- When canceling quantities like $\frac{\cancel{a}}{\cancel{a}} = 1$, to avoid the division by zero case we usually specify that $a \neq 0$.
- Remember that you can only cancel factors, not terms.

  Correct math canceling of factors: $\frac{x^2\cancel{(y+1)}}{z^3\cancel{(y+1)}} = \frac{x^2}{z^3}$, and $y+1 \neq 0$

  Incorrect math canceling of terms: $\frac{y+\cancel{1}}{x+\cancel{1}} \neq \frac{y}{x}$

  Factoring plays an important role in simplifying rational expressions.
- When adding or subtracting rational expressions you might have to do a lot of work. In general, you might need to
  - factor any polynomials in the expressions
  - get a common denominator for the rational expressions **(the critical step!)**
  - add or subtract using $\frac{a}{c} \pm \frac{b}{c} = \frac{a \pm b}{c}$
  - simplify the numerator (this could even involve another factoring!)
  - simplify further by canceling any common terms in the numerator and denominator
- Be careful, show all your work, and make sure minus signs get distributed correctly; for example, $-3x(x+4)$ is equal to $-3x^2 - 12x$ NOT $-3x^2 + 12x$.

Questions

1. Simplify $\frac{5x+2y}{35x+14y}$.
2. Simplify $\frac{9ab^2}{6a^2b^2(b+3a)}$.
3. Simplify $\frac{3x^2+7x-6}{x^2+7x+12}$.
4. Simplify $\frac{3x^2-8x+5}{4x^2-5x+1}$.
5. Simplify $\frac{10-2x}{4x^2-20x}$.
6. Simplify $\frac{6-2ab}{ab^2-3b}$.
7. Simplify $\frac{16x^2-24xy+9y^2}{16x^2-9y^2}$.
8. Simplify $\frac{bxy+bx^2-axy-ax^2}{ay^2+axy+2by^2+2bxy}$.
9. Simplify $\frac{x^2+3x-10}{x^2+x-20} \cdot \frac{x^2-3x-4}{x^2+4x+3}$.
10. Simplify $\frac{x^2-x-20}{x^2-3x-10} \cdot \frac{x^2+7x+10}{x^2+4x-5}$.
11. Simplify $(6x-5) \div \frac{36x^2-25}{6x^2+17x+10}$.
12. Simplify $\frac{4x^2-9}{4x^2+12x+9} \div (6x-9)$.
13. Simplify $\frac{3x^2+12xy+12y^2}{x^2+4xy+3y^2} \div \frac{4x+8y}{x+y}$.
14. Simplify $\frac{5y^2+17y+6}{10y^2+9y+2} \cdot \frac{4y^2-1}{2y^2+5y-3}$.
15. Simplify $\frac{x^2+8x+15}{2x^2+11x+5} \div \frac{x^2+6x+9}{2x^2-7x-4}$.
16. Simplify $\frac{8x+3}{5x+7} - \frac{6x+10}{5x+7}$.
17. Find the lowest common denominator for $\frac{1}{x^2-9}$ and $\frac{1}{x+3}$.
18. Find the lowest common denominator for $\frac{1}{2x^2-9x-35}$ and $\frac{1}{4x^2+20x+25}$.

19. Simplify $\dfrac{8}{cd}+\dfrac{9}{d}$.

20. Simplify $\dfrac{2}{y-1}+\dfrac{2}{y+1}$.

21. Simplify $\dfrac{2}{3xy}+\dfrac{1}{6yz}$.

22. Simplify $\dfrac{6}{3x-4}-\dfrac{5}{4x-3}$.

23. Simplify $\dfrac{x}{x^2+2x-3}-\dfrac{x}{x^2-5x+4}$.

24. Simplify $\dfrac{3x+5}{x^2+4x+3}+\dfrac{-x+5}{x^2+2x-3}$.

25. Simplify $\dfrac{2x}{x^2+5x+6}-\dfrac{x+1}{x^2+2x-3}$.

26. Simplify $\dfrac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{xy}}$.

27. Simplify $\dfrac{\frac{8}{x}-\frac{2}{3x}}{\frac{2}{3}+\frac{5}{x}}$.

28. Simplify $\dfrac{1-\frac{36}{x^2}}{1-\frac{6}{x}}$.

29. Simplify $\dfrac{x+\frac{4}{x}}{\frac{x^2+3}{4x}}$.

30. Simplify $\dfrac{\frac{y}{y+1}+1}{\frac{2y+1}{y-1}}$.

31. For what values of x is $\dfrac{\frac{5}{x-2}}{\frac{6}{x}+1}$ not defined?

32. Solve $\dfrac{8}{x}+\dfrac{2}{5}=-\dfrac{2}{x}$.

33. Solve $\dfrac{x+1}{2x}=\dfrac{2}{3}$.

34. Solve $\dfrac{2}{2x+5}=\dfrac{4}{x-4}$.

35. Solve $\dfrac{3}{x+5}=\dfrac{3}{3x-2}$.

36. Solve $7-\dfrac{x}{x+5}=\dfrac{5}{x+5}$.

37. Solve $\dfrac{8x}{4x^2-1}=\dfrac{3}{2x+1}+\dfrac{3}{2x-1}$.

38. Solve $\dfrac{6}{x-5}+\dfrac{3x+1}{x^2-2x-15}=\dfrac{5}{x+3}$.

39. Solve $\dfrac{6}{x-3}=\dfrac{-5}{x-2}-\dfrac{5}{x^2-5x+6}$.

40. The scale on a map is $\frac{3}{4}$ inch to 15 miles. If the distance between two cities on the map is 5.5 inches, how far apart are the two cities?

41. A recipe calls for $\frac{3}{4}$ cup of molasses. If the recipe is for 8 people, how many cups of molasses would you need if you expanded the recipe to be for 12 people?

42. Alfonse Elric and Winry Rockbell are traveling in Mexico. They know a speed of 100 kph is approximately the same as 62 mph. If the road they are traveling on has a speed limit of 90 kph, how many mph is the is the speed limit?

43. Jake is 6 feet tall, and notices that he casts a shadow of 8 feet. At the same time, the new public scuplture of the part casts a shadow of 23 feet. How tall is the sculpture?

44. It takes a person using a large lawn mower 4 hours to mow all the grass in the park. A person using the small mower takes 5 hours to mow all the lawns. How long would it take two people using these two mowers together to mow the all the grass in the park?

45. A tropical fish company has two employees, Juan and Chet, who alternate cleaning the fish tanks each week. Juan takes 5 hours to clean the tanks, but Chet takes 7 hours since he doesn't has as much experience as Juan. For a special promotional sale, the boss (Hugo) wants all the tanks cleaned Saturday morning just before the store opens. If Juan and Chet work together, how long will it take them to clean the tanks?

46. The carbon:hydrogen ratio in naphthalene is 5:4. If a sample of naphthalene has 12345 carbon atoms, how many hydrogen atoms does it have?

Solutions

1.

$$\frac{5x+2y}{35x+14y} = \frac{\cancel{5x+2y}}{7\cancel{(5x+2y)}} = \frac{1}{7} \text{ and } 5x+2y \neq 0$$

2.

$$\frac{9ab^2}{6a^2b^2(b+3a)} = \frac{3\cdot\cancel{3ab^2}}{2a\cdot\cancel{3ab^2}(b+3a)} = \frac{3}{2a(b+3a)} \text{ and } 3ab^2 \neq 0$$

3. Factor numerator using grouping method, look for two numbers whose product is -18 and sum is 7: $9, -2$.

$$\begin{aligned} \mathbf{3x^2+7x-6} &= 3x^2+9x-2x-6 \\ &= 3x(x+3)-2(x+3) \\ &= \mathbf{(3x-2)(x+3)} \end{aligned}$$

Factor denominator: look for two numbers whose product is 12 and sum is 7: 3,4.

$$\mathbf{x^2+7x+12} = \mathbf{(x+3)(x+4)}$$

Putting this into our rational expression (the colors here are used to track equivalent expressions):

$$\begin{aligned} \frac{\mathbf{3x^2+7x-6}}{\mathbf{x^2+7x+12}} &= \frac{\mathbf{(3x-2)}\cancel{\mathbf{(x+3)}}}{\cancel{\mathbf{(x+3)}}\mathbf{(x+4)}} \\ &= \frac{3x-2}{x+4} \text{ and } x+3 \neq 0 \end{aligned}$$

4. Factor numerator using grouping method, look for two numbers whose product is 15 and sum is -8: $-5, -3$.

$$\begin{aligned} \mathbf{3x^2-8x+5} &= 3x^2-5x-3x+5 \\ &= x(3x-5)-1(3x-5) \\ &= \mathbf{(x-1)(3x-5)} \end{aligned}$$

Factor denominator using grouping method, look for two numbers whose product is 4 and sum is -5: $-1, -4$.

$$\begin{aligned} \mathbf{4x^2-5x+1} &= 4x^2-x-4x+1 \\ &= x(4x-1)-1(4x-1) \\ &= \mathbf{(x-1)(4x-1)} \end{aligned}$$

Putting this into our rational expression:

$$\begin{aligned} \frac{\mathbf{3x^2-8x+5}}{\mathbf{4x^2-5x+1}} &= \frac{\mathbf{(3x-5)}\cancel{\mathbf{(x-1)}}}{\cancel{\mathbf{(x-1)}}\mathbf{(4x-1)}} \\ &= \frac{3x-5}{4x-1} \text{ and } x-1 \neq 0 \end{aligned}$$

5.

$$\begin{aligned}\frac{10-2x}{4x^2-20x} &= \frac{2(5-x)}{4x(x-5)} && \text{(common factor)}\\ &= \frac{-2\cancel{(x-5)}}{4x\cancel{(x-5)}} && \text{(factor } -1 \text{ in numerator)}\\ &= \frac{-2}{4x} = -\frac{1}{2x} \text{ and } x-5 \neq 0\end{aligned}$$

6.

$$\begin{aligned}\frac{6-2ab}{ab^2-3b} &= \frac{2(3-ab)}{b(ab-3)} && \text{(common factor)}\\ &= \frac{-2\cancel{(ab-3)}}{b\cancel{(ab-3)}} && \text{(factor } -1 \text{ in numerator)}\\ &= \frac{-2}{b} \text{ and } ab-3 \neq 0\end{aligned}$$

7. Numerator is a perfect square; denominator is a difference of squares.

$$\begin{aligned}\frac{16x^2-24xy+9y^2}{16x^2-9y^2} &= \frac{(4x-3y)^2}{(4x-3y)(4x+3y)}\\ &= \frac{\cancel{(4x-3y)}(4x-3y)}{\cancel{(4x-3y)}(4x+3y)}\\ &= \frac{4x-3y}{4x+3y} \text{ and } 4x-3y \neq 0\end{aligned}$$

8. This requires removing a greatest common factor from numerator and denominator, followed by factoring by grouping.

$$\begin{aligned}\frac{bxy+bx^2-axy-ax^2}{ay^2+axy+2by^2+2bxy} &= \frac{x(\boldsymbol{by+bx-ay-ax})}{y(\boldsymbol{ay+ax}+\boldsymbol{2by+2bx})} && \text{(common factor)}\\ &= \frac{x(\boldsymbol{b[y+x]-a[y+x]})}{y(\boldsymbol{a[y+x]}+\boldsymbol{2b[y+x]})} && \text{(common factor)}\\ &= \frac{x(b[\boldsymbol{y+x}]-a[\boldsymbol{y+x}])}{y(a[\boldsymbol{y+x}]+2b[\boldsymbol{y+x}])} && \text{(recopy to identify factors)}\\ &= \frac{x([b-a]\cancel{[\boldsymbol{y+x}]})}{y([a+2b]\cancel{[\boldsymbol{y+x}]})} && \text{(factor, cancel common factors)}\\ &= \frac{x(b-a)}{y(a+2b)} \text{ and } y+x \neq 0\end{aligned}$$

9. Simplify $\frac{x^2+3x-10}{x^2+x-20} \cdot \frac{x^2-3x-4}{x^2+4x+3}$. Factor all polynomials:

$$x^2+3x-10=(x-2)(x+5) \text{ two numbers whose product is } -10 \text{ sum is 3: } -2,5$$
$$x^2-3x-4=(x-4)(x+1) \text{ two numbers whose product is } -4 \text{ sum is } -3\text{: } -4,1$$
$$x^2+x-20=(x+5)(x-4) \text{ two numbers whose product is } -20 \text{ sum is 1: } -4,5$$
$$x^2+4x+3=(x+3)(x+1) \text{ two numbers whose product is 3 sum is 4: } 1,3$$

$$\begin{aligned}\frac{x^2+3x-10}{x^2+x-20} \cdot \frac{x^2-3x-4}{x^2+4x+3} &= \frac{(x^2+3x-10)(x^2-3x-4)}{(x^2+x-20)(x^2+4x+3)} \quad \text{(simplify polynomial multiplication)}\\ &= \frac{(x-2)\cancel{(x+5)}\cancel{(x-4)}\cancel{(x+1)}}{\cancel{(x+5)}\cancel{(x-4)}(x+3)\cancel{(x+1)}}\\ &= \frac{x-2}{x+3} \text{ and } x-4\neq 0, x+1\neq 0, x+5\neq 0\end{aligned}$$

10. Simplify $\frac{x^2-x-20}{x^2-3x-10} \cdot \frac{x^2+7x+10}{x^2+4x-5}$. Factor all polynomials:

$$x^2-x-20=(x-5)(x+4) \text{ two numbers whose product is } -20 \text{ sum is } -1\text{: } -5,4$$
$$x^2-3x-10=(x-5)(x+2) \text{ two numbers whose product is } -10 \text{ sum is } -3\text{: } -5,2$$
$$x^2+7x+10=(x+5)(x+2) \text{ two numbers whose product is 10 sum is 7: } 5,2$$
$$x^2+4x-5=(x+5)(x-1) \text{ two numbers whose product is } -5 \text{ sum is 4: } 5,-1$$

$$\begin{aligned}\frac{x^2-x-20}{x^2-3x-10} \cdot \frac{x^2+7x+10}{x^2+4x-5} &= \frac{(x^2-x-20)(x^2+7x+10)}{(x^2-3x-10)(x^2+4x-5)} \quad \text{(simplify polynomial multiplication)}\\ &= \frac{\cancel{(x-5)}(x+4)\cancel{(x+5)}\cancel{(x+2)}}{\cancel{(x-5)}\cancel{(x+2)}\cancel{(x+5)}(x-1)}\\ &= \frac{x+4}{x-1} \text{ and } x+5\neq 0, x-5\neq 0, x+2\neq 0\end{aligned}$$

11. Simplify $(6x-5) \div \frac{36x^2-25}{6x^2+17x+10}$. Factor all polynomials:

$$\begin{aligned}6x^2+\mathbf{17x}+10 &= 6x^2+\mathbf{12x+5x}+10 \text{ two numbers whose product is 60 sum is 17: } 12,5\\ &= 6x(x+2)+5(x+2) \text{ Factor by grouping}\\ &= (6x+5)(x+2)\\ 36x^2-25 &= (6x-5)(6x+5) \text{ Difference of squares}\end{aligned}$$

$$\begin{aligned}(6x-5) \div \frac{36x^2-25}{6x^2+17x+10} &= (6x-5) \cdot \frac{6x^2+17x+10}{36x^2-25} \quad \text{(simplify polynomial division)}\\ &= \frac{(6x-5)(6x^2+17x+10)}{(36x^2-25)} \quad \text{(simplify polynomial multiplication)}\\ &= \frac{\cancel{(6x-5)}\cancel{(6x+5)}(x+2)}{\cancel{(6x-5)}\cancel{(6x+5)}}\\ &= x+2 \text{ and } 6x+5\neq 0, 6x-5\neq 0\end{aligned}$$

12. Simplify $\dfrac{4x^2-9}{4x^2+12x+9} \div (6x-9)$. Factor all polynomials:

$$\begin{aligned}
4x^2+12x+9 &= 4x^2+6x+6x+9 \text{ two numbers whose product is 36 sum is 12: } 6,6\\
&= 2x(2x+3)+3(2x+3) \text{ Factor by grouping}\\
&= (2x+3)(2x+3) \text{ hey–this was a perfect square!}\\
4x^2-9 &= (2x+3)(2x-3) \text{ Difference of squares}\\
6x-9 &= 3(2x-3) \text{ common factor}
\end{aligned}$$

$$\begin{aligned}
\frac{4x^2-9}{4x^2+12x+9} \div (6x-9) &= \frac{4x^2-9}{4x^2+12x+9}\cdot\frac{1}{(6x-9)} \quad \text{(simplify polynomial division)}\\
&= \frac{(4x^2-9)}{(4x^2+12x+9)(6x-9)} \quad \text{(simplify polynomial multiplication)}\\
&= \frac{\cancel{(2x-3)}\cancel{(2x+3)}}{(2x+3)\cancel{(2x+3)}3\cancel{(2x-3)}}\\
&= \frac{1}{3(2x+3)} \text{ and } 2x-3\neq 0, 2x+3\neq 0
\end{aligned}$$

13. Simplify $\dfrac{3x^2+12xy+12y^2}{x^2+4xy+3y^2} \div \dfrac{4x+8y}{x+y}$. Factor all polynomials (let the y tag along with the constants):

$$\begin{aligned}
3x^2+12xy+12y^2 &= 3x^2+6xy+6xy+12y^2\\
&\qquad \text{two numbers whose product is 36 sum is 12: } 6,6\\
&= 3x(x+2y)+6y(x+2y) \text{ Factor by grouping}\\
&= (3x+6y)(x+2y) = 3(x+2y)(x+2y) \text{ hey–this was a perfect square!}\\
x^2+4xy+3y^2 &= (x+1y)(x+3y) = (x+y)(x+3y)\\
&\qquad \text{two numbers whose product is 3 sum is 4: } 1,3\\
4x+8y &= 4(x+2y) \text{ common factor}
\end{aligned}$$

$$\begin{aligned}
\frac{3x^2+12xy+12y^2}{x^2+4xy+3y^2} \div \frac{4x+8y}{x+y} &= \frac{3x^2+12xy+12y^2}{x^2+4xy+3y^2}\cdot\frac{x+y}{4x+8y} \quad \text{(simplify polynomial division)}\\
&= \frac{(3x^2+12xy+12y^2)(x+y)}{(x^2+4xy+3y^2)(4x+8y)} \quad \text{(simplify polynomial multiplication)}\\
&= \frac{3(x+2y)\cancel{(x+2y)}\cancel{(x+y)}}{\cancel{(x+y)}(x+3y)4\cancel{(x+2y)}}\\
&= \frac{3(x+2y)}{4(x+3y)} \text{ and } x+2y\neq 0, x+y\neq 0
\end{aligned}$$

14. Simplify $\dfrac{5y^2+17y+6}{10y^2+9y+2}\cdot\dfrac{4y^2-1}{2y^2+5y-3}$. Factor all polynomials:

$$\begin{aligned}5y^2+17y+6 &= 5y^2+15y+2y+6 \text{ two numbers whose product is 30 sum is 17: } 15,2\\ &= 5y(y+3)+2(y+3) \text{ Factor by grouping}\\ &= (5y+2)(y+3)\\ 10y^2+9y+2 &= 10y^2+5y+4y+2 \text{ two numbers whose product is 20 sum is 9: } 5,4\\ &= 5y(2y+1)+2(2y+1) \text{ Factor by grouping}\\ &= (5y+2)(2y+1)\\ 2y^2+5y-3 &= 2y^2+6y-1y-3 \text{ two numbers whose product is } -6 \text{ sum is 5: } 6,-1\\ &= 2y(y+3)-1(y+3) \text{ Factor by grouping}\\ &= (2y-1)(y+3)\\ 4y^2-1 &= (2y-1)(2y+1) \text{ difference of squares}\end{aligned}$$

$$\begin{aligned}\frac{5y^2+17y+6}{10y^2+9y+2}\cdot\frac{4y^2-1}{2y^2+5y-3} &= \frac{(5y^2+17y+6)(4y^2-1)}{(10y^2+9y+2)(2y^2+5y-3)} \quad \text{(simplify polynomial multiplication)}\\ &= \frac{\cancel{(5y+2)}\cancel{(y+3)}\cancel{(2y+1)}\cancel{(2y-1)}}{\cancel{(5y+2)}\cancel{(2y+1)}\cancel{(2y-1)}\cancel{(y+3)})}\\ &= 1 \text{ and } 5y+2\neq 0, y+3\neq 0, 2y+1\neq 0, 2y-1\neq 0\end{aligned}$$

15. Simplify $\dfrac{x^2+8x+15}{2x^2+11x+5}\div\dfrac{x^2+6x+9}{2x^2-7x-4}$. Factor all polynomials:

$$\begin{aligned}x^2+8x+15 &= (x+5)(x+3) \text{ two numbers whose product is 15 sum is 8: } 5,3\\ 2x^2+11x+5 &= 2x^2+10x+1x+5 \text{ two numbers whose product is 10 sum is 11: } 10,1\\ &= 2x(x+5)+1(x+5) \text{ Factor by grouping}\\ &= (2x+1)(x+5)\\ 2x^2-7x-4 &= 2x^2-8x+1x-4 \text{ two numbers whose product is } -8 \text{ sum is } -7\text{: } -8,1\\ &= 2x(x-4)+1(x-4) \text{ Factor by grouping}\\ &= (2x+1)(x-4)\\ x^2+6x+9 &= (x+3)(x+3) \text{ two numbers whose product is 9 sum is 6: } 3,3\end{aligned}$$

$$\begin{aligned}\frac{x^2+8x+15}{2x^2+11x+5}\div\frac{x^2+6x+9}{2x^2-7x-4} &= \frac{x^2+8x+15}{2x^2+11x+5}\cdot\frac{2x^2-7x-4}{x^2+6x+9} \quad \text{(simplify polynomial division)}\\ &= \frac{(x^2+8x+15)(2x^2-7x-4)}{(2x^2+11x+5)(x^2+6x+9)}\cdot\frac{2x^2-7x-4}{x^2+6x+9}\\ &\quad \text{Simplify polynomial multiplication.}\\ &= \frac{\cancel{(x+5)}\cancel{(x+3)}\cancel{(2x+1)}(x-4)}{\cancel{(2x+1)}\cancel{(x+5)}\cancel{(x+3)}(x+3)}\\ &= \frac{x-4}{x+3} \text{ and } x+5\neq 0, x+3\neq 0, 2x+1\neq 0\end{aligned}$$

16. The denominators are the same, so we can subtract immediately.

$$\begin{aligned}\frac{8x+3}{5x+7}-\frac{6x+10}{5x+7}&=\frac{(8x+3)-(6x+10)}{5x+7} && \text{(subtract rational expressions with common denominators)}\\ &=\frac{8x+3-6x-10}{5x+7}\\ &=\frac{2x-7}{5x+7}\end{aligned}$$

17. To find lowest common denominator we need to factor.

$$\begin{aligned}x^2-9&=\boldsymbol{(x+3)}(x-3) && \text{(difference of squares)}\\ x+3&=\boldsymbol{(x+3)}\end{aligned}$$

The lowest common denominator is $(x+3)(x-3)$. I've highlighted the overlap in red.

18. Factor everything first.

$$\begin{aligned}2x^2-9x-35&=2x^2-14x+5x-35 \text{ need two numbers whose product is } -70 \text{ and sum is } -9\text{: } -14, 5\\ &=2x(x-7)+5(x-7) \text{ factor by grouping}\\ &=(2x+5)(x-7)\\ 4x^2+20x+25&=4x^2+10x+10x+25 \text{ need two numbers whose product is } 100 \text{ and sum is } 20\text{: } 10, 10\\ &=2x(2x+5)+5(2x+5) \text{ factor by grouping}\\ &=(2x+5)(2x+5) \text{ this was a perfect square}\end{aligned}$$

$$\begin{aligned}2x^2-9x-35&=\boldsymbol{(2x+5)}(x-7)\\ 4x^2+20x+25&=\boldsymbol{(2x+5)}(2x+5)=(2x+5)^2\end{aligned}$$

The lowest common denominator is $(2x+5)^2(x-7)$. I've highlighted the overlap in red.

19. Nothing needs to be factored.

$$\begin{aligned}\frac{8}{cd}+\frac{9}{d}&=\frac{8}{cd}+\frac{9\cdot \boldsymbol{c}}{d\cdot \boldsymbol{c}} && \text{(get LCD)}\\ &=\frac{8}{cd}+\frac{9c}{cd}\\ &=\frac{8+9c}{cd} && \text{(add rational expressions with common denominators)}\end{aligned}$$

20. Nothing needs to be factored.

$$\begin{aligned}\frac{2}{y-1}+\frac{2}{y+1}&=\frac{2\boldsymbol{(y+1)}}{(y-1)\boldsymbol{(y+1)}}+\frac{2\boldsymbol{(y-1)}}{(y+1)\boldsymbol{(y-1)}} && \text{(get LCD)}\\ &=\frac{2(y+1)+2(y-1)}{(y-1)(y+1)} && \text{(add)}\\ &=\frac{2y+2+2y-2}{(y-1)(y+1)} && \text{(simplify numerator)}\\ &=\frac{4y}{(y-1)(y+1)}\end{aligned}$$

21. Nothing needs to be factored.

$$\begin{aligned}\frac{2}{3xy}+\frac{1}{6yz}&=\frac{2\boldsymbol{(2z)}}{3xy\boldsymbol{(2z)}}+\frac{1\boldsymbol{(x)}}{6yz\boldsymbol{(x)}} && \text{(get LCD)}\\&=\frac{4z}{6xyz}+\frac{x}{6xyz} && \text{(add)}\\&=\frac{4z+x}{6xyz}\end{aligned}$$

22. Nothing needs to be factored.

$$\begin{aligned}\frac{6}{3x-4}-\frac{5}{4x-3}&=\frac{6\boldsymbol{(4x-3)}}{(3x-4)\boldsymbol{(4x-3)}}-\frac{5\boldsymbol{(3x-4)}}{(4x-3)\boldsymbol{(3x-4)}} && \text{(get LCD)}\\&=\frac{6(4x-3)-5(3x-4)}{(3x-4)(4x-3)}\\&=\frac{24x-18-15x+20}{(3x-4)(4x-3)} && \text{(simplify numerator)}\\&=\frac{9x+2}{(3x-4)(4x-3)}\end{aligned}$$

23. We need to factor here.

$x^2+2x-3=(x+3)(x-1)$ two numbers whose product is -3 sum is 2: $3,-1$
$x^2-5x+4=(x-4)(x-1)$ two numbers whose product is 4 sum is -5: $-4,-1$

$$\begin{aligned}\frac{x}{x^2+2x-3}-\frac{x}{x^2-5x+4}&=\frac{x}{(x+3)(x-1)}-\frac{x}{(x-4)(x-1)}\\&=\frac{x\boldsymbol{(x-4)}}{(x+3)(x-1)\boldsymbol{(x-4)}}-\frac{x\boldsymbol{(x+3)}}{(x-4)(x-1)\boldsymbol{(x+3)}} && \text{(get LCD)}\\&=\frac{x(x-4)-x(x+3)}{(x+3)(x-1)(x-4)} && \text{(subtract)}\\&=\frac{x^2-4x-x^2-3x}{(x+3)(x-1)(x-4)} && \text{(simplify)}\\&=\frac{-7x}{(x+3)(x-1)(x-4)}\end{aligned}$$

24. We need to factor here.

$x^2+4x+3=(x+3)(x+1)$ two numbers whose product is 3 sum is 4: $3,1$
$x^2+2x-3=(x+3)(x-1)$ two numbers whose product is -3 sum is 2: $3,-1$

$$\begin{aligned}
\frac{3x+5}{x^2+4x+3}+\frac{-x+5}{x^2+2x-3} &= \frac{3x+5}{(x+3)(x+1)}+\frac{5-x}{(x+3)(x-1)} \quad \text{(factor)}\\
&= \frac{(3x+5)\mathbf{(x-1)}}{(x+3)(x+1)\mathbf{(x-1)}}+\frac{(5-x)\mathbf{(x+1)}}{(x+3)(x-1)\mathbf{(x+1)}} \quad \text{(factor)}\\
&= \frac{(3x+5)(x-1)+(5-x)(x+1)}{(x+3)(x+1)(x-1)} \quad \text{(add)}\\
&= \frac{3x^2+2x-5-x^2+4x+5}{(x+3)(x+1)(x-1)} \quad \text{(simplify numerator: distribute)}\\
&= \frac{2x^2+6x}{(x+3)(x+1)(x-1)} \quad \text{(simplify numerator: collect like terms)}\\
&= \frac{2x\cancel{(x+3)}}{\cancel{(x+3)}(x+1)(x-1)} \quad \text{(simplify: factor numerator)}\\
&= \frac{2x}{(x+1)(x-1)} \text{ and } x+3 \neq 0
\end{aligned}$$

25. We need to factor here.

$x^2+5x+6=(x+3)(x+2)$ two numbers whose product is 6 sum is 5: $3, 2$
$x^2+2x-3=(x+3)(x-1)$ two numbers whose product is -3 sum is 2: $3, -1$

$$\begin{aligned}
\frac{2x}{x^2+5x+6}-\frac{x+1}{x^2+2x-3} &= \frac{2x}{(x+3)(x+2)}-\frac{x+1}{(x+3)(x-1)} \quad \text{(factor)}\\
&= \frac{2x\mathbf{(x-1)}}{(x+3)(x+2)\mathbf{(x-1)}}-\frac{(x+1)\mathbf{(x+2)}}{(x+3)(x-1)\mathbf{(x+2)}} \quad \text{(get LCD)}\\
&= \frac{2x(x-1)-(x+1)(x+2)}{(x+3)(x+2)(x-1)} \quad \text{(subtract)}\\
&= \frac{2x^2-2x-x^2-3x-2}{(x+3)(x+2)(x-1)} \quad \text{(simplify numerator: distribute)}\\
&= \frac{x^2-5x-2}{(x+3)(x+2)(x-1)} \quad \text{(simplify numerator: collect like terms)}
\end{aligned}$$

The numerator is prime. If we could factor it, we would.

26.

$$\frac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{xy}} = \frac{\frac{\boldsymbol{y}}{x\boldsymbol{y}}+\frac{\boldsymbol{x}}{\boldsymbol{x}y}}{\frac{1}{xy}} \qquad \text{(common denominator)}$$

$$= \frac{\left(\frac{y+x}{xy}\right)}{\left(\frac{1}{xy}\right)}$$

$$= \left(\frac{y+x}{xy}\right)\left(\frac{xy}{1}\right) \qquad \text{(invert and multiply)}$$

$$= \frac{(y+x)(xy)}{(xy)(1)} \qquad \text{(multiply)}$$

$$= \frac{(y+x)\cancel{(xy)}}{\cancel{(xy)}(1)} \qquad \text{(cancel common factors)}$$

$$= y+x, \text{ and } x \neq 0, y \neq 0$$

27.

$$\frac{\frac{8}{x}-\frac{2}{3x}}{\frac{2}{3}+\frac{5}{x}} = \frac{\frac{\mathbf{3}\times 8}{\mathbf{3}x}-\frac{2}{3x}}{\frac{2\boldsymbol{x}}{3\boldsymbol{x}}+\frac{\mathbf{3}\times 5}{\mathbf{3}x}} \qquad \text{(common denominator)}$$

$$= \frac{\frac{24-2}{3x}}{\frac{2x+15}{3x}} \qquad \text{(add)}$$

$$= \frac{\left(\frac{22}{3x}\right)}{\left(\frac{2x+15}{3x}\right)}$$

$$= \frac{22}{3x}\cdot\frac{3x}{2x+15} \qquad \text{(invert and multiply)}$$

$$= \frac{(22)\cancel{(3x)}}{\cancel{(3x)}(2x+15)}$$

$$= \frac{22}{2x+15} \text{ and } 3x \neq 0$$

28.

$$\frac{1-\frac{36}{x^2}}{1-\frac{6}{x}} = \frac{\frac{\boldsymbol{x^2}}{\boldsymbol{x^2}}-\frac{36}{x^2}}{\frac{\boldsymbol{x}}{\boldsymbol{x}}-\frac{6}{x}} \qquad \text{(common denominator)}$$

$$= \frac{\left(\frac{x^2-36}{x^2}\right)}{\left(\frac{x-6}{x}\right)}$$

$$= \frac{x^2-36}{x^2}\cdot\frac{x}{x-6} \qquad \text{(invert and multiply)}$$

$$= \frac{(x^2-36)(x)}{(x^2)(x-6)}$$

$$= \frac{\cancel{(x-6)}(x+6)\cancel{(x)}}{(x^{\cancel{2}})\cancel{(x-6)}} \qquad \text{(factor, cancel common factors)}$$

$$= \frac{x+6}{x} \text{ and } x+6 \neq 0$$

29.

$$\frac{x+\frac{4}{x}}{\frac{x^2+3}{4x}} = \frac{\frac{x\times \boldsymbol{x}}{\boldsymbol{x}}+\frac{4}{x}}{\frac{x^2+3}{4x}} \qquad \text{(common denominator)}$$

$$= \frac{\left(\frac{x^2+4}{x}\right)}{\left(\frac{x^2+3}{4x}\right)}$$

$$= \left(\frac{x^2+4}{x}\right)\cdot\left(\frac{4x}{x^2+3}\right) \qquad \text{(invert and multiply)}$$

$$= \frac{(x^2+4)(4\cancel{x})}{\cancel{x}(x^2+3)}$$

$$= \frac{4(x^2+4)}{x^2+3} \text{ and } x \neq 0$$

30.

$$\frac{\frac{y}{y+1}+1}{\frac{2y+1}{y-1}} = \frac{\frac{y}{y+1}+\frac{\boldsymbol{y+1}}{\boldsymbol{y+1}}}{\frac{2y+1}{y-1}} \qquad \text{(common denominator)}$$

$$= \frac{\left(\frac{y+y+1}{y+1}\right)}{\left(\frac{2y+1}{y-1}\right)}$$

$$= \left(\frac{2y+1}{y+1}\right)\cdot\left(\frac{y-1}{2y+1}\right) \qquad \text{(invert and multiply)}$$

$$= \frac{\cancel{(2y+1)}(y-1)}{(y+1)\cancel{(2y+1)}}$$

$$= \frac{y-1}{y+1} \text{ and } 2y+1 \neq 0$$

31. No denominator in the expression can equal zero. There are three denominators in the expression.

$$\frac{\frac{5}{\boldsymbol{x-2}}}{\frac{6}{x}+1} \Rightarrow x-2 \neq 0 \Rightarrow x \neq 2$$

$$\frac{\frac{5}{x-2}}{\frac{6}{\boldsymbol{x}}+1} \Rightarrow x \neq 0$$

$$\frac{\frac{5}{x-2}}{\boldsymbol{\frac{6}{x}+1}} \Rightarrow \frac{6}{x}+1 \neq 0 \Rightarrow x \neq -6$$

32. Lowest common denominator is $5x$. Multiply the equation by LCD.

$$\left(\frac{8}{\cancel{x}}\right)\mathbf{5\cancel{x}} + \left(\frac{2}{\cancel{5}}\right)\mathbf{\cancel{5}x} = \left(-\frac{2}{\cancel{x}}\right)\mathbf{5\cancel{x}}$$

$$40 + 2x = -10$$

$$2x = -10 - 40$$

$$x = -\frac{50}{2} = -25$$

Check:

$$\frac{8}{(-25)} + \frac{2}{5} = -\frac{2}{(-25)}$$

$$-\frac{8}{25} + \frac{10}{25} = \frac{2}{25}$$

$$\frac{2}{25} = \frac{2}{25} \quad \text{so } x = -25 \text{ is a solution.}$$

33. LCD is $6x$. Multiply the equation by LCD.

$$\left(\frac{x+1}{2x}\right)\mathbf{6x} = \left(\frac{2}{3}\right)\mathbf{6x}$$

$$3x + 3 = 4x$$

$$3 = 4x - 3x$$

$$3 = x$$

Check:

$$\frac{(3)+1}{2(3)} = \frac{2}{3}$$

$$\frac{4}{6} = \frac{2}{3}$$

$$\frac{2}{3} = \frac{2}{3} \quad \text{so } x = 3 \text{ is a solution!}$$

34. LCD is $(2x+5)(x-4)$. Multiply the equation by LCD.

$$\left(\frac{2}{\cancel{2x+5}}\right)\cancel{(2x+5)}(x-4) = \left(\frac{4}{\cancel{x-4}}\right)(2x+5)\cancel{(x-4)}$$
$$2(x-4) = 4(2x+5)$$
$$2x-8 = 8x+20$$
$$-6x = 28$$
$$x = \frac{28}{-6} = -\frac{14}{3}$$

Check:

$$\frac{2}{2(-14/3)+5} = \frac{4}{(-14/3)-4}$$
$$\frac{2}{-28/3+15/3} = \frac{4}{(-14/6)-12/3}$$
$$\frac{2}{-13/3} = \frac{4}{-26/3}$$
$$\frac{6}{-13} = \frac{12}{-26}$$
$$\frac{6}{-13} = \frac{6}{-13} \quad \text{so } x = -14/3 \text{ is a solution.}$$

35. LCD is $(x+5)(3x-2)$. Multiply the equation by LCD.

$$\left(\frac{3}{\cancel{x+5}}\right)\cancel{(x+5)}(3x-2) = \left(\frac{3}{\cancel{3x-2}}\right)(x+5)\cancel{(3x-2)}$$
$$3(3x-2) = 3(x+5)$$
$$3x-2 = \frac{\cancel{3}}{\cancel{3}}(x+5)$$
$$3x-2 = x+5$$
$$2x = 7$$
$$x = \frac{7}{2}$$

Check:

$$\frac{3}{(7/2)+5} = \frac{3}{3(7/2)-2}$$
$$\frac{3}{7/2+10/2} = \frac{3}{21/2-4/2}$$
$$\frac{3}{17/2} = \frac{3}{17/2} \quad \text{so } x = 7/2 \text{ is a solution.}$$

36. LCD is $x+5$. Multiply the equation by LCD.

$$(7)\,\mathbf{(x+5)} - \left(\frac{x}{\cancel{x+5}}\right)\cancel{\mathbf{(x+5)}} = \left(\frac{5}{\cancel{x+5}}\right)\cancel{\mathbf{(x+5)}}$$
$$7x+35-x=5$$
$$6x=-30$$
$$x=-5$$

As soon as you try to check this in the original equation you will get a division by zero. Therefore $x=-5$ is not a solution (it is an extraneous solution). Therefore, the original equation has no solution.

37. Factor polynomials.

$4x^2-1=(2x-1)(2x+1)$ difference of squares

Looking at the equation, we now see the LCD is $(2x-1)(2x+1)$. Multiply the equation by LCD.

$$\left(\frac{8x}{\cancel{(2x-1)}\cancel{(2x+1)}}\right)\cancel{\mathbf{(2x-1)}}\cancel{\mathbf{(2x+1)}} = \left(\frac{3}{\cancel{2x+1}}\right)\mathbf{(2x-1)}\cancel{\mathbf{(2x+1)}} + \left(\frac{3}{\cancel{2x-1}}\right)\cancel{\mathbf{(2x-1)}}\mathbf{(2x+1)}$$
$$8x=3(2x-1)+3(2x+1)$$
$$8x=6x-3+6x+3$$
$$8x=12x$$
$$-4x=0$$
$$x=\frac{0}{-4}=0$$

Check:

$$\frac{8(0)}{4(0)^2-1}=\frac{3}{2(0)+1}+\frac{3}{2(0)-1}$$
$$0=3-3 \quad \text{so } x=0 \text{ is a solution.}$$

38. Factor polynomials.

$x^2-2x-15=(x+3)(x-5)$ Need two numbers whose product is -15 sum is -2: $3,-5$

Looking at the equation, we now see the LCD is $(x+3)(x-5)$. Multiply the equation by LCD.

$$\left(\frac{6}{\cancel{x-5}}\right)\mathbf{(x+3)\cancel{(x-5)}} + \left(\frac{3x+1}{\cancel{(x+3)}\cancel{(x-5)}}\right)\mathbf{\cancel{(x+3)}\cancel{(x-5)}} = \left(\frac{5}{\cancel{x+3}}\right)\mathbf{\cancel{(x+3)}(x-5)}$$

$$\begin{aligned}6(x+3)+3x+1 &= 5(x-5)\\ 6x+18+3x+1 &= 5x-25\\ 4x &= -44\\ x &= -11\end{aligned}$$

Check:

$$\begin{aligned}\frac{6}{(-11)-5}+\frac{3(-11)+1}{(-11)^2-2(-11)-15} &= \frac{5}{(-11)+3}\\ \frac{6}{-16}+\frac{-32}{128} &= \frac{5}{-8}\\ \frac{3}{-8}+\frac{-1}{4} &= \frac{5}{-8}\\ -\frac{3}{8}-\frac{2}{8} &= -\frac{5}{8}\\ -\frac{5}{8} &= -\frac{5}{8} \quad \text{so } x=-11 \text{ is a solution}\end{aligned}$$

39. Factor polynomials.

$x^2-5x+6=(x-3)(x-2)$ Need two numbers whose product is 6 sum is -5: $-2,-3$

Looking at the equation, we now see the LCD is $(x-3)(x-2)$. Multiply the equation by LCD.

$$\frac{6}{x-3} = \frac{-5}{x-2} - \frac{5}{(x-3)(x-2)}$$

$$\left(\frac{6}{\cancel{x-3}}\right)\mathbf{\cancel{(x-3)}(x-2)} = \left(\frac{-5}{\cancel{x-2}}\right)\mathbf{(x-3)\cancel{(x-2)}} - \left(\frac{5}{\cancel{(x-3)}\cancel{(x-2)}}\right)\mathbf{\cancel{(x-3)}\cancel{(x-2)}}$$

$$\begin{aligned}6(x-2) &= -5(x-3)-5\\ 6x-12 &= -5x+15-5\\ 11x &= 22\\ x &= 2\end{aligned}$$

As soon as you try to check this in the original equation you will get a division by zero. Therefore $x=2$ is not a solution (it is an extraneous solution). Therefore, the original equation has no solution.

40. Use $\frac{3}{4}=0.75$ inches. Let x be the unknown distance (in miles) between the two cities.

$$\frac{\text{map measurement in inches}}{\text{actual distance in miles}} = \frac{\text{second map measurement in inches}}{\text{second actual distance in miles}}$$

$$\frac{0.75\ \cancel{\text{inch}}}{15\ \cancel{\text{miles}}} = \frac{5.5\ \cancel{\text{inch}}}{x\ \cancel{\text{miles}}} \quad \text{Solve for } x\text{. Note Units cancel.}$$

$$\frac{0.75}{15} = \frac{5.5}{x}$$

$$x = \frac{5.5\cdot 15}{0.75} = 110$$

The distance between the two cities is 110 miles.

41. Let x be the amount of molasses you need if the recipe is expanded to feed 12 people.

$$\frac{\text{initial amount of molasses}}{\text{initial number of people}} = \frac{\text{new amount of molasses}}{\text{new number of people}}$$
$$\frac{\frac{3}{4}\text{ cups}}{8\text{ people}} = \frac{x\text{ cups}}{12\text{ people}}$$
$$\frac{\left(\frac{3}{4}\right)}{8} = \frac{x}{12}$$
$$x = \frac{\frac{3}{4}\cdot 12}{8} = \frac{9}{8} = 1\frac{1}{8}$$

To expand the recipe for 12 people you will need $1\frac{1}{8}$ cups of molasses.

42. Let x be the speed in mph corresponding to 100 kph.

$$\frac{62\text{mph}}{100\text{kph}} = \frac{x\text{mph}}{90\text{kph}}$$
$$\frac{62}{100} = \frac{x}{90}$$
$$x = \frac{62\cdot 90}{100} = 55.8$$

The speed limit on the road is $55.8 \sim 56$ mph.

43. This is a similar triangle problem. Let x be the height of the sculpture.

$$\frac{\text{Jake's height}}{\text{Length of Jake's Shadow}} = \frac{\text{Sculpture's height}}{\text{Length of Sculpture's shadow}}$$
$$\frac{6\text{ft}}{8\text{ft}} = \frac{x\text{ft}}{23\text{ft}}$$
$$\frac{6}{8} = \frac{x}{23}$$
$$x = \frac{6\cdot 23}{8} = 17.25$$

The height of the sculpture is $17.25 \sim 17$ ft.

44. One person with large mower takes 4 hrs to do mowing. In one hour they complete $\frac{1}{4}$ of the job.
Second person with small mower takes 5 hrs to do the mowing. In one hour they complete $\frac{1}{5}$ of the job.
Working together, the job will take x hrs. In one hour they complete $\frac{1}{x}$ of the job.

$$\frac{1}{4} + \frac{1}{5} = \frac{1}{x}$$
$$\frac{5}{20} + \frac{4}{20} = \frac{1}{x}$$
$$\frac{9}{20} = \frac{1}{x}$$
$$\frac{20}{9} = \frac{x}{1}$$
$$x = \frac{20}{9} = 2\frac{2}{9}$$

It will take them $2\frac{2}{9}$ hours to complete the job together.

$$\frac{2}{9} \text{ hours} = \frac{2}{9} \cdot 60 \text{ minutes } = 13.33 \text{ minutes}$$

It will take them 2 hours and 13 minutes to complete the job together.

45. Juan takes 5 hrs. In one hour he complete $\frac{1}{5}$ of the job.

Chet takes 7 hrs. In one hour he complete $\frac{1}{7}$ of the job.

Working together, the job will take x hrs. In one hour they complete $\frac{1}{x}$ of the job.

$$\begin{aligned} \frac{1}{5} + \frac{1}{7} &= \frac{1}{x} \\ \frac{7}{35} + \frac{5}{35} &= \frac{1}{x} \\ \frac{12}{35} &= \frac{1}{x} \\ \frac{35}{12} &= \frac{x}{1} \\ x &= \frac{35}{12} = 2\frac{11}{12} \end{aligned}$$

It will take them $2\frac{11}{12}$ hours to complete the job together.

$$\frac{11}{12} \text{ hours} = \frac{11}{12} \cdot 60 \text{ minutes } = 55 \text{ minutes}$$

It will take them 2 hours and 55 minutes to complete the job together.

46. Let x be the number of hydrogen atoms in the sample.

$$\begin{aligned} \frac{5 \text{ carbon atoms}}{4 \text{ hydrogen atoms}} &= \frac{12345 \text{ carbon atoms}}{x \text{ hydrogen atoms}} \\ \frac{5}{4} &= \frac{12345}{x} \\ x &= \frac{12345 \cdot 4}{5} = 9876 \end{aligned}$$

The sample has 9876 hydrogen atoms.