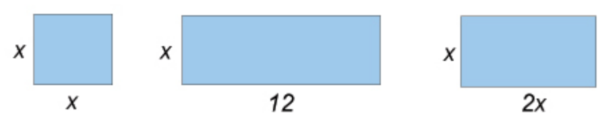
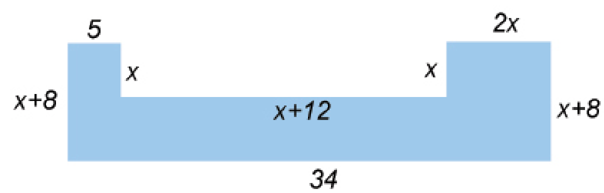
Questions

1. To use the rules of exponents (except for product raised to a power and quotient raised to a power), what must be true of the bases? Write down the rules of exponents.
2. Evaluate $3x^0$ and $(3x)^0$. Why are the results different?
3. Multiply $(7^4)(7^6)$.
4. Multiply $w^{12} \cdot w^{20}$.
5. Multiply $(-16x^2y^4)(-5xy^3)$.
6. Multiply $(-12x^4y)(-7x^5y^3)$.
7. Multiply $(-8x^3y^2)(3xy^5)$.
8. Divide. Assume all variables in denominators are nonzero. $\dfrac{48x^5y^3}{24xy^3}$.
9. Divide. Assume all variables in denominators are nonzero. $\dfrac{45a^4b^3}{15a^4b^2}$.
10. Divide. Assume all variables in denominators are nonzero. $\dfrac{16x^5y}{-32x^2y^3}$.
11. What expression can be multiplied by $-3x^3yz$ to obtain $81x^8y^2z^4$?
12. Simplify $\left(\dfrac{8}{y^5}\right)^2$.
13. Simplify $\left(\dfrac{a^3b}{c^5d}\right)^5$.
14. Simplify $(2x^{-3})^{-3}$.
15. Simplify $(4x^{-4})^{-2}$.
16. Simplify $\dfrac{x^{-2}y^{-3}}{x^4y^{-2}}$.
17. Simplify $\dfrac{a^{-6}b^3}{a^{-2}b^{-5}}$.
18. Write 8.137×10^7 in decimal notation.
19. Write 4.7×10^{-4} in decimal notation.
20. The tip of a 1/3-inch long hour hand on a watch travels at a speed of 0.000 00275 miles per hour. how far has it traveled in a day?
21. Avogadro's number says there are approximately 6.02×10^{23} molercules/mole. How many molecules can one expect in 0.00483 moles?
22. State the degree of the polynomial $5xy^2 - 3x^2y^3$, and whether it is a monomial, binomial, or trinomial.
23. State the degree of the polynomial $7x^3y + 5x^4y^4$, and whether it is a monomial, binomial, or trinomial.
24. Subtract $(2x - 19) - (-3x + 5)$.
25. Subtract $\left(\dfrac{3}{8}x^2 - \dfrac{2}{3}x - 7\right) - \left(\dfrac{2}{3}x^2 - \dfrac{1}{2}x + 2\right)$.
26. Simplify $(3x^4 - 4x^2 - 18) - (2x^4 + 3x^3 + 6)$.
27. Simplify $(2b^3 + 3b - 5) - (-3b^3 + 5b^2 + 7b)$.
28. The lengths and widths of three rectangles are labeled below. Create a polynomial that describes the the sum of the area of these three rectangles.



29. The dimensions of the sides of the following figure are labeled. Create a polynomial that describes the perimeter of the figure.



30. Multiply $-2x(6x^3 - x)$.
31. Multiply $\dfrac{1}{2}(2x + 3x^2 + 5x^3)$.
32. Multiply $(2b^3 - 5b^2 + 3ab)(-3b^2)$.
33. Multiply $(-4x^3 + 6x^2 - 5x)(-7xy^2)$.

34. Multiply $(3x+4)(5x-y)$.
35. Multiply $(4y+1)(5y-3)$.
36. Multiply $(5y+1)(6y-5)$.
37. What is wrong with this multiplication: $(x-2)(-3) = 3x-6$?
38. What is wrong with this multiplication: $-(3x-7) = -3x-7$?
39. Multiply $\left(\frac{1}{3}x+\frac{1}{5}\right)\left(\frac{1}{3}x-\frac{1}{2}\right)$.
40. Multiply $(x+6)(x-6)$.
41. Multiply $(4x-9)(4x+9)$.
42. Multiply $\left(5x-\frac{1}{5}\right)\left(5x+\frac{1}{5}\right)$.
43. Multiply $(6x+5)^2$.
44. Multiply $(8x-3)^2$.
45. Multiply $\left(\frac{3}{5}x-\frac{1}{3}\right)\left(\frac{3}{5}x+\frac{1}{3}\right)$.
46. Multiply $\left(\frac{3}{5}x-\frac{1}{3}\right)^2$.
47. Multiply $(a^2-3a+2)(a^2+4a-3)$.
48. Multiply $(x^2+4x-5)(x^2-3x+4)$.
49. Multiply $(x+7)(3x-2)(x-7)$.
50. Divide $\frac{8x^4-12x^3-4x^2}{4x^2}$.
51. Divide $\frac{49x^8-35x^6-56x^3}{7x^3}$.
52. Divide $\frac{12x^2+19x+5}{3x+1}$.
53. Divide $\frac{4x^3+4x^2-19x-15}{2x+5}$.
54. Divide $\frac{9x^3-30x^2+31x-4}{3x-5}$.
55. Divide $\frac{8x^3+3x-7}{4x-1}$.

Solutions

1. The bases must be the same.

   The rules of exponents are:

   - $x^0 = 1$ if $x \neq 0$ (0^0 is indeterminant and is dealt with in calculus).
   - Product Rule: $x^a \cdot x^b = x^{a+b}$.
   - Quotient Rule: $\dfrac{x^a}{x^b} = x^{a-b}$.
   - Power Rule: $(x^a)^b = x^{ab}$.
   - Product Raised to Power Rule: $(xy)^a = x^a y^a$.
   - Quotient Raised to a Power Rule: $\left(\dfrac{x}{y}\right)^a = \dfrac{x^a}{y^a}$ if $y \neq 0$.
   - Negative Exponent: $x^{-n} = \dfrac{1}{x^n}$, if $x \neq 0$.

2. The results are different because the bases are different (one has base x, the other base $3x$).

$$3x^0 = 3(1) = 3$$
$$(3x)^0 = 1$$

3. $(7^4)(7^6) = 7^{4+6} = 7^{10}$

4. $w^{12} \cdot w^{20} = w^{12+20} = w^{32}$

5. $(-16x^2y^4)(-5xy^3) = 80x^{2+1}y^{4+3} = 80x^3y^7$

6. $(-12x^4y)(-7x^5y^3) = 84x^{4+5}y^{1+3} = 84x^9y^4$

7. $(-8x^3y^2)(3xy^5) = -24x^{3+1}y^{2+5} = -24x^4y^7$

8. $\dfrac{48x^5y^3}{24xy^3} = 2x^{5-1}y^{3-3} = 2x^4y^0 = 2x^4$

9. $\dfrac{45a^4b^3}{15a^4b^2} = 3a^{4-4}b^{3-2} = 3a^0b^1 = 3b$

10. $\dfrac{16x^5y}{-32x^2y^3} = -\dfrac{1}{2}x^{5-2}y^{1-3} = -\dfrac{1}{2}x^3y^{-2} = -\dfrac{x^3}{2y^2}$

11.

$$(\text{expression})(-3x^3yz) = 81x^8y^2z^4$$
$$\text{expression} = \frac{81x^8y^2z^4}{-3x^3yz} = -27x^{8-3}y^{2-1}z^{4-1} = -27x^5yz^3$$

12. $\left(\dfrac{8}{y^5}\right)^2 = \dfrac{8^2}{(y^5)^2} = \dfrac{64}{y^{5\cdot 2}} = \dfrac{64}{y^{10}}$

13. $\left(\dfrac{a^3b}{c^5d}\right)^5 = \dfrac{(a^3b)^5}{(c^5d)^5} = \dfrac{(a^3)^5(b)^5}{(c^5)^5(d)^5} = \dfrac{a^{15}b^5}{c^{25}d^5}$

14. $(2x^{-3})^{-3} = 2^{-3}x^{(-3)(-3)} = \dfrac{1}{2^3}x^9 = \dfrac{1}{8}x^9 = \dfrac{x^9}{8}$

15. $(4x^{-4})^{-2} = 4^{-2}x^{(-4)(-2)} = \dfrac{1}{4^2}x^8 = \dfrac{1}{16}x^8 = \dfrac{x^8}{16}$

16. $\dfrac{x^{-2}y^{-3}}{x^4y^{-2}} = x^{-2-4}y^{-3+2} = x^{-6}y^{-1} = \dfrac{1}{x^6y}$

17. $\dfrac{a^{-6}b^3}{a^{-2}b^{-5}} = a^{-6+2}b^{3+5} = a^{-4}b^8 = \dfrac{b^8}{a^4}$

18. $8.137 \times 10^7 = 81\,370\,000.$ move decimal 7 places to the right.

19. $4.7 \times 10^{-4} = 0.000\,47$ move decimal 4 places to the left.

20.

$$\begin{aligned}(24\ \cancel{\text{hours}})(0.000\,00275\frac{\text{miles}}{\cancel{\text{hour}}}) &= (24)(2.75 \times 10^{-6})\text{ miles} \\ &= 66 \times 10^{-6}\text{ miles} \\ &= 6.6 \times 10^{-5}\text{ miles}\end{aligned}$$

21.

$$\begin{aligned}\text{Number of Molecules in 0.00483 moles} &= (0.00483\ \cancel{\text{moles}})(6.02 \times 10^{23}\frac{\text{molecules}}{\cancel{\text{moles}}}) \\ &= (0.00483)(6.02 \times 10^{23})\text{ molecules} \\ &= 0.0290766 \times 10^{23}\text{ molecules} \\ &= 2.90766 \times 10^{21}\text{ molecules} \\ &= 2.91 \times 10^{21}\text{ molecules}\end{aligned}$$

In the last step we rounded based on Avogadro's number having three significant digits. In your science courses you should determine what the conventions are for rounding answers. In general, round at the end of your calculation.

22. Two terms, so it is a binomial. Degree is 5, since term $3x^2y^3$ has sum of exponents of the variables which is 5 (other term has smaller sum of exponents).

23. Two terms, so it is a binomial. Degree is 8, since term $5x^4y^4$ has sum of exponents of the variables which is 8 (largest sum for all terms).

24. $(2x - 19) - (-3x + 5) = 2x - 19 + 3x - 5 = 5x - 24$

25.

$$\begin{aligned}
\left(\frac{3}{8}x^2 - \frac{2}{3}x - 7\right) - \left(\frac{2}{3}x^2 - \frac{1}{2}x + 2\right) &= \frac{3}{8}x^2 - \frac{2}{3}x - 7 - \frac{2}{3}x^2 + \frac{1}{2}x - 2 && \text{(distribute)} \\
&= \left(\frac{3}{8} - \frac{2}{3}\right)x^2 + \left(-\frac{2}{3} + \frac{1}{2}\right)x - 7 - 2 && \text{(collect like terms)} \\
&= \left(\frac{9}{24} - \frac{16}{24}\right)x^2 + \left(-\frac{4}{6} + \frac{3}{6}\right)x - 9 && \text{(common denominator)} \\
&= \left(-\frac{7}{24}\right)x^2 + \left(-\frac{1}{6}\right)x - 9 && \text{(add fractions)}
\end{aligned}$$

26. $(3x^4 - 4x^2 - 18) - (2x^4 + 3x^3 + 6) = 3x^4 - 4x^2 - 18 - 2x^4 - 3x^3 - 6 = x^4 - 3x^3 - 4x^2 - 24$

27. $(2b^3 + 3b - 5) - (-3b^3 + 5b^2 + 7b) = 2b^3 + 3b - 5 + 3b^3 - 5b^2 - 7b = 5b^3 - 5b^2 - 4b - 5$

28. Area $= x^2 + 12x + (2x)x = 3x^2 + 12x$.

29. Perimeter $= 34 + x + 8 + 2x + x + x + 12 + x + 5 + x + 8 = 7x + 67$.

30. $-2x(6x^3 - x) = -2x(6x^3) - 2x(-x) = -12x^4 + 2x^2$

31. $\frac{1}{2}(2x + 3x^2 + 5x^3) = \frac{1}{2}(2x) + \frac{1}{2}(3x^2) + \frac{1}{2}(5x^3) = x + \frac{3}{2}x^2 + \frac{5}{2}x^3$

32.

$$\begin{aligned}
(2b^3 - 5b^2 + 3ab)(-3b^2) &= (2b^3)(-3b^2) - (5b^2)(-3b^2) + (3ab)(-3b^2) && \text{(distribute)} \\
&= -6b^{3+2} + 15b^{2+2} - 9ab^{1+2} && \text{(product rule of exponents)} \\
&= -6b^5 + 15b^4 - 9ab^3
\end{aligned}$$

33.

$$\begin{aligned}
(-4x^3 + 6x^2 - 5x)(-7xy^2) &= (-4x^3)(-7xy^2) + (6x^2)(-7xy^2) - (5x)(-7xy^2) && \text{(distribute)} \\
&= 28x^{3+1}y^2 - 42x^{2+1}y^2 + 35x^{1+1}y^2 && \text{(rule of exponents)} \\
&= 28x^4y^2 - 42x^3y^2 + 35x^2y^2
\end{aligned}$$

34.

$$\begin{aligned}(3x+4)(5x-y) &= (3x)(5x-y)+(4)(5x-y) && \text{(distribute the } 5x-y\text{)}\\ &= (3x)(5x)-(3x)(y)+(4)(5x)-(4)(y) && \text{(distribute the } 3x \text{ and } 4\text{)}\\ &= 15x^2-3xy+20x-4y\end{aligned}$$

Sometimes it helps to use arrows to indicate how you are distributing, to make sure you get all the terms when you distribute. Notice that the order you distribute in does not matter:

$$\begin{aligned}(3x+4)(5x-y) &= 3x(5x-y)+4(5x-y)\\ &= (3x)(5x)-3x(y)+4(5x)-4y\\ &= 15x^2-3xy+20x-4y\end{aligned}$$

OR

$$\begin{aligned}(3x+4)(5x-y) &= (3x+4)5x-(3x+4)y\\ &= (3x)(5x)+4(5x)-(3x)y-4y\\ &= 15x^2+20x-3xy-4y\end{aligned}$$

35.

$$\begin{aligned}(4y+1)(5y-3) &= (4y+1)5y-(4y+1)3 && \text{distribute}\\ &= (4y)(5y)+(1)(5y)-(4y)(3)-(1)(3) && \text{distribute}\\ &= 20y^2+5y-12y-3 && \text{simplify}\\ &= 20y^2-7y-3 && \text{collect like terms}\end{aligned}$$

36.

$$\begin{aligned}(5y+1)(6y-5) &= (5y+1)(6y)+(5y+1)(-5) && \text{distribute}\\ &= 5y(6y)+1(6y)+5y(-5)+1(-5) && \text{distribute}\\ &= 30y^2+6y-25y-5 && \text{simplify}\\ &= 30y^2-19y-5 && \text{collect like terms}\end{aligned}$$

37. Let's do the multiplication and then compare to find the error.

$$\begin{aligned}(x-2)(-3) &= (x)(-3)-(2)(-3)\\ &= -3x-(-6) = -3x+6\end{aligned}$$

Signs are incorrect.

38. Let's do the multiplication and then compare to find the error.

$$\begin{aligned}-(3x-7) &= -1(3x-7)\\ &= -1(3x)+(-1)(-7)\\ &= -3x+7\end{aligned}$$

The minus sign was not distributed to the last term.

39.

$$\begin{aligned}\left(\frac{1}{3}x+\frac{1}{5}\right)\left(\frac{1}{3}x-\frac{1}{2}\right) &= \left(\frac{1}{3}x+\frac{1}{5}\right)\left(\frac{1}{3}x\right)+\left(\frac{1}{3}x+\frac{1}{5}\right)\left(-\frac{1}{2}\right) && \text{(distribute)}\\ &= \left(\frac{1}{3}x\right)\left(\frac{1}{3}x\right)+\left(\frac{1}{5}\right)\left(\frac{1}{3}x\right)+\left(\frac{1}{3}x\right)\left(-\frac{1}{2}\right)+\left(\frac{1}{5}\right)\left(-\frac{1}{2}\right) && \text{(distribute)}\\ &= \frac{1}{9}x^2+\frac{2}{30}x-\frac{5}{30}x-\frac{1}{10} && \text{(like terms)}\\ &= \frac{1}{9}x^2-\frac{3}{30}x-\frac{1}{10} && \text{(simplify)}\\ &= \frac{1}{9}x^2-\frac{1}{10}x-\frac{1}{10}\end{aligned}$$

40. Difference of squares, $(a+b)(a-b)=a^2-b^2$.

$$(x+6)(x-6)=x^2-36$$

41. Difference of squares, $(a+b)(a-b)=a^2-b^2$.

$$(4x-9)(4x+9)=16x^2-81$$

42. Difference of squares, $(a+b)(a-b)=a^2-b^2$.

$$\left(5x-\frac{1}{5}\right)\left(5x+\frac{1}{5}\right)=25x^2-\frac{1}{25}$$

43. A binomial squared with addition, $(a+b)^2=a^2+2ab+b^2$.

$$(6x+5)^2=36x^2+60x+25$$

44. A binomial squared with subtraction, $(a-b)^2=a^2-2ab+b^2$.

$$(8x-3)^2=64x^2-48x+9$$

45. Difference of squares, $(a+b)(a-b) = a^2 - b^2$.

$$\left(\frac{3}{5}x - \frac{1}{3}\right)\left(\frac{3}{5}x + \frac{1}{3}\right) = \frac{9}{25}x^2 - \frac{1}{9}$$

46. A binomial squared with subtraction, $(a-b)^2 = a^2 - 2ab + b^2$.

$$\left(\frac{3}{5}x - \frac{1}{3}\right)^2 = \frac{9}{25}x^2 - \frac{2}{5}x + \frac{1}{9}$$

47. Use distributive property.

$$(a^2-3a+2)(a^2+4a-3) = (a^2-3a+2)(a^2) + (a^2-3a+2)(4a) + (a^2-3a+2)(-3)$$
$$= (a^2)(a^2) - 3a(a^2) + 2(a^2) + a^2(4a) - 3a(4a) + 2(4a) + a^2(-3) - 3a(-3) + 2(-3)$$
$$= a^4 - 3a^3 + 2a^2 + 4a^3 - 12a^2 + 8a - 3a^2 + 9a - 6$$
$$= a^4 + a^3 - 13a^2 + 17a - 6$$

48. Use distributive property.

$$(x^2+4x-5)(x^2-3x+4) = (x^2+4x-5)(x^2) + (x^2+4x-5)(-3x) + (x^2+4x-5)(4)$$
$$= x^2(x^2) + 4x(x^2) - 5(x^2) + x^2(-3x) + 4x(-3x) - 5(-3x) + x^2(4) + 4x(4) - 5(4)$$
$$= x^4 + 4x^3 - 5x^2 - 3x^3 - 12x^2 + 15x + 4x^2 + 16x - 20$$
$$= x^4 + x^3 - 13x^2 + 31x - 20$$

49. Use distributive property.

$$
\begin{aligned}
(x+7)(3x-2)(x-7) &= \left[(x+7)3x + (x+7)(-2)\right](x-7) \\
&= \left[x(3x) + 7(3x) + x(-2) + 7(-2)\right](x-7) \\
&= \left[3x^2 + 21x - 2x - 14\right](x-7) \\
&= \left[3x^2 + 19x - 14\right](x-7) \\
&= (3x^2 + 19x - 14)(x) + (3x^2 + 19x - 14)(-7) \\
&= 3x^3 + 19x^2 - 14x - 21x^2 - 133x + 98 \\
&= 3x^3 - 2x^2 - 147x + 98
\end{aligned}
$$

50. Since the denominator is a monomial, we can use $\dfrac{a+b}{c} = \dfrac{a}{c} + \dfrac{b}{c}$.

$$\frac{8x^4 - 12x^3 - 4x^2}{4x^2} = \frac{8x^4}{4x^2} - \frac{12x^3}{4x^2} - \frac{4x^2}{4x^2} = 2x^2 - 3x - 1, \quad x \neq 0.$$

Polynomial long division will also work, but it is a bit more work.

$$
\begin{array}{r}
2x^2 - 3x - 1 \\
4x^2 \overline{\big) \; 8x^4 - 12x^3 - 4x^2} \\
\underline{-\,8x^4} \qquad\qquad\quad \\
-\,12x^3 \qquad\quad \\
\underline{12x^3} \qquad\quad \\
-\,4x^2 \\
\underline{4x^2} \\
0
\end{array}
$$

51. Since the denominator is a monomial, we can use $\dfrac{a+b}{c} = \dfrac{a}{c} + \dfrac{b}{c}$.

$$\frac{49x^8 - 35x^6 - 56x^3}{7x^3} = \frac{49x^8}{7x^3} - \frac{35x^6}{7x^3} - \frac{56x^3}{7x^3} = 7x^5 - 5x^3 - 8.$$

Polynomial long division will also work, but it is a bit more work.

$$\begin{array}{r|l}
 & \quad 7x^5 \quad - 5x^3 - 8 \\ \hline
7x^3 & 49x^8 - 35x^6 \qquad - 56x^3 \\
 & -49x^8 \\ \hline
 & \qquad -35x^6 \\
 & \qquad\ \ 35x^6 \\ \hline
 & \qquad\qquad\qquad -56x^3 \\
 & \qquad\qquad\qquad\ \ 56x^3 \\ \hline
 & \qquad\qquad\qquad\qquad 0
\end{array}$$

52.
$$\begin{array}{r|l}
 & \qquad\qquad 4x + 5 \\ \hline
3x+1 & 12x^2 + 19x + 5 \\
 & -12x^2 - 4x \\ \hline
 & \qquad\quad 15x + 5 \\
 & \qquad -15x - 5 \\ \hline
 & \qquad\qquad\quad 0
\end{array}$$

This means $\dfrac{12x^2 + 19x + 5}{3x + 1} = 4x + 5, \quad 3x + 1 \neq 0.$

53.
$$\begin{array}{r|l}
 & \qquad\quad 2x^2 \ - 3x \ - 2 \\ \hline
2x+5 & 4x^3 \ + 4x^2 - 19x - 15 \\
 & -4x^3 - 10x^2 \\ \hline
 & \qquad -6x^2 - 19x \\
 & \qquad\ \ 6x^2 + 15x \\ \hline
 & \qquad\qquad\quad -4x - 15 \\
 & \qquad\qquad\qquad 4x + 10 \\ \hline
 & \qquad\qquad\qquad\quad -5
\end{array}$$

This means $\dfrac{4x^3 + 4x^2 - 19x - 15}{2x + 5} = 2x^2 - 3x - 2 - \dfrac{5}{2x + 5}.$

54.

$$
\begin{array}{r|rrrr}
 & 3x^2 & -5x & +2 & \\
\cline{2-5}
3x-5 & 9x^3 & -30x^2 & +31x & -4 \\
 & -9x^3 & +15x^2 & & \\
\cline{2-3}
 & & -15x^2 & +31x & \\
 & & 15x^2 & -25x & \\
\cline{3-4}
 & & & 6x & -4 \\
 & & & -6x & +10 \\
\cline{4-5}
 & & & & 6
\end{array}
$$

This means $\dfrac{9x^3-30x^2+31x-4}{3x-5} = 3x^2-5x+2+\dfrac{6}{3x-5}$.

55.

$$
\begin{array}{r|rrrr}
 & 2x^2 & +\frac{1}{2}x & +\frac{7}{8} & \\
\cline{2-5}
4x-1 & 8x^3 & & +3x & -7 \\
 & -8x^3 & +2x^2 & & \\
\cline{2-3}
 & & 2x^2 & +3x & \\
 & & -2x^2 & +\frac{1}{2}x & \\
\cline{3-4}
 & & & \frac{7}{2}x & -7 \\
 & & & -\frac{7}{2}x & +\frac{7}{8} \\
\cline{4-5}
 & & & & -\frac{49}{8}
\end{array}
$$

This means $\dfrac{8x^3+3x-7}{4x-1} = 2x^2+\dfrac{1}{2}x+\dfrac{7}{8}-\dfrac{49/8}{4x-1}$.