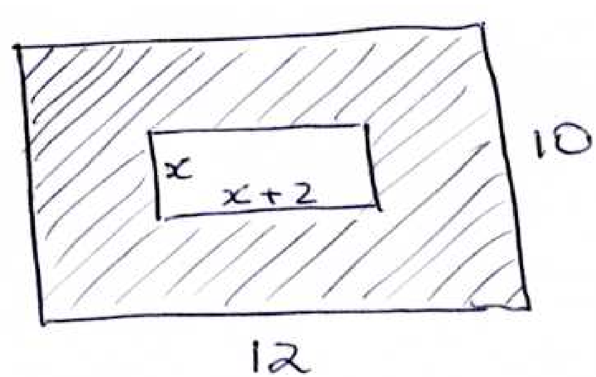
- Factoring polynomials is the distributive property done in reverse!
- To check your answers, use the distributive property to multiply our your final answer.
- Remember, your solution can be different in detail from mine and still be completely correct.

Questions

1. Remove the largest possible common factor from $3a^2 + 3a$.
2. Remove the largest possible common factor from $12xy - 18yz - 36xz$.
3. Remove the largest possible common factor from $16x^5 + 24x^3 - 32x^2$.
4. Remove the largest possible common factor from $36x^6 + 45x^4 - 18x^2$.
5. Factor $7a(x + 2y) - b(x + 2y)$.
6. Factor $3b(y^2 - x) - 4a(y^2 - x) + 6c(y^2 - x)$.
7. Factor $3c(bc - 3a) - 2(bc - 3a) - 6b(bc - 3a)$.
8. Find a formula for the total cost of all purchases by four people. Each person went to the local wholesale warehouse and spent \$29.95 per item. Harry bought a items, Tim bought b items, Larry bought c items and Dougie bought d items. Write the formula in factored form.
9. Factor by grouping $3x^2 - 6xy + 5x - 10y$.
10. Factor by grouping $4x + 8y - 3wx - 6wy$.
11. Factor by grouping $4u^2 + v + 4uv + u$.
12. Factor by grouping $6tx + r - 3t - 2rx$.
13. Factor by grouping $x^2 - 2x - xy + 2y$.
14. Tim was trying to factor $5x^2-3xy-10x+6y$. In his first step he wrote down $x(5x-3y)+2(-5x+3y)$. Was he doing the problem correctly? What is the answer?
15. A train was traveling 73 miles per hour when the engineer spotted a stalled truck on the crossing ahead. He jammed on the brakes but was unable to stop the train before it collided with the truck. For every second the engineer applied the brakes, the train slowed down by 4 miles per hour. The accident reconstruction team found that the train was still traveling at 41 miles per hour at the time of impact. For how many second were the brakes applied?
16. Factor $x^2 + 11x + 30$.
17. Factor $x^2 - 6x + 8$.
18. Factor $x^2 + 18x + 72$.
19. Factor $x^2 + 9x + 20$.
20. Factor $a^2 - 13a + 30$.
21. Factor $x^2 + 9x - 36$.
22. Factor $x^2 - 16xy + 63y^2$.
23. Factor $4x^2 + 28x + 40$.
24. Factor $3x^2 - 12x - 63$.
25. Factor $3x^2 - 33x + 54$.
26. Find a polynomial in factored form for the shaded area in the following rectangular region:



27. Factor $9x^2 + 9x + 2$.
28. Factor $4x^2 + 11x + 6$.

29. Factor $15x^2 - 34x + 15$.
30. Factor $3a^2 - 10a - 8$.
31. Factor $12x^2 + 28x + 15$.
32. Factor $4x^4 - 11x^2 - 3$.
33. Factor $8x^2 + 16x - 10$.
34. Factor $16x^2 + 36x - 10$.
35. Factor $25a^2 - 16$.
36. Factor $9x^2 - 49y^2$.
37. Factor $25x^2 + 10x + 1$.
38. Factor $y^2 - 12y + 36$.
39. Factor $16x^2 - 72x + 81$.
40. Factor $49x^2 - 42x + 9$.
41. Factor $169a^2 + 26ab + b^2$.
42. Factor $9x^2 - 25$.
43. Factor $r^3 - 1$.
44. Factor $8x^3 + 27$.
45. Jerome says he can find two values of b so that $100x^2 + bx - 9$ will be a perfect square. Kesha says there is only one that fits, and Larry says there are none. Who is correct and why?
46. Factor $128x^2 + 96x + 18$.
47. Factor $12x^2 - 2x - 18x^3$.
48. Factor $4x^2 - 28x - 72$.
49. Factor $7x^2 + 3x - 2$.
50. Factor $14x^2 - x^3 + 32x$.
51. Factor $30x^3 - 25x^2y - 30xy^2$.
52. Factor $27x^4 - 64x$.
53. Solve $x^2 - x - 20 = 0$.
54. Solve $x^2 + 11x + 18 = 0$.
55. Solve $8x^2 = 72$.
56. Solve $(x - 5)(x + 4) = 2(x - 5)$.
57. Solve $\dfrac{x^2 + 5x}{6} = 4$.
58. Solve $\dfrac{12x^2 - 4x}{5} = 8$.
59. The area of a rectangular garden is 140 square meters. The width is 3 meters longer than one-half of the length. Find the length and width of the garden.
60. Jules is standing on a platform 6 meters high and throws a ball straight up as high as he can at a velocity of 13 meters per second. At what time t will the ball hit the ground? How far from the ground is the ball 2 seconds after Jules threw the ball (assume the ball is 6 meters from the ground when it leaves Jules' hand).

Solutions

1. Largest numerical factor is 3. Largest variable factor is a. Write each term with the factor 3a.

$$\begin{aligned} 3a^2 + 3a &= \mathbf{3a}(a) + \mathbf{3a}(1) && \text{(identify common factor)} \\ &= \mathbf{3a}(a+1) && \text{(factor)} \end{aligned}$$

2. Largest numerical factor is 6. There is no variable factor. Write each term with the factor 6.

$$\begin{aligned} 12xy - 18yz - 36xz &= \mathbf{6}(2xy) - \mathbf{6}(3yz) - \mathbf{6}(6xz) && \text{(identify common factor)} \\ &= \mathbf{6}(2xy - 3yz - 6xz) && \text{(factor)} \end{aligned}$$

3. Largest numerical factor is 8. Largest variable factor is x^2. Write each term with the factor $8x^2$.

$$\begin{aligned} 16x^5 + 24x^3 - 32x^2 &= \mathbf{8x^2}(2x^3) + \mathbf{8x^2}(3x) - \mathbf{8x^2}(4) && \text{(identify common factor)} \\ &= \mathbf{8x^2}(2x^3 + 3x - 4) && \text{(factor)} \end{aligned}$$

4. Largest numerical factor is 9. Largest variable factor is x^2. Write each term with the factor $9x^2$.

$$\begin{aligned} 36x^6 + 45x^4 - 18x^2 &= \mathbf{9x^2}(4x^4) + \mathbf{9x^2}(5x^2) - \mathbf{9x^2}(2) && \text{(identify common factor)} \\ &= \mathbf{9x^2}(4x^4 + 5x^2 - 2) && \text{(factor)} \end{aligned}$$

5. Identify common factor in each term. Each term has a common factor of $x + 2y$.

$$\begin{aligned} 7a(x+2y) - b(x+2y) &= 7a\mathbf{(x+2y)} - b\mathbf{(x+2y)} && \text{(identify common factor)} \\ &= (7a - b)\mathbf{(x+2y)} && \text{(factor)} \end{aligned}$$

6. Identify common factor in each term. Each term has a common factor of $y^2 - x$.

$$\begin{aligned} 3b(y^2 - x) - 4a(y^2 - x) + 6c(y^2 - x) &= 3b\mathbf{(y^2 - x)} - 4a\mathbf{(y^2 - x)} + 6c\mathbf{(y^2 - x)} \\ &= (3b - 4a + 6c)\mathbf{(y^2 - x)} && \text{(factor)} \end{aligned}$$

7. Identify common factor in each term. Each term has a common factor of $bc - 3a$.

$$\begin{aligned} 3c(bc - 3a) - 2(bc - 3a) - 6b(bc - 3a) &= 3c\mathbf{(bc - 3a)} - 2\mathbf{(bc - 3a)} - 6b\mathbf{(bc - 3a)} \\ &= (3c - 2 - 6b)\mathbf{(bc - 3a)} && \text{(factor)} \end{aligned}$$

8. cost $= \$29.95(a + b + c + d)$.

   Check:

$$\begin{aligned} (3x+5)(x-2y) &= (3x+5)(x) + (3x+5)(-2y) \\ &= (3x)(x) + (5)(x) + (3x)(-2y) + (5)(-2y) \\ &= 3x^2 + 5x - 6xy - 10y, && \text{(our answer is correct)} \end{aligned}$$

9. Identify common factors in pairs of terms. First two terms: Factor of $3x$. Last two terms: Factor of 5.

$$\begin{aligned} 3x^2 - 6xy + 5x - 10y &= \mathbf{3x}(x) - \mathbf{3x}(2y) + \mathbf{5}(x) - \mathbf{5}(2y) \\ &= \mathbf{3x}(x - 2y) + \mathbf{5}(x - 2y) \\ &= 3x(x - 2y) + 5(x - 2y) && \text{(remove coloring, now factor this)} \\ &= 3x\mathbf{(x - 2y)} + 5\mathbf{(x - 2y)} && \text{(factor of } x - 2y\text{)} \\ &= (3x + 5)\mathbf{(x - 2y)} \end{aligned}$$

Check:

$$\begin{aligned} (3x + 5)(x - 2y) &= (3x + 5)(x) + (3x + 5)(-2y) \\ &= (3x)(x) + (5)(x) + (3x)(-2y) + (5)(-2y) \\ &= 3x^2 + 5x - 6xy - 10y, && \text{(our answer is correct)} \end{aligned}$$

10. Identify common factors in pairs of terms. First two terms: Factor of 4. Last two terms: Factor of $-3w$.

$$\begin{aligned} 4x + 8y - 3wx - 6wy &= \mathbf{4}(x) + \mathbf{4}(2y) + \mathbf{-3w}(x) + \mathbf{-3w}(2y) \\ &= \mathbf{4}(x + 2y) + \mathbf{-3w}(x + 2y) \\ &= 4\mathbf{(x + 2y)} + (-3w)\mathbf{(x + 2y)} \\ &= (4 - 3w)\mathbf{(x + 2y)} \end{aligned}$$

11. Identify common factors in pairs of terms. Reorder to get factors. First two terms: Factor of u. Last two terms: Factor of v.

$$\begin{aligned} 4u^2 + v + 4uv + u &= 4u^2 + u + v + 4uv && \text{(reorder to get a factor in first two terms)} \\ &= \mathbf{u}(4u) + \mathbf{u}(1) + \mathbf{v}(1) + \mathbf{v}(4u) && \text{(identify factors)} \\ &= u(4u + 1) + v(1 + 4u) && \text{(factor)} \\ &= u\mathbf{(4u + 1)} + v\mathbf{(4u + 1)} && \text{(rearrange to clearly identify factor of } 4u + 1\text{)} \\ &= \mathbf{(4u + 1)}(u + v) && \text{(factor)} \end{aligned}$$

12.

$$\begin{aligned} 6tx + r - 3t - 2rx &= 6tx - 3t + r - 2rx && \text{(reorder)} \\ &= \mathbf{3t}(2x) - \mathbf{3t}(1) + \mathbf{r}(1) + \mathbf{r}(-2x) && \text{(identify factors)} \\ &= \mathbf{3t}(2x - 1) + \mathbf{r}(1 - 2x) && \text{(factor)} \\ &= 3t(2x - 1) + (-r)(-1 + 2x) && \text{(factor } -1 \text{ out of last term)} \\ &= 3t\mathbf{(2x - 1)} + (-r)\mathbf{(2x - 1)} && \text{(reorder to clearly identify factor)} \\ &= (3t - r)\mathbf{(2x - 1)} && \text{(factor)} \end{aligned}$$

13.

$$\begin{aligned} x^2 - 2x - xy + 2y &= x(x - 2) + y(-x + 2) \\ &= x(x - 2) - y(x - 2) \\ &= (x - y)(x - 2) \end{aligned}$$

14. Damn straight Tim is on the right track! His approach is similar to what I did in the previous problem. Here's the complete solution:

$$\begin{aligned} 5x^2 - 3xy - 10x + 6y &= x(5x - 3y) + 2(-5x + 3y) \\ &= x(5x - 3y) - 2(5x - 3y) \\ &= (x - 2)(5x - 3y) \end{aligned}$$

15. A nice work problem to chew on.

    The train slowed by $73 - 41 = 32$ mph.

    Brakes reduce the speed by $4\ \frac{\text{mph}}{\text{sec}}$.

    Time brakes were applied is $\dfrac{32\cancel{\text{mph}}}{4\frac{\cancel{\text{mph}}}{\text{sec}}} = 8$ sec.

16. Two numbers whose product is 30 and sum is 11: 5, 6.

$$x^2 + 11x + 30 = (x + 5)(x + 6)$$

17. Two numbers whose product is 8 and sum is -6: -2, -4.

$$x^2 - 6x + 8 = (x - 2)(x - 4)$$

18. Two numbers whose product is 72 and sum is 18: 12, 6.

$$x^2 + 18x + 72 = (x + 12)(x + 6)$$

19. Two numbers whose product is 20 and sum is 9: 5, 4.

$$x^2 + 9x + 20 = (x + 5)(x + 4)$$

20. Two numbers whose product is 30 and sum is -13: -3, -10.

$$a^2 - 13a + 30 = (a - 3)(a - 10)$$

21. Two numbers whose product is -36 and sum is 9: 12, -3.

$$x^2 + 9x - 36 = (x + 12)(x - 3)$$

22. Just let y carry along. Two numbers whose product is $63y^2$ and sum is $-16y$: $-7y$, $-9y$.

$$x^2 - 16xy + 63y^2 = (x - 7y)(x - 9y)$$

23. Factor first to make numbers smaller, which gives $a = 1$. Two numbers whose product is 10 and sum is 7: 5, 2.

$$\begin{aligned} 4x^2 + 28x + 40 &= 4(x^2 + 7x + 10) \\ &= 4(x + 5)(x + 2) \end{aligned}$$

24. Factor first to get $a = 1$. Two numbers whose product is -21 and sum is -4: -7, 3.

$$\begin{aligned} 3x^2 - 12x - 63 &= 3(x^2 - 4x - 21) \\ &= 3(x-7)(x+3) \end{aligned}$$

25. Factor first to get $a = 1$. Two numbers whose product is 18 and sum is -11: -2, -9.

$$\begin{aligned} 3x^2 - 33x + 54 &= 3(x^2 - 11x + 18) \\ &= 3(x-2)(x-9) \end{aligned}$$

26.

$$\begin{aligned} \text{shaded area } &= = (12)(10) - x(x+2) \\ &= 120 - x^2 - 2x \\ &= -x^2 - 2x + 120 \\ &= -1(x^2 + 2x - 120) \text{ find two numbers product is } -120 \text{ sum is 2: } 12, -10 \\ &= -1(x+12)(x-10) = (x+12)(10-x) \end{aligned}$$

27. Factor $9x^2 + 9x + 2$.

Since the coefficient of x^2 is not 1, and there are no common factors we try *trial and error* or *the grouping method.*

<u>Trial and Error</u>

Factors of 9: 9 and 1
3 and 3
Factors of 2: 1 and 2

Possible Factors	Middle Term	Correct?
$(9x+1)(x+2)$	$19x$	No
$(9x+2)(x+1)$	$11x$	No
$(3x+1)(3x+2)$	$9x$	Yes

Check: $(3x + 1)(3x + 2) = 9x^2 + 3x + 6x + 2 = 9x^2 + 9x + 2$.

<u>Grouping Method</u>

$\mathbf{9}x^2 \mathbf{+9}x + \mathbf{2}$ has grouping number $\mathbf{9 \times 2 = 18}$.

Find two numbers whose product is **18** and whose sum is **9**: 3 and 6.

Now write the $9x$ term as two terms based on the numbers you found.

$$\begin{aligned} 9x^2 + 9x + 2 &= \mathbf{9x^2 + 3x + 6x + 2} \\ &\quad \text{(red terms have a factor of } 3x) \\ &\quad \text{(blue terms have a factor of 2)} \\ &= 3x\mathbf{(3x+1)} + 2\mathbf{(3x+1)} \\ &\quad \text{(both terms have a factor of } 3x+1) \\ &= (3x+2)\mathbf{(3x+1)} \end{aligned}$$

Check: $(3x+1)(3x+2) = 9x^2 + 3x + 6x + 2 = 9x^2 + 9x + 2$.

You might also have written the following, which is entirely correct.

$$\begin{aligned}
9x^2 + 9x + 2 &= \mathbf{9x^2 + 6x + 3x + 2} \\
&\qquad \text{(red terms have a factor of } 3x\text{)} \\
&\qquad \text{(blue terms have no factor (it appears))} \\
&= 3x(3x+2) + (3x+2) \\
&= 3x\mathbf{(3x+2)} + 1\mathbf{(3x+2)} \\
&\qquad \text{(both terms have a factor of } 3x+2\text{)} \\
&= (3x+1)\mathbf{(3x+2)}
\end{aligned}$$

28. Factor $4x^2 + 11x + 6$.

Since the coefficient of x^2 is not 1, and there are no common factors we try *trial and error* or *the grouping method.*

Trial and Error

Factors of 4:	4 and 1
	2 and 2
Factors of 6:	1 and 6
	2 and 3

Possible Factors	Middle Term	Correct?
$(4x+1)(x+6)$	$25x$	No
$(4x+2)(x+3)$	$14x$	No
$(2x+1)(2x+6)$	$14x$	No
$(2x+2)(2x+3)$	$10x$	No
$(4x+6)(x+1)$	$20x$	No
$(4x+3)(x+2)$	$11x$	Yes

Check: $(4x+3)(x+2) = 4x^2 + 3x + 8x + 6 = 4x^2 + 11x + 6$.

Grouping Method

$\mathbf{4}x^2\mathbf{+11}x + \mathbf{6}$ has grouping number $\mathbf{4 \times 6 = 24}$.

Find two numbers whose product is **24** and whose sum is **11**: 3 and 8.

Now write the $11x$ term as two terms based on the numbers you found.

$$\begin{aligned}
4x^2 + 11x + 6 &= \mathbf{4x^2 + 3x + 8x + 6} \\
&\qquad \text{(red terms have a factor of } x\text{)} \\
&\qquad \text{(blue terms have a factor of 2)} \\
&= x\mathbf{(4x+3)} + 2\mathbf{(4x+3)} \\
&\qquad \text{(both terms have a factor of } 4x+3\text{)} \\
&= (x+2)\mathbf{(4x+3)}
\end{aligned}$$

Check: $(4x+3)(x+2) = 4x^2 + 3x + 8x + 6 = 4x^2 + 11x + 6$.

29. Factor $15x^2 - 34x + 15$.

Since the coefficient of x^2 is not 1, and there are no common factors we try *trial and error* or *the grouping method.*

Trial and Error

Factors of 15: 15 and 1, 3 and 5. Signs must be negative since the middle term is negative $-34x$.

Possible Factors	Middle Term	Correct?
$(15x-15)(1x-1)$	$-30x$	No
$(15x-3)(1x-5)$	$-78x$	No
$(3x-15)(5x-1)$	$-78x$	No
$(3x-3)(5x-5)$	$-30x$	No
$(15x-1)(1x-15)$	$-226x$	No
$(15x-5)(1x-3)$	$-50x$	No
$(3x-1)(5x-15)$	$-50x$	No
$(3x-5)(5x-3)$	$-34x$	Yes (finally!)

Check: $(3x-5)(5x-3) = 15x^2 - 25x - 9x + 15 = 15x^2 - 34x + 15$.

Grouping Method

$\mathbf{15}x^2\mathbf{-34}x + \mathbf{15}$ has grouping number $\mathbf{15 \times 15 = 225}$.

Find two numbers whose product is **225** and whose sum is **−34**: -9 and -25.

Hint: Look for numbers "in the middle" rather than on the edges (this would help in the trial and error as well). What I mean is, don't start with $-1 \times (-225)$ since that does equal 225, but obviously won't have a sum of -34. This will just speed things up, you can always examine all the factors of 225.

Now write the $-34x$ term as two terms based on the numbers you found.

$$
\begin{aligned}
15x^2 - 34x + 15 &= \mathbf{15x^2 - 9x - 25x + 15} \\
&\quad \text{(red terms have a factor of } 3x) \\
&\quad \text{(blue terms have a factor of } 5) \\
&= 3x(5x-3) + 5(-5x+3) \\
&\quad \text{(factor a } -1 \text{ out of second term to get common factor in each term)} \\
&= 3x\mathbf{(5x-3)} - 5\mathbf{(5x-3)} \\
&\quad \text{(both terms have a factor of } 5x-3) \\
&= (3x-5)\mathbf{(5x-3)}
\end{aligned}
$$

Check: $(3x-5)(5x-3) = 15x^2 - 25x - 9x + 15 = 15x^2 - 34x + 15$.

30. Factor $3a^2 - 10a - 8$.

Since the coefficient of a^2 is not 1, and there are no common factors we try *trial and error* or *the grouping method.*

Trial and Error

Factors of 3: 3 and 1
Factors of 8: 2 and 4 Signs must be opposite since the last term is negative (-8).
1 and 8

Possible Factors	Middle Term	Correct?
$(3a-2)(1a+4)$	$+10x$	No, but only out by sign, so switch them
$(3a+2)(1a-4)$	$-10x$	Yes

Check: $(3a+2)(a-4) = 3a^2 - 12a + 2s - 8 = 3a^2 - 10a - 8$.

Grouping Method

$\mathbf{3}a^2\mathbf{-10}a \mathbf{- 8}$ has grouping number $\mathbf{3 \times (-8) = -24}$.

Find two numbers whose product is $\mathbf{-24}$ and whose sum is $\mathbf{-10}$: -12 and 2.

Now write the $-10a$ term as two terms based on the numbers you found.

$$\begin{aligned}3a^2 - 10a - 8 &= \mathbf{3a^2 - 12a + 2a - 8}\\ &\quad \text{(red terms have a factor of } 3a)\\ &\quad \text{(blue terms have a factor of 2)}\\ &= 3a\mathbf{(a-4)} + 2\mathbf{(a-4)}\\ &\quad \text{(both terms have a factor of } a-4)\\ &= (3a+2)\mathbf{(a-4)}\end{aligned}$$

Check: $(3a+2)(a-4) = 3a^2 - 12a + 2s - 8 = 3a^2 - 10a - 8$.

31. Factor $12x^2 + 28x + 15$.

Since the coefficient of x^2 is not 1, and there are no common factors we try *trial and error* or *the grouping method.*

Trial and Error

Factors of 12: 12 and 1
6 and 2
3 and 4
Factors of 15: 15 and 1
5 and 3

Signs must be the same since all terms are positive.

Possible Factors	Middle Term	Correct?
$(12x+15)(1x+1)$	$27x$	No
$(6x+5)(2x+3)$	$28x$	Yes

Check: $(6x+5)(2x+3) = 12x^2 + 10x + 18x + 15 = 12x^2 + 28x + 15$.

Grouping Method

$\mathbf{12}x^2\mathbf{+28}x \mathbf{+ 15}$ has grouping number $\mathbf{12 \times (15) = 180}$.

Find two numbers whose product is **180** and whose sum is **28**: 10 and 18.

Now write the $28x$ term as two terms based on the numbers you found.

$$\begin{aligned} 12x^2 + 28x + 15 &= \mathbf{12x^2 + 10x + 18x + 15} \\ &\quad \text{(red terms have a factor of } 2x\text{)} \\ &\quad \text{(blue terms have a factor of 3)} \\ &= 2x\mathbf{(6x+5)} + 3\mathbf{(6x+5)} \\ &\quad \text{(both terms have a factor of } 6x+5\text{)} \\ &= (2x+3)\mathbf{(6x+5)} \end{aligned}$$

Check: $(2x + 3)(6x + 5) = 12x^2 + 10x + 18x + 15 = 12x^2 + 28x + 15$.

32. Factor $4x^4 - 11x^2 - 3$.

Note: We can work with $z = x^2$ in this problem. The problem has been cooked so $4z^2 - 11z - 3$ is one we can solve with our current techniques.

Since the coefficient of z^2 is not 1, and there are no common factors we try *trial and error* or *the grouping method.*

Trial and Error

Factors of 4:	4 and 1
	2 and 2
Factors of 3:	3 and 1

Signs must be the opposite since the last term is negative.

Possible Factors	Middle Term	Correct?
$(4z - 3)(1z + 1)$	z	No
$(2z - 3)(2z + 1)$	$-4z$	No
$(4z - 1)(1z + 3)$	$11z$	No, but only differs by sign, so switch signs.
$(4z + 1)(1z - 3)$	$-11z$	Yes.

Check: $(4z + 1)(1z - 3) = 4z^2 - 11z - 3$, or $(4x^2 + 1)(x^2 - 3) = 4x^4 - 11x^2 - 3$.

Grouping Method

$\mathbf{4}x^4 \mathbf{-11}x^2 \mathbf{-3}$ has grouping number $\mathbf{4 \times (-3) = -12}$.

Find two numbers whose product is $\mathbf{-12}$ and whose sum is $\mathbf{-11}$: -12 and 1.

Now write the $-11x^2$ term as two terms based on the numbers you found.

$$\begin{aligned} 4x^4 - 11x^2 - 3 &= \mathbf{4x^4 - 12x^2 + x^2 - 3} \\ &\quad \text{(red terms have a factor of } 4x^2\text{)} \\ &\quad \text{(blue terms have a factor of 1)} \\ &= 4x^2\mathbf{(x^2-3)} + \mathbf{(x^2-3)} \\ &\quad \text{(both terms have a factor of } x^2-3\text{)} \\ &= (4x^2+1)\mathbf{(x^2-3)} \end{aligned}$$

Check: $(4x^2 + 1)(x^2 - 3) = 4x^4 + x^2 - 12x^2 - 3 = 4x^4 - 11x^2 - 3$.

Note that in the grouping method, you didn't have to introduce $z = x^2$.

33. Factor $8x^2 + 16x - 10$.

There is a common factor: $8x^2 + 16x - 10 = 2(4x^2 + 8x - 5)$.

Since the coefficient of x^2 is not 1, and there are no common factors we try *trial and error* or *the grouping method.*

Trial and Error

Factors of 4:	2 and 2
	4 and 1
Factors of 5:	5 and 1

Signs must be opposite since the last term is negative.

Possible Factors	Middle Term	Correct?
$(2x + 5)(2x - 1)$	$8x$	Yes

Check: $2(2x + 5)(2x - 1) = 2(4x^2 + 10x - 2x - 5) = 2(4x^2 + 8x - 5) = 8x^2 + 16x - 10$.

Grouping Method

$\mathbf{8}x^2\mathbf{+16}x\mathbf{-10}$ has grouping number $\mathbf{8 \times (-10) = -80}$.

Find two numbers whose product is $\mathbf{-80}$ and whose sum is $\mathbf{16}$: -4 and 20.

Now write the $16x$ term as two terms based on the numbers you found.

$$
\begin{aligned}
8x^2 + 16x - 10 &= \boldsymbol{8x^2 + 20x - 4x - 10} \\
&\quad \text{(red terms have a factor of } 4x\text{)} \\
&\quad \text{(blue terms have a factor of } -2\text{)} \\
&= 4x\boldsymbol{(2x + 5)} + (-2)\boldsymbol{(2x + 5)} \\
&\quad \text{(both terms have a factor of } 2x + 5\text{)} \\
&= (4x - 2)\boldsymbol{(2x + 5)} \\
&= 2(2x - 1)(2x + 5)
\end{aligned}
$$

Check: $2(2x + 5)(2x - 1) = 2(4x^2 + 10x - 2x - 5) = 2(4x^2 + 8x - 5) = 8x^2 + 16x - 10$.

Note that in the grouping method, you didn't have to factor the 2 out at the beginning!

34. Factor $16x^2 + 36x - 10$.

Since the coefficient of x^2 is not 1, we try *trial and error* or *the grouping method.* Let's see what happens with the trial and error method if we don't factor out the common factor of 2 at the beginning.

Trial and Error

Factors of 16:	8 and 2
	4 and 4
	16 and 1
Factors of 10:	2 and 5
	10 and 1

Signs must be opposite since the last term is negative.

Possible Factors	Middle Term	Correct?
$(8x + 2)(2x - 5)$	$-36x$	No, but only differs in sign, so change the signs
$(8x - 2)(2x + 5)$	$36x$	Yes

Check: $(8x-2)(2x+5) = 2(4x-1)(2x+5) = 2(8x^2 - 2x + 20x - 5) = 2(8x^2 + 18x - 5) = 16x^2 + 36x - 10$.

Note that in the trial and error method, you didn't have to factor the 2 out at the beginning!

Factoring out the common factor at the beginning just makes the problem easier since you are working with smaller numbers.

Grouping Method

$\mathbf{16}x^2 \mathbf{+36}x \mathbf{- 10}$ has grouping number $\mathbf{16 \times (-10) = -160}$.

Find two numbers whose product is $\mathbf{-160}$ and whose sum is $\mathbf{36}$: 40 and -4.

Now write the $36x$ term as two terms based on the numbers you found.

$$\begin{aligned}
16x^2 + 36x - 10 &= \mathbf{16x^2 + 40x - 4x - 10} \\
&\quad \text{(red terms have a factor of } 8x) \\
&\quad \text{(blue terms have a factor of } -2) \\
&= 8x\mathbf{(2x + 5)} + (-2)\mathbf{(2x + 5)} \\
&\quad \text{(both terms have a factor of } 2x + 5) \\
&= (8x - 2)\mathbf{(2x + 5)} \\
&= 2(4x - 1)(2x + 5)
\end{aligned}$$

Check: $(8x-2)(2x+5) = 2(4x-1)(2x+5) = 2(8x^2 - 2x + 20x - 5) = 2(8x^2 + 18x - 5) = 16x^2 + 36x - 10$.

35. Difference of squares, with $5a$ and 4 being squared.

$$25a^2 - 16 = (5a + 4)(5a - 4)$$

Check:

$$\begin{aligned}
(5a + 4)(5a - 4) &= (5a + 4)(5a) + (5a + 4)(-4) \\
&= (5a)(5a) + (4)(5a) + (5a)(-4) + (4)(-4) \\
&= 25a^2 + 20a - 20a - 16 = 25a^2 - 16
\end{aligned}$$

36. Difference of squares, with $3x$ and $7y$ being squared.

$$9x^2 - 49y^2 = (3x - 7y)(3x + 7y)$$

Check:

$$\begin{aligned}(3x - 7y)(3x + 7y) &= (3x)(3x + 7y) + (-7y)(3x + 7y)\\ &= (3x)(3x) + (3x)(7y) + (-7y)(3x) + (-7y)(7y)\\ &= 9x^2 + 21xy - 21xy - 49y^2 = 9x^2 - 49y^2\end{aligned}$$

37. With $5x$ and 1 being squared, cross term should be $2 \cdot 5x \cdot 1 = 10x$, which it is. Since the cross term is positive, this is a Perfect Square (sum).

$$25x^2 + 10x + 1 = (5x + 1)^2$$

Check:

$$\begin{aligned}(5x + 1)^2 &= (5x + 1)(5x + 1)\\ &= (5x)(5x + 1) + (1)(5x + 1)\\ &= (5x)(5x) + (5x)(1) + 5x + 1\\ &= 25x^2 + 10x + 1\end{aligned}$$

38. With y and 6 being squared, cross term should be $2 \cdot y \cdot 6 = 12y$, which it is. Since the cross term is negative, this is a Perfect Square (difference).

$$y^2 - 12y + 36 = (y - 6)^2$$

Check:

$$\begin{aligned}(y - 6)^2 &= (y - 6)(y - 6)\\ &= (y - 6)(y) + (y - 6)(-6)\\ &= (y)(y) + (-6)(y) + (y)(-6) + (-6)(-6)\\ &= y^2 - 12y + 36\end{aligned}$$

39. With $4x$ and 9 being squared, cross term should be $2 \cdot 4x \cdot 9 = 72x$, which it is. Since the cross term is negative, this is a Perfect Square (difference).

$$16x^2 - 72x + 81 = (4x - 9)^2$$

Check:

$$\begin{aligned}(4x - 9)^2 &= (4x - 9)(4x - 9)\\ &= (4x - 9)(4x) + (4x - 9)(-9)\\ &= (4x)(4x) + (-9)(4x) + (4x)(-9) + (-9)(-9)\\ &= 16x^2 - 72x + 81\end{aligned}$$

40. With $7x$ and 3 being squared, cross term should be $2 \cdot 7x \cdot 3 = 42x$, which it is. Since the cross term is negative, this is a Perfect Square (difference).

$$49x^2 - 42x + 9 = (7x-3)^2$$

Check:

$$\begin{aligned}(7x-3)^2 = (7x-3)(7x-3) &= (7x-3)(7x-3)\\ &= (7x)(7x-3) + (-3)(7x-3)\\ &= (7x)(7x) + (7x)(-3) + (-3)(7x) + (-3)(-3)\\ &= 49x^2 - 42x + 9\end{aligned}$$

41. With $13a$ and b being squared, cross term should be $2 \cdot 13a \cdot b = 26ab$, which it is. Since the cross term is positive, this is a Perfect Square (sum).

$$169a^2 + 26ab + b^2 = (13a+b)^2$$

Check:

$$\begin{aligned}(5x+1)^2 &= (13a+b)(13a+b)\\ &= (13a+b)(13a) + (13a+b)(b)\\ &= (13a)(13a) + (b)(13a) + (13a)(b) + (b)(b)\\ &= 169a^2 + 26ab + b^2\end{aligned}$$

42. Difference of squares, with $3x$ and 5 being squared.

$$9x^2 - 25 = (3x-5)(3x+5)$$

Check:

$$\begin{aligned}(3x-5)(3x+5) &= (3x)(3x+5) + (-5)(3x+5)\\ &= (3x)(3x) + (3x)(5) + (-5)(3x) + (-5)(5)\\ &= 9x^2 - 25\end{aligned}$$

43. Difference of cubes, where r and 1 are being cubed.

$$r^3 - 1 = (r-1)(r^2+r+1)$$

Check:

$$\begin{aligned}(r-1)(r^2+r+1) &= (r-1)(r^2) + (r-1)(r) + (r-1)(1)\\ &= r^3 - r^2 + r^2 - r + r - 1 = r^3 - 1\end{aligned}$$

44. Sum of cubes, where $2x$ and 3 are being cubed.

$$8x^3 + 27 = (2x-3)((2x)^2 + (2x)(3) + (3)^2) = (2x-3)(4x^2+6x+9)$$

Check:

$$\begin{aligned}(2x-3)(4x^2+6x+9) &= (2x-3)(4x^2) + (2x-3)(6x) + (2x-3)(9)\\ &= 8x^3 - 12x^2 + 12x^2 - 18x + 18x - 27 = 8x^3 - 27\end{aligned}$$

45. Larry is right. In both the perfect square formulas, the last term is positive ($a^2 \pm 2ab\underline{+b^2}$), and since $100x^2 + bx - 9 = (10x)^2 + bx - (3)^2$ has the last term negative, there is no way to make this into a perfect square.

46. Start by removing a common factor, so $128x^2 + 96x + 18 = 2(64x^2 + 48x + 9)$. Now factor $64x^2 + 48x + 9$. With $8x$ and 3 being squared, cross term should be $2 \cdot 8x \cdot 3 = 48x$, which it is. Since the cross term is positive, this is a Perfect Square (sum).

$$\begin{aligned} 64x^2 + 48x + 9 &= (8x+3)^2 \\ 128x^2 + 96x + 18 &= 2(8x+3)^2 \end{aligned}$$

Check:

$$\begin{aligned} 2(8x+3)^2 &= 2(8x+3)(8x+3) \\ &= (16x+6)(8x+3) \\ &= (16x+6)(8x) + (16x+6)(3) \\ &= (16x)(8x) + (6)(8x) + (16x)(3) + (6)(3) \\ &= 128x^2 + 96x + 18 \end{aligned}$$

47.

$$\begin{aligned} 12x^2 - 2x - 18x^3 &= 2x(6x - 1 - 9x^2) && \text{(Factor } 2x\text{)} \\ &= -2x(9x^2 - 6x + 1) && \text{(Reorder and factor } -1\text{)} \\ &= -2x(3x-1)^2 && \text{(Perfect square (difference), } 3x \text{ and } 1\text{)} \end{aligned}$$

48.

$$\begin{aligned} 4x^2 - 28x - 72 &= 4(x^2 - 7x - 18) \text{ Factor 4. Need two numbers: sum is } -7\text{, product is } -18\text{: } -9, 2 \\ &= -2x(x-9)(x-2) \end{aligned}$$

49. $7x^2 + 3x - 2$ is a prime polynomial. You cannot find two integers whose sum is 3 and product is -14.

50.

$$\begin{aligned} 14x^2 - x^3 + 32x &= -x(-14x + x^2 - 32) && \text{(Factor } x\text{)} \\ &\quad \text{(Reorder. Need two numbers:)} \\ &\quad \text{(sum is } -14\text{, product is } -32\text{: } -16, 2\text{)} \\ &= -x(x^2 - 14x - 32) \\ &= -x(x-16)(x+2) \end{aligned}$$

51.

$$\begin{aligned}30x^3 - 25x^2y - 30xy^2 &= 5x(6x^2 - 5xy - 6y^2)\\ &\quad \text{Factor } 5x\text{. Grouping Method is next, let } y \text{ follow along with constant.}\\ &= 5x(6x^2 - 5xy - 6y^2)\\ &\quad \text{Need two numbers: sum is } -5y\text{, product is } -36y^2\text{: } -9y, 4y\\ &= 5x\left[\underline{6x^2 - 9yx} + \underline{4yx - 6y^2}\right]\\ &\quad \text{find greatest common factor in first two terms and last two terms.}\\ &= 5x\left[3x\mathbf{(2x - 3y)} + 2y\mathbf{(2x - 3y)}\right]\\ &= 5x\left[(3x + 2y)\mathbf{(2x - 3y)}\right]\\ &= 5x(3x + 2y)(2x - 3y)\end{aligned}$$

52.

$$\begin{aligned}27x^5 - 64x^2 &= x^2(27x^3 - 64)\\ &\quad \text{Factor } x^2\text{. Difference of cubes with } (3x)^3 = 27x^3 \text{ and } 4^3 = 64.\\ &= x^2(3x - 4)((3x)^2 + (3x)(4) + 4^2)\\ &= x^2(3x - 4)(9x^2 + 12x + 16)\end{aligned}$$

53.

$$\begin{aligned}x^2 - x - 20 &= 0 && \text{(Find two numbers product is } -20 \text{ and sum is } -1\text{: } -5, 4)\\ (x - 5)(x + 4) &= 0 && \text{(Zero Factor Property)}\\ (x - 5) = 0 \text{ or } (x + 4) &= 0 && \text{(Solve each linear equation)}\\ x = 5 \text{ or } x &= -4\end{aligned}$$

Check:

$$\begin{aligned}(5)^2 - (5) - 20 &= 25 - 25 = 0\\ (-4)^2 - (-4) - 20 &= 16 - 16 = 0\end{aligned}$$

54.

$$\begin{aligned}x^2 + 11x + 18 &= 0 && \text{(Find two numbers product is 18 and sum is 11: } 2, 9)\\ (x + 2)(x + 9) &= 0\\ (x + 2) = 0 \text{ or } (x + 9) &= 0\\ x = -2 \text{ or } x &= -9\end{aligned}$$

Check:

$$\begin{aligned}(-2)^2 + 11(-2) + 18 &= 4 - 22 + 18 = 0\\ (-9)^2 + 11(-9) + 18 &= 81 - 99 + 18 = 0\end{aligned}$$

55.

$$
\begin{aligned}
8x^2 - 72 &= 0 && \text{(Factor)} \\
8(x^2 - 9) &= 0 && \text{(Factor)} \\
x^2 - 9 &= 0 && \text{(Divide by 8. Difference of Squares.)} \\
(x+3)(x-3) &= 0 \\
(x+3) = 0 \text{ or } (x-3) &= 0 && \text{(zero factor property)} \\
x = -3 \text{ or } x &= 3
\end{aligned}
$$

Check:

$$
\begin{aligned}
8(-3)^2 &= 8(9) = 72 \\
8(3)^2 &= 8(9) = 72
\end{aligned}
$$

Alternate solution, which only works because there was no x term:

$$
\begin{aligned}
8x^2 &= 72 \\
x^2 &= 9 \\
\sqrt{x^2} &= \pm\sqrt{9} \text{ when taking square root of both sides of equation, one side can be } \pm. \\
x &= \pm 3
\end{aligned}
$$

56. Start by multiplying everything to get in form $ax^2 + bx + c = 0$.

$$
\begin{aligned}
(x-5)(x+4) &= 2(x-5) \\
x^2 - x - 20 &= 2x - 10 \\
x^2 - x - 20 - 2x + 10 &= 0 \\
x^2 - 3x - 10 &= 0 && \text{(two numbers product is } -10 \text{ and sum is } -3\text{: } -5, 2) \\
(x-5)(x+2) &= 0 \\
(x-5) = 0 \text{ or } (x+2) &= 0 && \text{(zero factor property)} \\
x = 5 \text{ or } x &= -2
\end{aligned}
$$

Check:

$$
\begin{aligned}
((5)-5)((5)+4) - 2((5)-5) &= 0 \\
((-2)-5)((-2)+4) - 2((-2)-5) &= -14 + 14 = 0
\end{aligned}
$$

57. Start by multiplying everything to get in form $ax^2 + bx + c = 0$.

$$
\begin{aligned}
\frac{x^2 + 5x}{6} &= 4 \\
x^2 + 5x &= 24 \\
x^2 + 5x - 24 &= 0 && \text{(Find two numbers product is } -24 \text{ and sum is 5: } 8, -3) \\
(x+8)(x-3) &= 0 \\
(x+8) = 0 \text{ or } (x-3) &= 0 && \text{(zero factor property)} \\
x = -8 \text{ or } x &= 3
\end{aligned}
$$

Check:

$$\frac{(-8)^2+5(-8)}{6}=\frac{64-40}{6}=\frac{24}{6}=4$$
$$\frac{(3)^2+5(3)}{6}=\frac{9+15}{6}=\frac{24}{6}=4$$

58. Start by multiplying everything to get in form $ax^2+bx+c=0$.

$$\frac{12x^2-4x}{5}=8$$
$$12x^2-4x=40$$
$$12x^2-4x-40=0$$
$$3x^2-x-10=0$$

Grouping Method:

Find two numbers product is -30 and sum is -1: $-6, 5$.

$$\underline{3x^2-6x}+\underline{5x-10}=0$$

Factor by grouping.

$$3x(x-2)+5(x-2)=0$$
$$(3x+5)(x-2)=0$$
$$(3x+5)=0 \text{ or } (x-2)=0$$
$$x=-\frac{5}{3} \text{ or } x=2$$

Check:

$$\frac{12(-5/3)^2-4(-5/3)}{5}=\frac{12(25/9)+20/3}{5}=\frac{100/3+20/3}{5}=\frac{120/3}{5}=\frac{40}{5}=8$$
$$\frac{12(2)^2-4(2)}{5}=\frac{48-8}{5}=\frac{40}{5}=8$$

59. Let x be the length (in meters). Then the width is $\frac{x}{2}+3$ meters. Area is 140 m^2.

$$\text{Area}=(\text{length})(\text{width})$$
$$140=x\left(\frac{x}{2}+3\right)$$
$$140=\frac{x^2}{2}+3x \text{ write in form } ax^2+bx+c=0.$$
$$280=x^2+6x$$
$$0=x^2+6x-280$$
$$x^2+6x-280=0 \text{ Find two numbers product is 6 and sum is } -280\text{: } -14, 20.$$
$$(x-14)(x+20)=0$$
$$(x-14)=0 \text{ or } (x+20)=0$$
$$x=14 \text{ or } x=-20$$

Exclude the $x=-20$ as unphysical (can't have negative length). So The length is $x=14$ meters. Width is 10 meters.

60. Set $h = 6$ and $v = 13$ in our model equation $S = -5t^2 + vt + h$ (see handout).

$$\begin{aligned}
-5t^2 + 13t + 6 &= 0 \text{ Ball hits ground when } S = 0. \text{ Use Grouping Method to factor.} \\
-5t^2 + 13t + 6 &= 0 \text{ Find two numbers product is } -30 \text{ and sum is 13: } 15, -2. \\
-5t^2 + 15t - 2t + 6 &= 0 \\
-5t(t-3) - 2(t-3) &= 0 \\
(-5t-2)(t-3) &= 0 \\
(-5t-2) = 0 \text{ or } (t-3) &= 0 \\
t = -2/5 \text{ or } t &= 3
\end{aligned}$$

Exclude the $t = -5/3$ as unphysical, so the ball hits the ground after 3 seconds.

Two second after throwing the ball, it it $S = -5(2)^2 + 13(2) + 6 = -20 + 26 + 6 = 12$ meters above the ground.