To solve by sketching, we should use graph paper, or be very careful with the scale as we sketch by hand.

Whenever you are reading a solution off of a graph you need to be very precise! That's why we prefer algebra to solve systems of equations.

You can sketch using techniques from previous sections (slope and y-intercept, or getting two points).

Questions

1. Solve the system by graphing:

$$\begin{aligned} 3x + y &= 2 \\ 2x - y &= 3 \end{aligned}$$

2. Solve the system by graphing:

$$\begin{aligned} y &= \frac{1}{3}x - 2 \\ -x + 3y &= 9 \end{aligned}$$

3. Solve the system by graphing:

$$\begin{aligned} y &= -2x + 5 \\ 3y + 6x &= 15 \end{aligned}$$

4. Solve the system algebraically, using any method you like:

$$\begin{aligned} 4x + 3y &= 9 \\ 3y + 6 &= x \end{aligned}$$

5. Solve the system algebraically, using any method you like:

$$\begin{aligned} 5x + 2y &= 5 \\ 3x + y &= 4 \end{aligned}$$

6. Solve the system algebraically, using any method you like:

$$\begin{aligned} 4x + 2y &= 4 \\ 3x + y &= 4 \end{aligned}$$

7. Solve the system algebraically, using any method you like:

$$\begin{aligned} 9x + 2y &= 2 \\ 3x + 5y &= 5 \end{aligned}$$

8. Solve the system algebraically, using any method you like:

$$\begin{aligned} 6s - 3t &= 1 \\ 5s + 6t &= 15 \end{aligned}$$

9. Solve the system algebraically, using any method you like:

$$\begin{aligned} 0.2x &= 0.1y - 1.2 \\ 2x - y &= 6 \end{aligned}$$

10. Ninety-eight passengers rode in a train from Boston to Denver. Tickets for regular coach seats cost \$120, and sleeper car seats cost \$290. The receipts from the trip totaled \$19,750. How many passengers purchased each type of ticket?

11. Ventex makes auto radar detectors. Ventex has found that its basic model requires 3 hours of manufacturing for the inside components and 2 hours for the housing and controls. Its advanced model requires 5 hours to manufacture the inside components and 3 hours for the housing and controls. This week, the production division has available 1050 hours for producing inside components and 660 hours for housing and controls. How many detectors of each type can be made?

12. Against the wind a small plane flew 210 miles in 1 hour and 10 minutes. The return trip took only 50 minutes. What was the speed of teh wind? What was the speed of the plane in still air?

13. Tim Duncun scored 32 points in an NBA basketball game without scoring any 3-point shots. He scored 21 times. He made several free throws worth 1 point each and several regular shots from the floor, which were worth 2 points each. How many free throws did he make? How many 2-point shots did he make?

14. Graph the solution to the system of inequalities:

$$\begin{aligned} y &\geq 2x - 1 \\ x + y &\leq 6 \end{aligned}$$

15. Graph the solution to the system of inequalities:

$$\begin{aligned} x + 2y &< 6 \\ y &< 3 \end{aligned}$$

16. Graph the solution to the system of inequalities:

$$\begin{aligned} y &> -3 \\ x &< 2 \end{aligned}$$

17. Graph the solution to the system of inequalities, and find the vertex of the solution:

$$\begin{aligned} x + y &\geq 2 \\ y + 4x &\leq -1 \end{aligned}$$

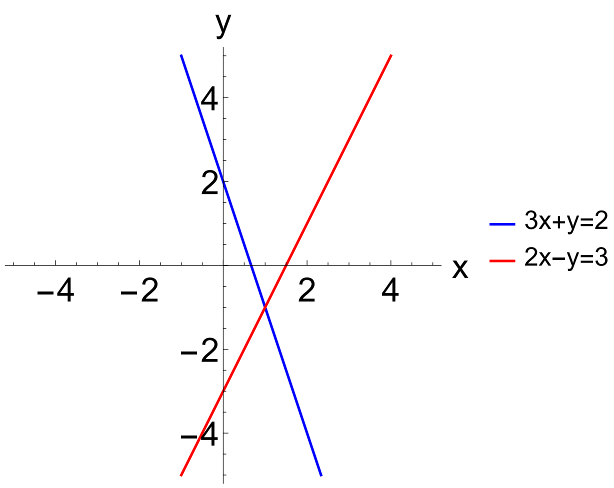
Solutions

1. Sketch $3x + y = 2$:

   When $x = 0 \Rightarrow 3(0) + y = 2 \Rightarrow y = 2,$ so the ordered pair is $(0, 2)$.
   When $y = 0 \Rightarrow 3x + (0) = 2 \Rightarrow x = 2/3,$ so the ordered pair is $(2/3, 0)$.

   Sketch $2x - y = 3$:

   When $x = 0 \Rightarrow 2(0) - y = 3 \Rightarrow y = -3,$ so the ordered pair is $(0, -3)$.
   When $y = 0 \Rightarrow 2x - (0) = 3 \Rightarrow x = 3/2,$ so the ordered pair is $(3/2, 0)$.



   The solution to the system appears to be $(1, -1)$. Check by substituting into the original equations:

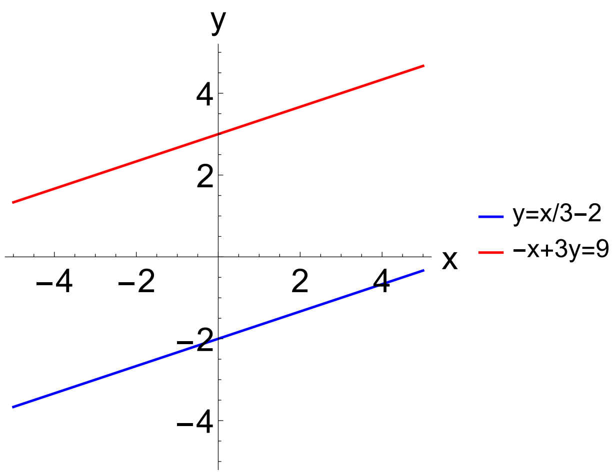
   $3(1) + (-1) = 2$ True
   $2(1) - (-1) = 3$ True

2. Sketch $y = \frac{1}{3}x - 2$:

   When $x = 0 \Rightarrow y = \dfrac{1}{3}(0) - 2 \Rightarrow y = -2,$ so the ordered pair is $(0, -2)$.
   When $y = 0 \Rightarrow 0 = \dfrac{1}{3}x - 2 \Rightarrow x = 6,$ so the ordered pair is $(6, 0)$.

   Sketch $-x + 3y = 9$:

   When $x = 0 \Rightarrow -(0) + 3y = 9 \Rightarrow y = 3,$ so the ordered pair is $(0, 3)$.
   When $y = 0 \Rightarrow -x + 3(0) = 9 \Rightarrow x = -9,$ so the ordered pair is $(-9, 0)$.



The system has no solution, since the lines are parallel. Check by computing the slope of each line (parallel lines have the same slope).

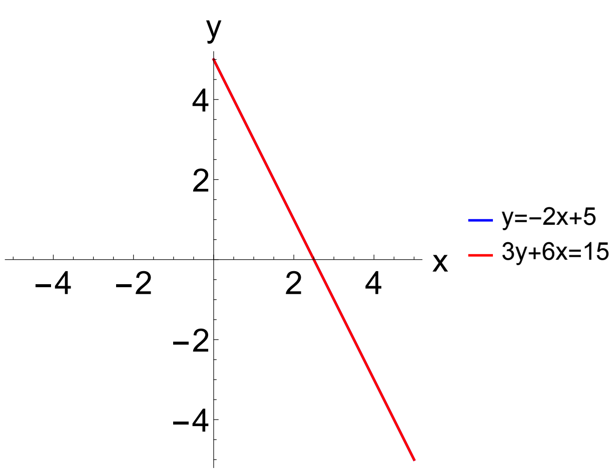
$$y = \frac{1}{3}x - 2 \text{ has slope } m = \frac{1}{3},$$
$$-x + 3y = 9 \Rightarrow y = \frac{1}{3}x + 3 \text{ has slope } m = \frac{1}{3}.$$

3. Sketch $y = -2x + 5$:

   When $x = 0 \Rightarrow y = -2(0) + 5 \Rightarrow y = 5,$ so the ordered pair is $(0, 5)$.
   When $y = 0 \Rightarrow 0 = -2x + 5 \Rightarrow x = 5/2,$ so the ordered pair is $(5/2, 0)$.

   Sketch $3y + 6x = 15$:

   When $x = 0 \Rightarrow 3y + 6(0) = 15 \Rightarrow y = 5,$ so the ordered pair is $(0, 5)$.
   When $y = 0 \Rightarrow 3(0) + 6x = 15 \Rightarrow x = 5/2,$ so the ordered pair is $(5/2, 0)$.



The system has an infinite number of solutions, since the lines are identical. Check by showing the lines have the same equation. We can see that second equation is just the first equation multiplied by 3.

$$y = -2x + 5$$
$$3y + 6x = 15$$

4. Let's use the substitution method.

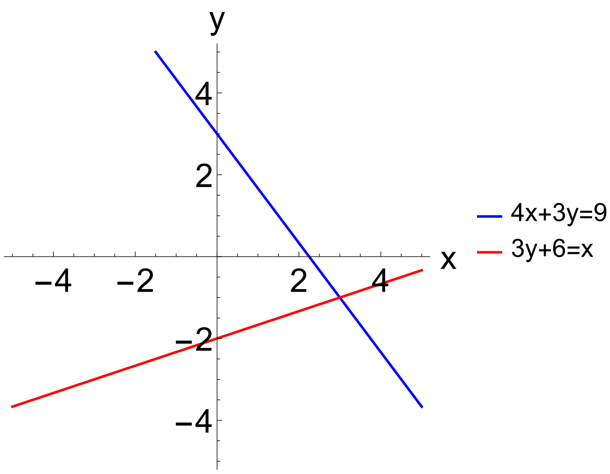
   From the second equation, we can solve for $x = 3y + 6$. Substitute this into the first equation:

$$\begin{aligned} 4\boldsymbol{x} + 3y &= 9 \\ 4\boldsymbol{(3y+6)} + 3y &= 9 && \text{(now, solve for } y\text{)} \\ 12y + \cancel{24} + 3y \boldsymbol{-\cancel{24}} &= 9 \boldsymbol{-24} && \text{(addition principle)} \\ \frac{\cancel{15}y}{\boldsymbol{\cancel{15}}} &= \frac{-15}{\boldsymbol{15}} && \text{(division principle)} \\ y &= -1 \end{aligned}$$

   Now, use this value of y in $x = 3y + 6$ to determine x:

$$\begin{aligned} x &= 3\boldsymbol{y} + 6 \\ x &= 3\boldsymbol{(-1)} + 6 \\ x &= 3 \end{aligned}$$

   The solution to the system is the ordered pair $(3, -1)$. You can check by substituting this back into both original equations. They should both be true when $x = 3$ and $y = -1$. A sketch should also verify this result.



5. Let's use the substitution method.

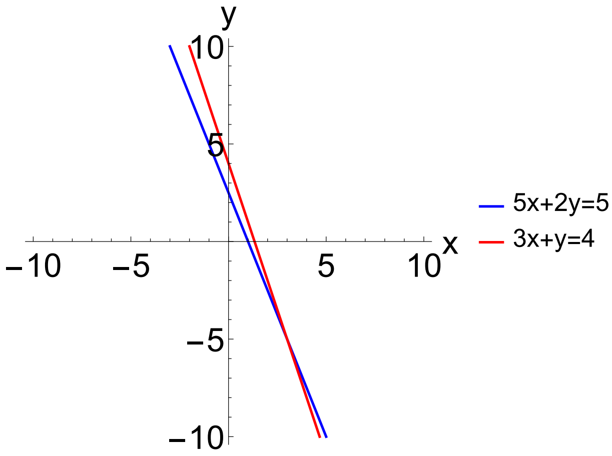
   From the second equation, we can solve for $y = 4 - 3x$. Substitute this into the first equation:

$$\begin{aligned} 5x + 2\boldsymbol{y} &= 5 \\ 5x + 2\boldsymbol{(4-3x)} &= 5 && \text{(now, solve for } x\text{)} \\ 5x + 8 - 6x &= 5 && \text{(distribute 2)} \\ 5x + \cancel{8} - 6x \boldsymbol{-\cancel{8}} &= 5 \boldsymbol{-8} && \text{(addition principle)} \\ -x &= -3 \\ x &= 3 \end{aligned}$$

   Now, use this value of x in $y = 4 - 3x$ to determine y:

$$\begin{aligned} y &= 4 - 3\boldsymbol{x} \\ y &= 4 - 3\boldsymbol{(3)} \\ y &= -5 \end{aligned}$$

The solution to the system is the ordered pair $(3, -5)$. A sketch should also verify this result.



6. Let's use the substitution method.

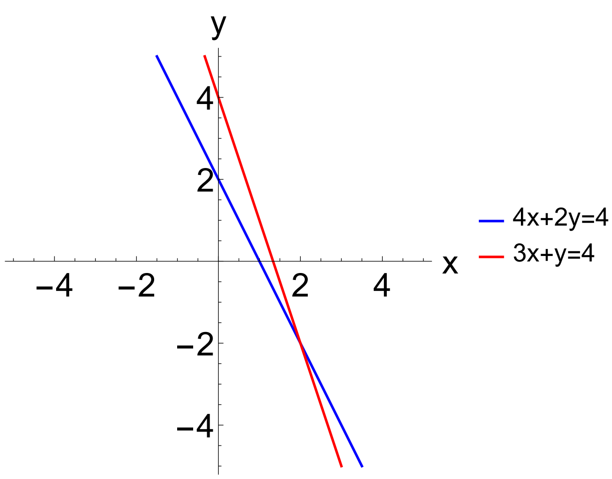
From the second equation, we can solve for $y = 4 - 3x$. Substitute this into the first equation:

$$\begin{aligned} 4x + 2\boldsymbol{y} &= 4 \\ 4x + 2(\mathbf{4 - 3}\boldsymbol{x}) &= 4 && \text{(now, solve for } x\text{)} \\ 4x + 8 - 6x &= 4 && \text{(distribute the 2)} \\ \cancel{8} - 2x \mathbf{-\cancel{8}} &= 4 \mathbf{-8} && \text{(collect like terms, addition principle)} \\ \frac{\cancel{-2}x}{\cancel{\mathbf{-2}}} &= \frac{-4}{\mathbf{-2}} && \text{(division principle)} \\ x &= 2 \end{aligned}$$

Now, use this value of x in $y = 4 - 3x$ to determine y:

$$\begin{aligned} y &= 4 - 3\boldsymbol{x} \\ y &= 4 - 3(\mathbf{2}) \\ y &= -2 \end{aligned}$$

The solution to the system is the ordered pair $(2, -2)$. A sketch should also verify this result.



7. Let's use the elimination method.

   Multiply the second equation by -3 to make the coefficient of x the same in both equations, but with opposite sign.

$$\begin{aligned} \mathbf{9x + 2y} &= \mathbf{2} \\ \mathbf{-9x - 15y} &= \mathbf{-15} \end{aligned}$$

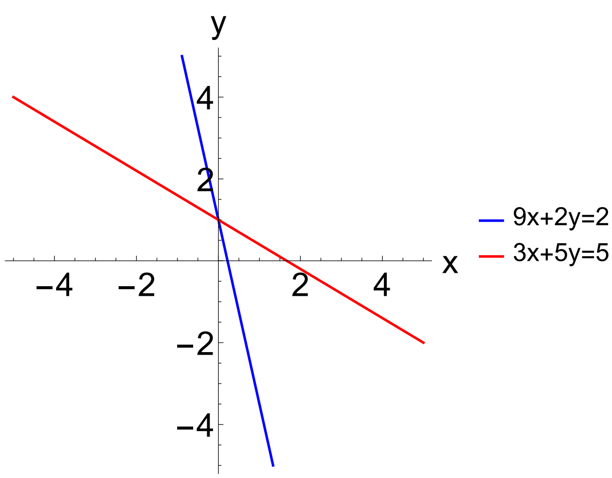
   Now add the two equations to eliminate the x (since $9x - 9x = 0$):

$$\begin{aligned} \cancel{-9x} - 15y + \cancel{9x} + 2y &= -15 + 2 \\ -13y &= -13 && \text{(collect like terms, now solve for } y\text{)} \\ y &= 1 \end{aligned}$$

   Now, use this value of y in any of the earlier equations to determine x:

$$\begin{aligned} 9x + 2\mathbf{y} &= 2 \\ 9x + 2\mathbf{(1)} &= 2 \\ 9x + 2 &= 2 \\ 9x &= 0 \\ x &= 0 \end{aligned}$$

   The solution to the system is the ordered pair $(0, 1)$.



8. Let's use the elimination method.

   Multiply the first equation by 2 to make the coefficient of t the same in both equations, but with opposite sign.

$$\begin{aligned} 12s - 6t &= 2 \\ 5s + 6t &= 15 \end{aligned}$$

   Now add the two equations to eliminate the t (since $-6t + 6t = 0$):

$$\begin{aligned} \mathbf{12s - 6t} &= \mathbf{2} \\ \mathbf{5s + 6t} &= \mathbf{15} \end{aligned}$$

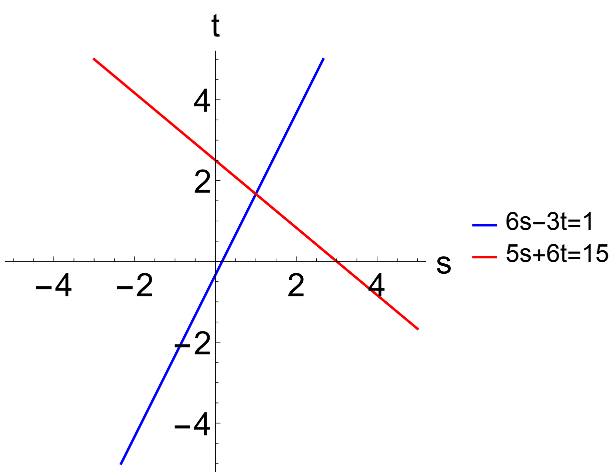
Adding the equations:

$$\begin{aligned} \mathbf{12s} \cancel{\mathbf{-6t}} + \mathbf{5s} \cancel{\mathbf{+6t}} &= \mathbf{2} + \mathbf{15} \\ 17s &= 17 && \text{(now solve for } s\text{)} \\ s &= 1 \end{aligned}$$

Now, use this value of s in any of the earlier equations to determine t:

$$\begin{aligned} 5\boldsymbol{s} + 6t &= 15 \\ 5(\mathbf{1}) + 6t &= 15 \\ \cancel{5} + 6t \mathbf{-\cancel{5}} &= 15 \mathbf{-5} && \text{(addition principle)} \\ \frac{\cancel{6}t}{\cancel{\mathbf{6}}} &= \frac{10}{\mathbf{6}} && \text{(division principle)} \\ t &= \frac{10}{6} = \frac{5}{3} \end{aligned}$$

The solution to the system is the ordered pair $(s, t) = \left(1, \frac{5}{3}\right)$.



9. Let's use the elimination method.

Multiply the first equation by -10 to make the coefficient of x the same in both equations, but with opposite sign.

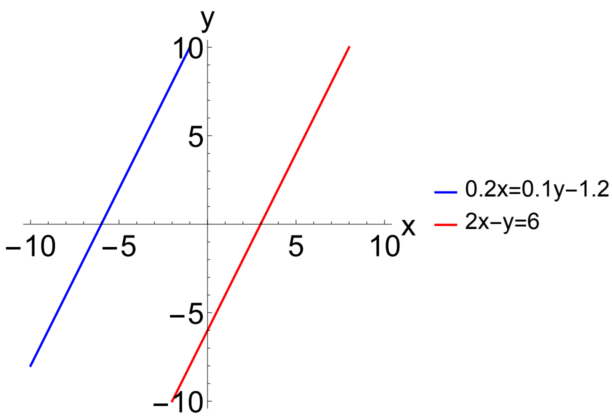
$$\begin{aligned} \mathbf{-2x} &= \mathbf{-y + 12} \\ \mathbf{2x - y} &= \mathbf{6} \end{aligned}$$

Now add the two equations to eliminate the x (since $-2x + 2x = 0$):

$$\begin{aligned} \cancel{\mathbf{-2x}} + \cancel{\mathbf{2x}} \mathbf{- y} &= \mathbf{-y + 12} + \mathbf{6} \\ -\cancel{y} \mathbf{+ \cancel{y}} &= -\cancel{y} + 18 \mathbf{+ \cancel{y}} && \text{(addition principle)} \\ 0 &= 18 \end{aligned}$$

You might think you've made a mistake, but you just need to interpret what you've found.

Since 0 can never equal 18, there is no solution to the system of equations. Graphically, the two equations represent two parallel lines.



10. **Assign variables.**

Let x be the number of coach seats sold (i.e., coach passengers).
Let y the number of sleeper car seats sold (i.e., sleeper passengers).

Gather Facts.

98 passengers.
The cost for coach is \$120.
The cost for sleeper is \$290.
Total collected was \$19,750.

Write equations.

$$\$120x + \$290y = \$19750. \qquad \text{(comes from total receipts)}$$
$$x + y = 98. \qquad \text{(comes from how many passengers)}$$

Solve and state the answer.

System of linear equations in two unknowns.
Solve by substitution, so second equation tells us $x = 98 - y$. Put this into the first equation:

$$120\boldsymbol{x} + 290y = 19750$$
$$120(\mathbf{98 - \boldsymbol{y}}) + 290y = 19750 \qquad \text{(distribute 120)}$$
$$\cancel{11760} - 120y + 290y \mathbf{-\cancel{11760}} = 19750 \mathbf{-11760} \qquad \text{(addition principle)}$$
$$\frac{\cancel{170}y}{\mathbf{\cancel{170}}} = \frac{7990}{\mathbf{170}} \qquad \text{(division principle)}$$
$$y = 47$$

Now, use any earlier equation to solve for x when $y = 47$:

$$x = 98 - \boldsymbol{y}$$
$$x = 98 - \mathbf{47}$$
$$x = 51$$

So there were 47 sleeper seats sold and 51 coach seats sold.

11. **Assign variables.**

Let x be the number of basic models produced this week.
Let y the number of advanced models produced this week.

Gather Facts.

Inside components:

Basic model require 3 hrs manufacturing.
Advanced model require 5 hrs manufacturing.
1050 hours available.

Housing:

Basic model require 2 hrs manufacturing.
Advanced model require 3 hrs manufacturing.
660 hours available.

Write equations.

$$\begin{aligned} 1050 &= 3x + 5y. && \text{(comes from inside components)} \\ 660 &= 2x + 3y. && \text{(comes from housing and controls)} \end{aligned}$$

Solve and state the answer.

System of linear equations in two unknowns.
Solve by substitution, so second equation tells us $x = 330 - \frac{3}{2}y$. Put this into the first equation:

$$\begin{aligned} 1050 &= 3\boldsymbol{x} + 5y \\ 1050 &= 3\left(\mathbf{330 - \frac{3}{2}y}\right) + 5y && \text{(solve for } y\text{)} \\ 1050 \mathbf{-990} &= \cancel{990} - \frac{9}{2}y + 5y \mathbf{\cancel{-990}} && \text{(addition principle)} \\ \mathbf{2\times} 60 &= \cancel{\mathbf{2}}\mathbf{\times}\frac{1}{\cancel{2}}y && \text{(multiplication principle)} \\ 120 &= y \end{aligned}$$

Now, use any earlier equation to solve for x when $y = 120$:

$$\begin{aligned} x &= 330 - \frac{3}{2}\boldsymbol{y} \\ x &= 330 - \frac{3}{2}\mathbf{(120)} \\ x &= 330 - 180 = 150 \end{aligned}$$

So they can make 150 basic models and 120 advanced models.

12. **Assign variables.**

Let speed of plane in still air be x.
Let speed of wind be y.

Gather Facts.

Use the formula speed $= \dfrac{\text{distance}}{\text{time}}$.
Units (my choice) will be miles and hours, so 70 minutes $= 7/6$ hours, and 50 minutes $= 5/6$ hours.

$$\begin{aligned} \text{Speed for plane flying against wind } &= x - y = \frac{210 \text{ miles}}{7/6 \text{ hours}} = 180 \text{ mph} \\ \text{Speed for plane flying with the wind } &= x + y = \frac{210 \text{ miles}}{5/6 \text{ hours}} = 252 \text{ mph} \end{aligned}$$

Write equations.

$$\begin{aligned} x - y &= 180 \\ x + y &= 252 \end{aligned}$$

Solve and state the answer.

System of linear equations in two unknowns.
Solve by substitution, so second equation tells us $x = 252 - y$. Put this into the first equation:

$$\begin{aligned} \boldsymbol{x} - y &= 180 \\ \mathbf{252} - \boldsymbol{y} - y &= 180 && \text{(solve for } y\text{)} \\ \cancel{252} - 2y \, \cancel{\mathbf{-252}} &= 180 \, \mathbf{-252} && \text{(addition principle)} \\ \frac{\cancel{-2}y}{\cancel{\mathbf{-2}}} &= \frac{-72}{\mathbf{-2}} && \text{(division principle)} \\ y &= 36 \end{aligned}$$

Now, use any earlier equation to solve for x when $y = 36$:

$$\begin{aligned} x &= 252 - \boldsymbol{y} \\ x &= 252 - \mathbf{36} = 216 \end{aligned}$$

So the plane had speed in still air of 216 mph, and the wind speed was 36 mph.

13. **Assign variables.**

Let x be the number of free throws.
Let y be the number of regular shots.

Gather Facts and Write Equations.

$$\begin{aligned} \text{Number of times Duncun scored } &= 21 = x + y \\ \text{Total points Duncun scored } &= 32 = x + 2y \end{aligned}$$

Solve equations.

System of linear equations in two unknowns.
Solve by substitution, so second equation tells us $x = 32 - 2y$. Put this into the first equation:

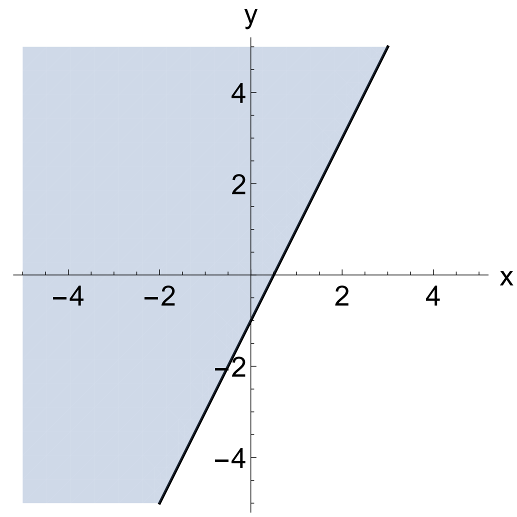
$$\begin{aligned} 21 &= \boldsymbol{x} + y \\ 21 &= \mathbf{32 - 2}\boldsymbol{y} + y \\ 21 &= 32 - y && \text{(collect like terms)} \\ 21 \mathbf{-32} &= \cancel{32} - y \mathbf{-\cancel{32}} && \text{(addition principle)} \\ -11 &= -y \\ 11 &= y \end{aligned}$$

Now, use any earlier equation to solve for x when $y = 11$:

$$\begin{aligned} x &= 32 - 2\boldsymbol{y} \\ x &= 32 - 2\mathbf{(11)} = 10 \end{aligned}$$

So Duncun scored 11 regular shots and 10 free throws.

14. **Sketch** $y \geq 2x - 1$**:**



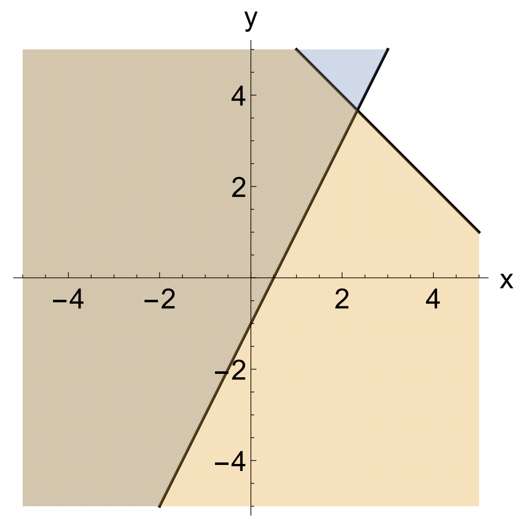
Since line satisfies inequality,
sketch solid line $y = 2x - 1$.

When $x = 0$, then $y = 0 - 1$, so $y = -1$.
Point on line is $(0, -1)$.

When $y = 0$, then $0 = 2x - 1$, so $x = 1/2$.
Point on line is $(1/2, 0)$.

Test point $(0, 0)$: $0 \geq 0 - 1$ is False,
so shade side opposite $(0, 0)$.

Sketch $x + y \leq 6$**:**



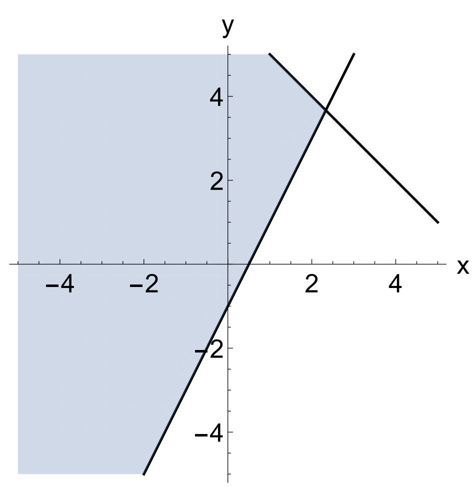
Since line satisfies inequality,
sketch solid line $x + y = 6$.

When $x = 0$, then $0 + y = 6$, so $y = 6$.
Point on line is $(0, 6)$.

When $y = 0$, then $x + 0 = 6$, so $x = 6$.
Point on line is $(6, 0)$.

Test point $(0, 0)$: $0 + 0 \leq 6$ is False,
so shade side opposite $(0, 0)$.

Here is the region that satisfies both inequalities:



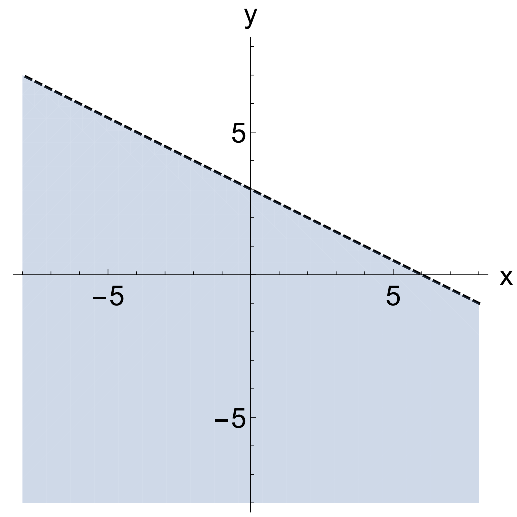
15. **Sketch** $x + 2y < 6$**:**

Since line does not satisfy inequality, sketch dashed line $x + 2y = 6$.

When $x = 0$, then $0 + 2y = 6$, so $y = 3$. Point on line is $(0, 3)$.

When $y = 0$, then $x + 0 = 6$, so $x = 6$. Point on line is $(6, 0)$.

Test point $(0, 0)$: $0 + 0 < 6$ is True, so shade side with $(0, 0)$.

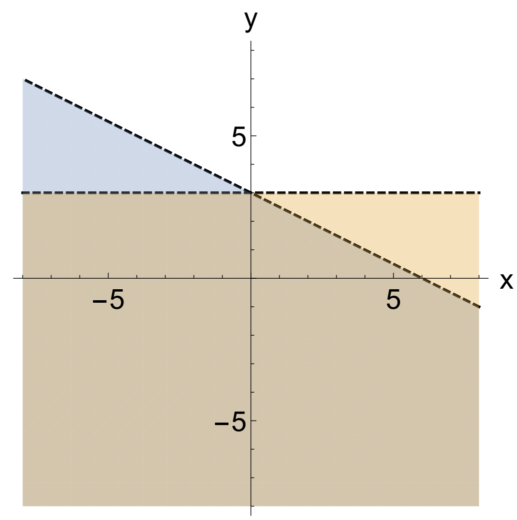


Sketch $y < 3$**:**

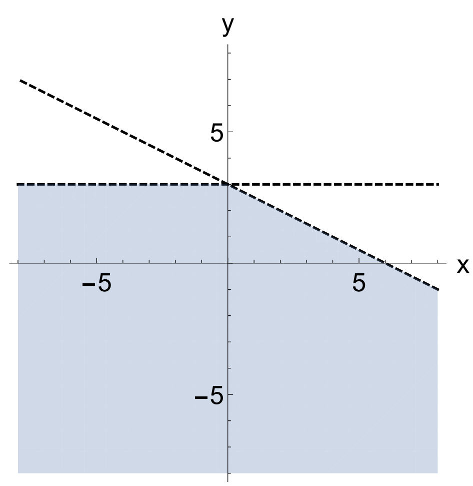
Since line does not satisfy inequality, sketch dashed line $y = 3$.

This is a horizontal line.

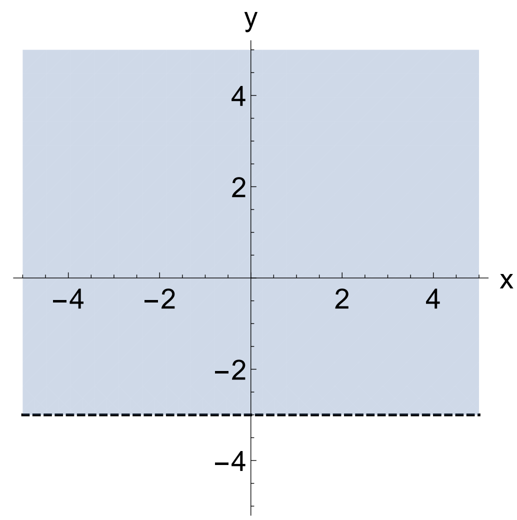
Test point $(0, 0)$: $0 < 3$ is True, so shade side with $(0, 0)$.



Here is the region that satisfies both inequalities:



16. **Sketch** $y > -3$**:**

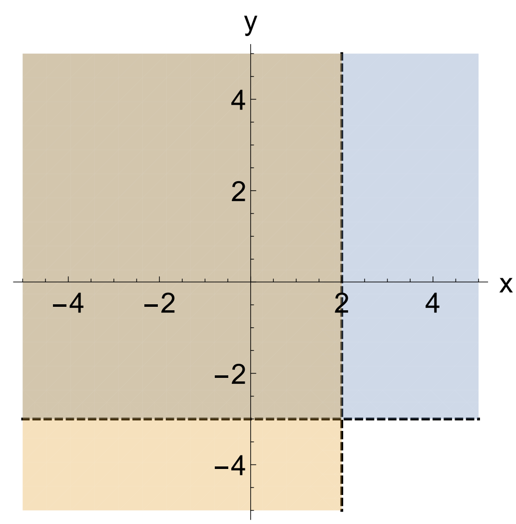


Since line does not satisfy inequality,
sketch dashed line $y = -3$.

This is a horizontal line.

Test point $(0, 0)$: $0 > -3$ is True,
so shade side with $(0, 0)$.

Sketch $x < 2$**:**

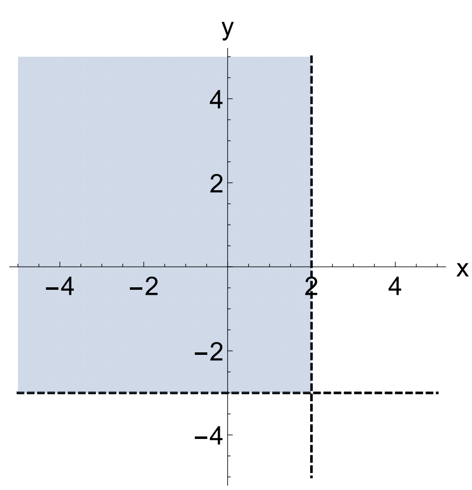


Since line does not satisfy inequality
, sketch dashed line $y = 3$.

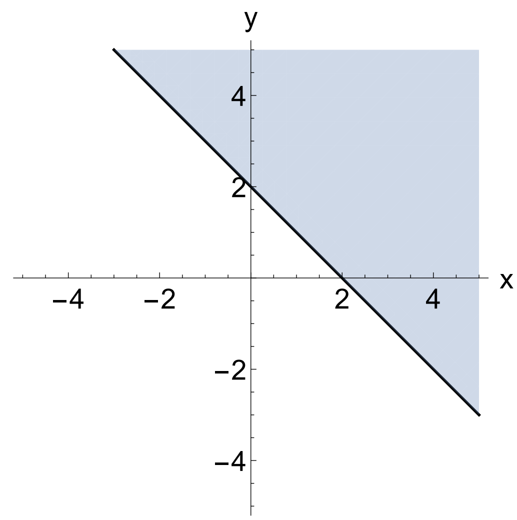
This is a vertical line.

Test point $(0, 0)$: $0 < 2$ is True,
so shade side with $(0, 0)$.

Here is the region that satisfies both inequalities:



17. **Sketch** $x + y \geq 2$**:**



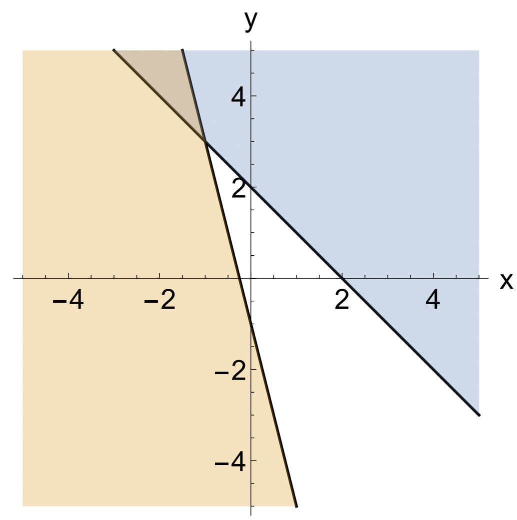
Since line satisfies inequality,
sketch solid line $x + y = 2$.

When $x = 0$, then $0 + y = 2$, so $y = 2$.
Point on line is $(0, 2)$.

When $y = 0$, then $x + 0 = 2$, so $x = 2$.
Point on line is $(2, 0)$.

Test point $(0, 0)$: $0 + 0 \geq 2$ is False,
so shade side opposite $(0, 0)$.

Sketch $y + 4x \leq -1$**:**



Since line satisfies inequality,
sketch solid line $y + 4x = -1$.

When $x = 0$, then $y + 0 = -1$, so $y = -1$.
Point on line is $(0, -1)$.

When $y = 0$, then $0 + 4x = -1$, so $x = -1/4$.
Point on line is $(-1/4, 0)$.

Test point $(0, 0)$: $0 + 0 \leq -1$ is False,
so shade side opposite $(0, 0)$.

Here is the region that satisfies both inequalities: