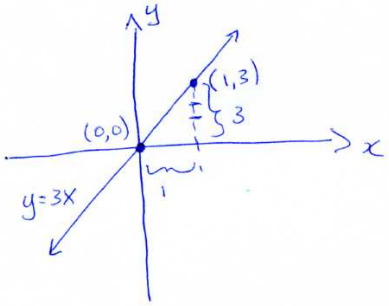


19. $y = -\dfrac{3}{2}x + 4.$



20. $y = 3x$.

Intercept is $b = 0$. Slope is $m = \dfrac{\text{rise}}{\text{run}} = \dfrac{3}{1}$.



21. Parallel: $\dfrac{11}{4}$. Perpendicular: $-\dfrac{4}{11}$.

22. Parallel: $\dfrac{3}{5}$. Perpendicular: $-\dfrac{5}{3}$.

23. $y = 14(4x + 35) = 56x + 490 \Rightarrow$ slope $= 56$ and y-intercept is $(0, 490)$.

The slope is the amount of increase in income of the federal budget in billions of dollars per year.

Aside: This equation is not as good as it could be, since x represents the number of years since 1980. The equation would be improved if the independent variable represented the year. We can make this change by introducing a change in variables.

Let z be the year. Then $z = 1980 + x$. Therefore, $x = z - 1980$. The equation becomes

$$y = 56x + 490$$
$$y = 56(z - 1980) + 490 = 56z - 110,390$$

What was the federal budget in 1987? Answer: $y = 56z - 110,390 = 56(1987) - 110,390 = 882$ billion dollars. This is the same answer you get if you use $y = 56x + 490$ with $x = 7$.

24. Use the slope-point equation of a line.

$$y - y_1 = m(x - x_1)$$
$$y - (-3) = -\frac{2}{5}(x - 5)$$
$$y + 3 = -\frac{2}{5}x + 2$$
$$y = -\frac{2}{5}x - 1$$

25. slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{\frac{5}{6} - \frac{3}{2}}{1-3} = \dfrac{\left(-\frac{4}{6}\right)}{-2} = \dfrac{1}{-2} \cdot \left(-\dfrac{4}{6}\right) = \dfrac{1}{3}.$

Now use the slope-point equation of a line.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - \frac{5}{6} &= \frac{1}{3}(x - 1) \\ y - \frac{5}{6} &= \frac{1}{3}x - \frac{1}{3} \\ y &= \frac{1}{3}x - \frac{1}{3} + \frac{5}{6} \\ y &= \frac{1}{3}x - \frac{2}{6} + \frac{5}{6} \\ y &= \frac{1}{3}x + \frac{3}{6} \\ y &= \frac{1}{3}x + \frac{1}{2} \end{aligned}$$

26. slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{0 - \frac{1}{2}}{2 - \frac{3}{2}} = \dfrac{\left(-\frac{1}{2}\right)}{\left(\frac{1}{2}\right)} = \dfrac{2}{1} \cdot \left(-\dfrac{1}{2}\right) = -1.$

Now use the slope-point equation of a line.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 0 &= -1(x - 2) \\ y &= -x + 2 \end{aligned}$$

27. Use the slope-point equation of a line.

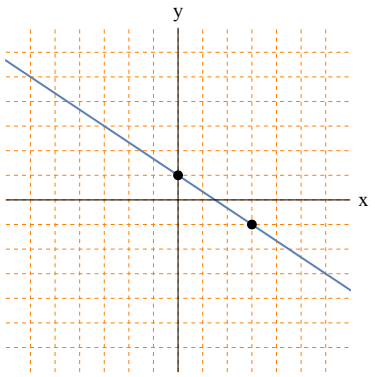
$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (3) &= -2(x - 4) \\ y - 3 &= -2x + 8 \\ y &= -2x + 8 + 3 \\ y &= -2x + 11 \end{aligned}$$

28. slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{-8 - (-14)}{1 - 2} = \dfrac{-8 + 14}{-1} = \dfrac{6}{-1} = -6.$

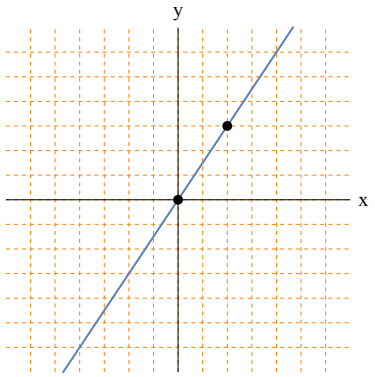
Now use the slope-point equation of a line.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-8) &= -6(x - 1) \\ y + 8 &= -6x + 6 \\ y &= -6x + 6 - 8 \\ y &= -6x - 2 \end{aligned}$$

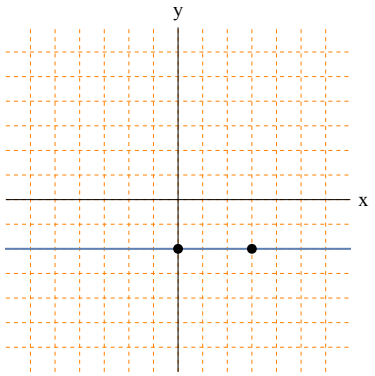
29. You need to be able to read these off the sketch. Look for two points that the line crosses a grid line intersection. Two points: $(0, 1)$ and $(3, -1)$.
Rise $= -2$, Run $= 3$.
slope $= \dfrac{\text{rise}}{\text{run}} = \dfrac{-2}{3} = -\dfrac{2}{3}$.
y-intercept $b = 1$.
$y = mx + b \Rightarrow y = -\dfrac{2}{3}x + 1$.



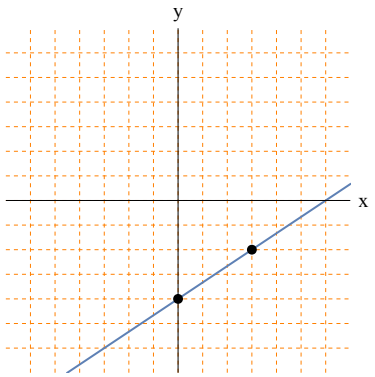
30. Two points: $(0, 0)$ and $(2, 3)$.
Rise $= 3$, Run $= 2$.
slope $= \dfrac{\text{rise}}{\text{run}} = \dfrac{3}{2}$.
y-intercept $b = 0$.
$y = mx + b \Rightarrow y = \dfrac{3}{2}x$.



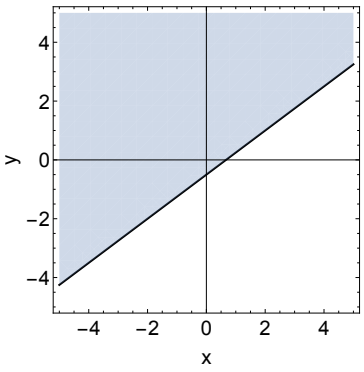
31. This is a horizontal line, so it's equation is just $y = -2$.



32. Two points: $(0, -4)$ and $(3, -2)$.
Rise $= 2$, Run $= 3$.
slope $= \dfrac{\text{rise}}{\text{run}} = \dfrac{2}{3}$.
y-intercept $b = -4$.
$y = mx + b \Rightarrow y = \dfrac{2}{3}x - 4$.



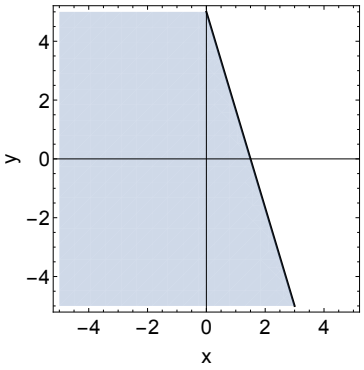
33. Sketch the linear inequality $3x \leq 4y + 2$.



Sketch $3x = 4y + 2$:
If $x = 0$ then $3(0) = 4y + 2$, so $y = -1/2$. Point is $(0, -1/2)$.
If $y = 0$ then $3x = 4(0) + 2$, so $x = 2/3$. Point is $(2/3, 0)$.

Test Point $(0, 0)$: $3(0) \leq 4(0) + 2$, so $0 \leq 2$ True, so shade side of line with $(0, 0)$.

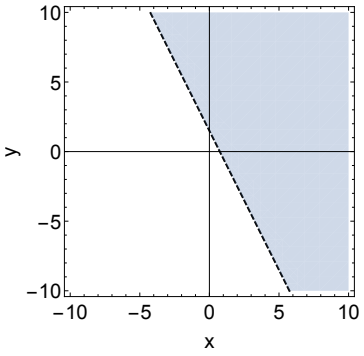
34. Sketch the linear inequality $\dfrac{1}{5}y + \dfrac{2}{3}x \leq 1$.



Sketch $\frac{1}{5}y + \frac{2}{3}x = 1$:
If $x = 0$ then $\frac{1}{5}y + \frac{2}{3}(0) = 1$, so $y = 5$. Point is $(0, 5)$.
If $y = 0$ then $\frac{1}{5}(0) + \frac{2}{3}(x) = 1$, so $x = 3/2$. Point is $(3/2, 0)$.

Test Point $(0, 0)$: $\frac{1}{5}(0) + \frac{2}{3}(0) \leq 1$, so $0 \leq 1$ True, so shade side of line with $(0, 0)$.

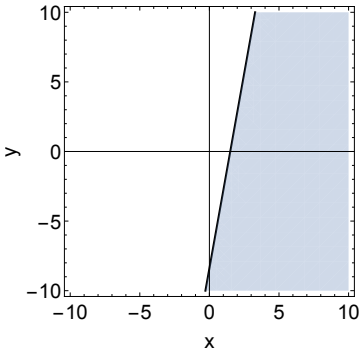
35. Sketch the linear inequality $4x + 2y > 3$.



Sketch $4x + 2y = 3$ (dashed as the line does not satisfy inequality):
If $x = 0$ then $4(0) + 2y = 3$, so $y = 3/2$. Point is $(0, 3/2)$.
If $y = 0$ then $4x + 2(0) = 3$, so $x = 3/4$. Point is $(3/4, 0)$.

Test Point $(0, 0)$: $4(0) + 2(0) > 3$, so $0 > 3$ False, so shade side of line opposite $(0, 0)$.

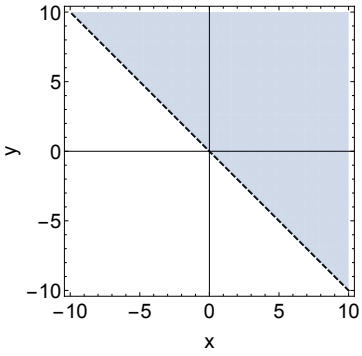
36. Sketch the linear inequality $4x - \dfrac{5}{7}y \geq 6$.



Sketch $4x - \frac{5}{7}y = 6$:
If $x = 0$ then $4(0) - \frac{5}{7}y = 6$, so $y = -42/5$. Point is $(0, -42/5)$.
If $y = 0$ then $4x - \frac{5}{7}(0) = 6$, so $x = 6/4 = 3/2$. Point is $(3/2, 0)$.

Test Point $(0, 0)$: $4(0) - \frac{5}{7}(0) \geq 6$, so $0 \geq 6$ False, so shade side of line opposite $(0, 0)$.

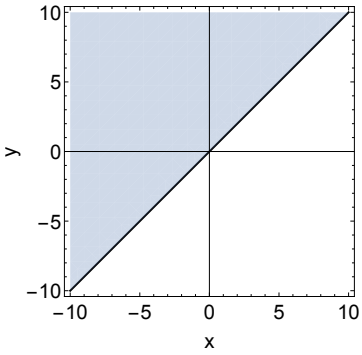
37. Sketch the linear inequality $x + y > 0$.



Sketch $x + y = 0$ (dashed as the line does not satisfy inequality):
If $x = 0$ then $0 + y = 0$, so $y = 0$. Point is $(0, 0)$.
If $x = 1$ then $1 + y = 0$, so $y = -1$. Point is $(1, -1)$.

Test Point $(1, 0)$: $1 + 0 > 0$, so $1 > 0$ True, so shade side of line with $(1, 0)$.

38. Sketch the linear inequality $x - y \leq 0$.



Sketch $x - y = 0$:
If $x = 0$ then $0 - y = 0$, so $y = 0$. Point is $(0, 0)$.
If $x = 1$ then $1 - y = 0$, so $y = 1$. Point is $(1, 1)$.

Test Point $(1, 0)$: $1 - 0 \leq 0$, so $1 \leq 0$ False, so shade side of line opposite $(1, 0)$.