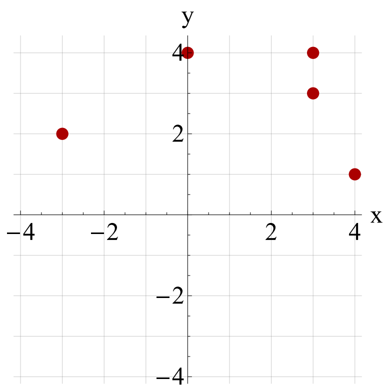
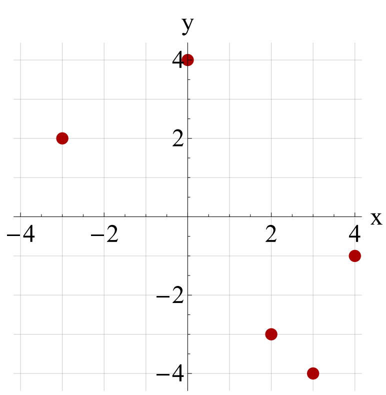
Questions

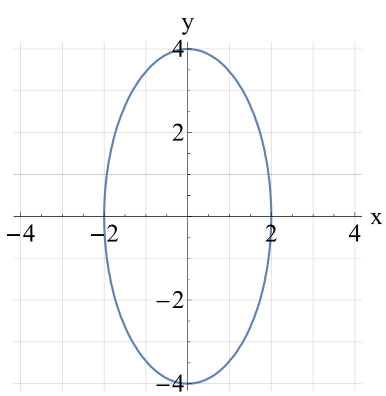
1. From the plot of the relation, determine the domain and range. Is the relation a function? Explain why or why not.



2. From the plot of the relation, determine the domain and range. Is the relation a function? Explain why or why not.



3. From the plot of the relation, determine the domain and range. Is the relation a function? Explain why or why not.



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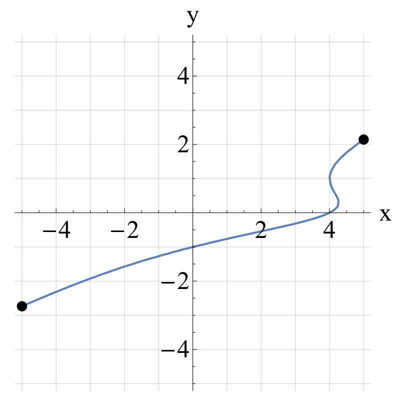

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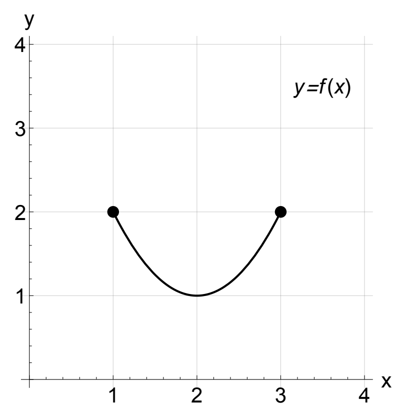
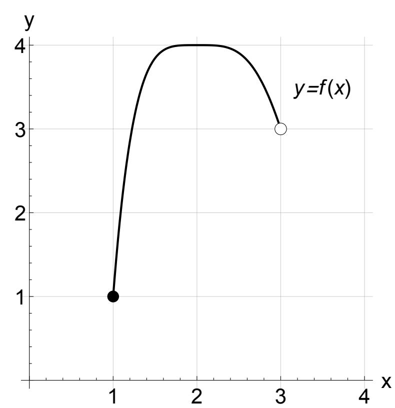
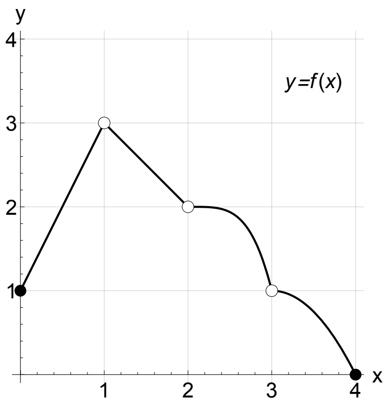


6. From the plot of the relation, determine the domain and range. Is the relation a function? Explain why or why not.



7. From the plot of the relation, determine the domain and range. Is the relation a function? Explain why or why not.



8. Given the relation $y = \frac{x}{3} - \frac{1}{2}$, determine the ordered pairs associated with $x = -3$, $x = -2$, $x = -3/2$, $x = 0$, $x = 1$, and $x = 2$.

   Sketch the ordered pairs and determine a line which represents all the ordered pairs in the relation.

   What is the domain and range for the relation?

   Is the relation a function? Explain your answer.

9. Given the relation $y = 14 - \frac{x^2}{2}$, determine the ordered pairs associated with $x = -3$, $x = -2$, $x = -3/2$, $x = 0$, $x = 1$, and $x = 2$, $x = 7/2$.

   Sketch the ordered pairs and determine a line which represents all the ordered pairs in the relation.

   What is the domain and range for the relation?

   Is the relation a function? Explain your answer.

10. Given the relation $\frac{y^2}{4} = 9 - x$, determine the ordered pairs associated with $y = -6$, $y = -4$, $y = -2$, $y = 0$, $y = 2$, $y = 4$, $y = 6$.

    Sketch the ordered pairs and determine a line which represents all the ordered pairs in the relation.

    What is the domain and range for the relation?

    Is the relation a function? Explain your answer.

11. Given the relation $\frac{1}{3}|2x| = y$, determine the ordered pairs associated with $x = -3$, $x = -2$, $x = -1$, $x = 0$, $x = 1$, $x = 2$, $x = 3$.

    Sketch the ordered pairs and determine a line which represents all the ordered pairs in the relation.

    What is the domain and range for the relation?

    Is the relation a function? Explain your answer.

12. Determine the domain for the rational function

$$f(x) = \frac{1}{x}.$$

13. Determine the domain for the rational function

$$f(x) = \frac{x-1}{(3x+1)(4x+2)(5x-6)}.$$

14. Determine the domain for the rational function

$$f(x) = \frac{1}{9x^2 + 12x + 4}.$$

15. Determine the domain for the rational function

$$f(x) = \frac{x}{-12x^2 + 4x + 8}.$$

16. Evaluate the function

$$f(x) = \frac{x}{-12x^2 + 4x + 8}$$

    at $x = \frac{1}{2}$.

17. Evaluate the function

$$f(x) = \frac{x}{x^2 - 9}$$

    at $x = \frac{7}{3}$.

18. Evaluate the function

$$f(x) = 4x^2 + 2$$

    at $x = 3 + h$ and simplify as much as possible.

Solutions

1. Domain: $x \in \{-3, 0, 3, 4\}$
   Range: $y \in \{1, 2, 3, 4\}$
   This relation is not a function since the x value $x = 3$ has two outputs, $y = 3$ and $y = 4$.

2. Domain: $x \in \{-3, 0, 2, 3, 4\}$
   Range: $y \in \{-4, -3, -1, 2, 4\}$
   This relation is a function since all the x values correspond to only one output y.

3. Domain: $-2 \leq x \leq 2$
   Range: $-4 \leq y \leq 4$
   This relation is not a function since the curve fails the vertical line test.

4. Domain: $-5 \leq x \leq 5$
   Range: $-3.7 \leq y \leq 2$. Note the left endpoint of the interval is estimated from the graph.
   This relation is not a function since the curve fails the vertical line test (look near $x = 4$).

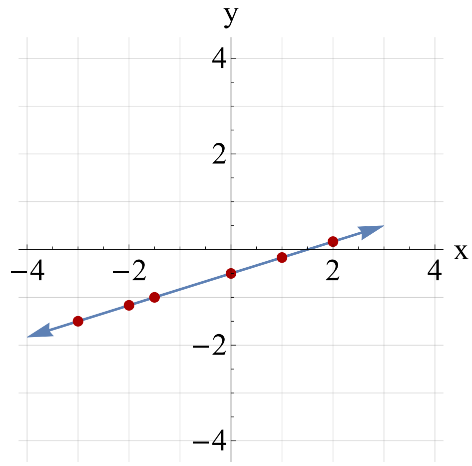
5. Domain: $1 \leq x \leq 3$. Remember filled dots mean the point is included.
   Range: $1 \leq y \leq 2$.
   This relation is a function since the curve passes the vertical line test.

6. Domain: $1 \leq x < 3$. Remember open dots mean the point is not included.
   Range: $1 \leq y \leq 4$. From the graph it looks like the line gets as high as $y =$, but this is an estimate.
   This relation is a function since the curve passes the vertical line test.

7. Domain: Remember open dots mean the point is not included. This is best shown using interval notation. $x \in [0, 1) \cup (1, 2) \cup (2, 3) \cup (3, 4]$
   Range: $0 \leq y < 3$.
   This relation is a function since the curve passes the vertical line test.

8. This is a function since we can write $y = f(x) = \frac{x}{3} - \frac{1}{2}$.

$$f(\mathbf{-3}) = \frac{\mathbf{-3}}{3} - \frac{1}{2} = -1 - \frac{1}{2} = -\frac{3}{2} \quad \Rightarrow \quad \text{ordered pair is } (-3, -3/2)$$
$$f(\mathbf{-2}) = \frac{\mathbf{-2}}{3} - \frac{1}{2} = -\frac{4}{6} - \frac{3}{6} = -\frac{7}{6} \quad \Rightarrow \quad \text{ordered pair is } (-2, -7/6)$$
$$f(\mathbf{-3/2}) = \frac{\mathbf{-3/2}}{3} - \frac{1}{2} = -\frac{1}{2} - \frac{1}{2} = -1 \quad \Rightarrow \quad \text{ordered pair is } (-3/2, -1)$$
$$f(\mathbf{0}) = \frac{\mathbf{0}}{3} - \frac{1}{2} = 0 - \frac{1}{2} = -\frac{1}{2} \quad \Rightarrow \quad \text{ordered pair is } (0, -1/2)$$
$$f(\mathbf{1}) = \frac{\mathbf{1}}{3} - \frac{1}{2} = \frac{2}{6} - \frac{3}{6} = -\frac{1}{6} \quad \Rightarrow \quad \text{ordered pair is } (1, -1/6)$$
$$f(\mathbf{2}) = \frac{\mathbf{2}}{3} - \frac{1}{2} = \frac{4}{6} - \frac{3}{6} = \frac{1}{6} \quad \Rightarrow \quad \text{ordered pair is } (2, 1/6)$$



9. This is a function since we can write $y = f(x) = 14 - \frac{x^2}{2}$.

$$f(\mathbf{-3}) = 14 - \frac{(\mathbf{-3})^2}{2} = \frac{28}{2} - \frac{9}{2} = \frac{17}{2} \sim 8.5 \quad \Rightarrow \quad \text{ordered pair is } (-3, 17/2)$$

$$f(\mathbf{-2}) = 14 - \frac{(\mathbf{-2})^2}{2} = \frac{28}{2} - \frac{4}{2} = \frac{24}{2} = 12 \quad \Rightarrow \quad \text{ordered pair is } (-2, 12)$$

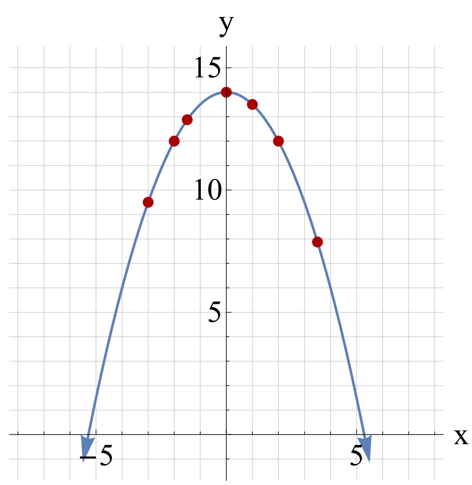
$$f(\mathbf{-3/2}) = 14 - \frac{(\mathbf{-3/2})^2}{2} = \frac{112}{8} - \frac{9}{8} = \frac{103}{8} \sim 12.875 \quad \Rightarrow \quad \text{ordered pair is } (-3/2, 103/8)$$

$$f(\mathbf{0}) = 14 - \frac{(\mathbf{0})^2}{2} = 14 - \frac{0}{2} = 14 \quad \Rightarrow \quad \text{ordered pair is } (0, 14)$$

$$f(\mathbf{1}) = 14 - \frac{(\mathbf{1})^2}{2} = \frac{28}{2} - \frac{1}{2} = \frac{27}{2} \sim 13.5 \quad \Rightarrow \quad \text{ordered pair is } (1, 27/2)$$

$$f(\mathbf{2}) = 14 - \frac{(\mathbf{2})^2}{2} = 14 - 2 = 12 \quad \Rightarrow \quad \text{ordered pair is } (2, 12)$$

$$f(\mathbf{7/2}) = 14 - \frac{(\mathbf{7/2})^2}{2} = \frac{112}{8} - \frac{49}{8} = \frac{63}{8} \sim 7.875 \quad \Rightarrow \quad \text{ordered pair is } (7/2, 63/8)$$



10. This is not a function since we can write $y = f(x)$ and get the entire curve. We can solve for $x = 9 - \frac{y^2}{4}$. This problem was chosen specifically since it is not a function but we can still get ordered pairs.

$$y = -6 \text{ then } x = 9 - \frac{(-6)^2}{4} = 9 - 9 = 0 \quad \Rightarrow \quad \text{ordered pair is } (0, -6)$$

$$y = -4 \text{ then } x = 9 - \frac{(-4)^2}{4} = 9 - 4 = 5 \quad \Rightarrow \quad \text{ordered pair is } (5, -4)$$

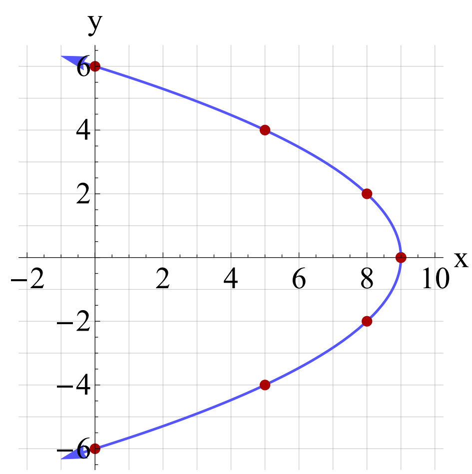
$$y = -2 \text{ then } x = 9 - \frac{(-2)^2}{4} = 9 - 1 = 8 \quad \Rightarrow \quad \text{ordered pair is } (8, -2)$$

$$y = 0 \text{ then } x = 9 - \frac{(0)^2}{4} = 9 - 0 = 9 \quad \Rightarrow \quad \text{ordered pair is } (9, 0)$$

$$y = 2 \text{ then } x = 9 - \frac{(2)^2}{4} = 9 - 1 = 8 \quad \Rightarrow \quad \text{ordered pair is } (8, 2)$$

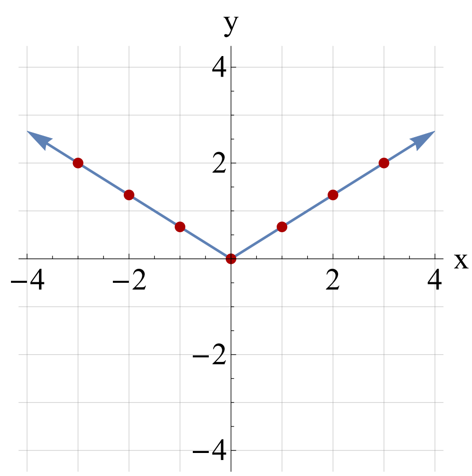
$$y = 4 \text{ then } x = 9 - \frac{(4)^2}{4} = 9 - 4 = 5 \quad \Rightarrow \quad \text{ordered pair is } (5, 4)$$

$$y = 6 \text{ then } x = 9 - \frac{(6)^2}{4} = 9 - 9 = 0 \quad \Rightarrow \quad \text{ordered pair is } (0, 6)$$



11. This is a function since we can write $y = f(x) = \frac{1}{3}\left|2x\right|$.

$$f(\mathbf{-3}) = \frac{1}{3}\left|2(\mathbf{-3})\right| = \frac{1}{3}\left|-6\right| = \frac{1}{3}(6) = 2 \quad \Rightarrow \quad \text{ordered pair is } (-3, 2)$$
$$f(\mathbf{-2}) = \frac{1}{3}\left|2(\mathbf{-2})\right| = \frac{1}{3}\left|-4\right| = \frac{1}{3}(4) = \frac{4}{3} \quad \Rightarrow \quad \text{ordered pair is } (-2, 4/3)$$
$$f(\mathbf{-1}) = \frac{1}{3}\left|2(\mathbf{-1})\right| = \frac{1}{3}\left|-2\right| = \frac{1}{3}(2) = \frac{2}{3} \quad \Rightarrow \quad \text{ordered pair is } (-1, 2/3)$$
$$f(\mathbf{0}) = \frac{1}{3}\left|2(\mathbf{0})\right| = \frac{1}{3}\left|0\right| = \frac{1}{3}(0) = 0 \quad \Rightarrow \quad \text{ordered pair is } (0, 0)$$
$$f(\mathbf{1}) = \frac{1}{3}\left|2(\mathbf{1})\right| = \frac{1}{3}\left|2\right| = \frac{1}{3}(2) = \frac{2}{3} \quad \Rightarrow \quad \text{ordered pair is } (1, 2/3)$$
$$f(\mathbf{2}) = \frac{1}{3}\left|2(\mathbf{2})\right| = \frac{1}{3}\left|4\right| = \frac{1}{3}(4) = \frac{4}{3} \quad \Rightarrow \quad \text{ordered pair is } (2, 4/3)$$
$$f(\mathbf{3}) = \frac{1}{3}\left|2(\mathbf{3})\right| = \frac{1}{3}\left|6\right| = \frac{1}{3}(6) = 2 \quad \Rightarrow \quad \text{ordered pair is } (3, 2)$$



12. Domain of rational function is all x where the denominator it not equal to zero, so $x \neq 0$.

    Interval notation: $x \in (-\infty, 0) \cup (0, \infty)$.

13. Domain of rational function is all x where the denominator it not equal to zero, so solve

$$(3x+1)(4x+2)(5x-6) = 0$$
$$3x+1=0 \quad \text{or} \quad 4x+2=0 \quad \text{or} \quad 5x-6=0 \qquad \text{(zero factor property)}$$
$$x=-1/3 \quad \text{or} \quad x=-2 \quad \text{or} \quad x=6/5 \qquad \text{(solve)}$$

    So domain is all x except $x = -1/3, x = -2, x = 6/5$.

    Interval notation: $x \in (-\infty, -2) \cup (-2, -\frac{1}{3}) \cup (-\frac{1}{3}, \frac{6}{5}) \cup (\frac{6}{5}, \infty)$.

14. We need to factor the denominator.

Two numbers whose product is 36 and sum is 12: 6,6.

$$\begin{aligned}
9x^2 + \mathbf{12x} + 4 &= 9x^2 + \mathbf{6x + 6x} + 4 && \text{(rewrite middle term)}\\
&= \big(9x^2 + 6x\big) + \big(6x + 4\big)\\
&= 3x\big(\mathbf{3x+2}\big) + 2\big(\mathbf{3x+2}\big) && \text{(factor)}\\
&= (3x+2)\big(\mathbf{3x+2}\big) && \text{(factor)}\\
&= (3x+2)^2
\end{aligned}$$

So the denominator is a perfect square! I missed that.

So domain should exclude $3x + 2 = 0$ to avoid division by zero, which is $x = -2/3$.

Interval notation: $x \in (-\infty, -2/3) \cup (-2/3, \infty)$.

15. We need to factor the denominator, $-12x^2 + 4x + 8 = 4(-3x^2 + x + 2)$.

    Two numbers whose product is -6 and sum is 1: $3, -2$.

$$\begin{aligned}
-12x^2 + 4x + 8 &= -4\Big(-3x^2 + \mathbf{x} + 2\Big)\\
&= 4\Big(-3x^2 + \mathbf{3x - 2x} + 2\Big) && \text{(rewrite middle term)}\\
&= 4\Big(\big(-3x^2 + 3x\big) + \big(-2x + 2\big)\Big)\\
&= 4\Big(3x\big(\mathbf{-x+1}\big) + 2\big(\mathbf{-x+1}\big)\Big) && \text{(factor)}\\
&= 4\Big(\big(3x+2\big)\big(\mathbf{-x+1}\big)\Big) && \text{(factor)}\\
&= 4\big(3x+2\big)\big(1-x\big)
\end{aligned}$$

    So domain should exclude $3x + 2 = 0$ and $1 - x = 0$ to avoid division by zero, which is $x = -2/3, 1$.

    Interval notation: $x \in (-\infty, -2/3) \cup (-2/3, 1) \cup (1, \infty)$.

16.

$$f(\mathbf{1/2}) = \frac{\mathbf{(1/2)}}{-12\mathbf{(1/2)}^2 + 4\mathbf{(1/2)} + 8} = \frac{(1/2)}{-3 + 2 + 8} = \frac{(1/2)}{7} = \frac{1}{2} \cdot \frac{1}{7} = \frac{1}{14}$$

17.

$$f(\mathbf{7/3}) = \frac{\mathbf{(7/3)}}{\mathbf{(7/3)}^2 - 9} = \frac{(7/3)}{49/9 - 81/9} = \frac{(7/3)}{(-32/9} = \frac{7}{3} \cdot \left(-\frac{9}{32}\right) = -\frac{21}{32}$$

18. This is an important type of evaluation you will see more of with average rate of change of a function over an interval.

$$\begin{aligned}
f(\mathbf{3+h}) &= 4\mathbf{(3+h)}^2 + 2 && \text{(functional evaluation)}\\
&= 4\mathbf{(9 + 6h + h^2)} + 2 && \text{(expand the square)}\\
&= 36 + 24h + 4h^2 + 2 && \text{(distribute the 4)}\\
&= 38 + 24h + 4h^2 && \text{(collect like terms)}
\end{aligned}$$