Questions

1. A number is doubled and then increased by seven. The result is ninety-three. What is the original number?

2. Six less than five times a number is the same as seven times the number. What is the number?

3. Brad is a waiter, and he gets paid \$5.75 per hour, and he can keep his tips. He knows his tips average \$8.80 per table. If he worked an eight-hour shift and took home \$169.20, how many tables did he serve?

4. On May 18, 1990 the fastest speed of any national railroad was achieved by the French high speed train *Train á Grande Vitess* as it traveled over a distance from Cortalain to Tours, France. A commentator said that this speed was so fast that if it continued at that rate, the train would travel 6404 miles in 20 hours. How fast did the train travel on that date?

5. Two trains leave a train station at the same time. One train travels east at 50 mph. The other train travels west at 55mph. In how many hours will the two trains be 315 miles apart?

6. In warmer climates, approximate temperature predictions can be made by counting the number of chirps a cricket makes during a minute. The temperature (in Fahrenheit) decreased by 40 is equivalent to one-forth of the number of cricket chirps in a minute.

(a) Write an equation for this relationship.
(b) Approximately how many chirps per minute should be recorded if the temperature is 90 F?
(c) If a person recorded 48 cricket chirps in a minute, what would the temperature be?

Solutions

I am showing lots of work here for those who need the organizational structure to get the answer. You should show as much work as you need to communicate the result effectively to your peers, and so that you can come back to the problem as understand your solution quickly. Your solution may look significantly different from mine and still be correct. Check with me or a TA if you have any questions.

1. A number is doubled and then increased by seven. The result is ninety-three. What is the original number?

- **Understand the problem**. We are looking for a number, let's call it x.
- **Write an equation.**
  The number is doubled ($2x$) *and then increased by seven* ($2x + 7$). *The result is ninety-three* ($2x + 7 = 93$).
- **Solve and state the answer.**

$$\begin{aligned} 2x + 7 &= 93 \\ 2x &= 86 \\ x &= 43 \end{aligned}$$

  The number is 43.
- **Check.** Is seven more than two times forty-three ninety-three?

$$2(43) + 7 = 86 + 7 = 93\checkmark$$

2. Six less than five times a number is the same as seven times the number. What is the number?

- **Understand the problem**. We are looking for a number, let's call it x.
- **Write an equation.**
  Six less than five times a number: $5x - 6$
  Seven times the same number: $7x$
  These things are the same: $5x - 6 = 7x$.
- **Solve and state the answer.**

$$\begin{aligned} 5x - 6 &= 7x \\ -2x &= 6 \\ x &= -3 \end{aligned}$$

  The number is -3.

- **Check.**

$$5(-3) - 6 \stackrel{?}{=} 7(-3) \Rightarrow -21 \stackrel{?}{=} -21\checkmark$$

3. Brad is a waiter, and he gets paid \$5.75 per hour, and he can keep his tips. He knows his tips average \$8.80 per table. If he worked an eight-hour shift and took home \$169.20, how many tables did he serve?

Since this is more complicated, let's use the Mathematics Blueprint to organize the information and understand the problem. Start filling information in where ever it seems appropriate. Don't worry about putting something in the "wrong" column, your goal is just to collect information from the problem until you figure out what you need to do to solve it.

Gather Facts	Assign Variables	Basic Formula or Equation	Key Points
Brad is paid \$5.75 per hour. His tips average \$8.80 per table. He worked 8 hours. He took home \$169.20.	We need to know the number of tables Brad waited on, so let that be x.	The amount of money he earns from tips is $\$8.80x$. The amount of money he earned in salary in 8 hours is $\$5.75 \times 8 = \$46$. This money must add up to \$169.20: $8.80x+46 = 169.20$.	The number of tables should probably be an integer, and positive.

Now, we can solve the equation:

$$\begin{aligned} 8.80x + 46 &= 169.20 \\ 8.80x &= 123.20 \\ x &= 14 \end{aligned}$$

Brad waited on 14 tables.

Check by working backwards. If he waited on 14 tables he would have earned $\$5.75 \times 8 + 14 \times \$8.80 = \$169.20\checkmark$.

4. On May 18, 1990 the fastest speed of any national railroad was achieved by the French high speed train *Train á Grande Vitess* as it traveled over a distance from Cortalain to Tours, France. A commentator said that this speed was so fast that if it continued at that rate, the train would travel 6404 miles in 20 hours. How fast did the train travel on that date?

Gather Facts	Assign Variables	Basic Formula or Equation	Key Points
Train would have traveled 6404 miles in 20 hours. The distance is 6404 miles. The time is 20 hours.	We need to know the speed of the train, so let that be x.	Speed is given by the formula $x = \frac{\text{distance}}{\text{time}}$.	The speed should have units of miles per hour.

Now, we can solve the equation:

$$\begin{aligned} x &= \frac{\text{distance}}{\text{time}} \\ x &= \frac{6404\text{miles}}{20\text{hours}} \\ x &= 320.2\frac{\text{miles}}{\text{hours}} \end{aligned}$$

The speed of the train was 320.2 mph.

Check: In 20 hours, a train traveling at 320.2 miles per hour would travel a distance of $20 \times 320.2 = 6404$ miles. ✓

5. Two trains leave a train station at the same time. One train travels east at 50 mph. The other train travels west at 55mph. In how many hours will the two trains be 315 miles apart?

Gather Facts	Assign Variables	Basic Formula or Equation	Key Points
In 1 hours, EAST bound Train would have traveled 50×1 miles. In 2 hours, EAST bound Train would have traveled 50×2 miles. In t hours, EAST bound Train would have traveled $50t$ miles. In t hours, WEST bound Train would have traveled $55t$ miles.	We need to know the time it takes for something to happen, so let that be t (hours).	The distance between the two trains at time t is $50t + 55t$ miles. We want the distance to be 315 miles. Set the two expressions for the distance equal to get an equation: $50t + 55t = 315$.	

Now, we can solve the equation:

$$\begin{aligned} 50t + 55t &= 315 \\ 105t &= 315 \\ t &= 3 \end{aligned}$$

After 3 hours the trains will be 315 miles apart.

Check: After 3 hours, the EAST bound train will be $50(3) = 150$ miles from the station, and the WEST bound train will be $55(3) = 165$ miles from the station, so they will be $150 + 165 = 315$ miles apart. ✓

6. In warmer climates, approximate temperature predictions can be made by counting the number of chirps a cricket makes during a minute. The temperature (in Fahrenheit) decreased by 40 is equivalent to one-forth of the number of cricket chirps in a minute.

(a) Write an equation for this relationship.
(b) Approximately how many chirps per minute should be recorded if the temperature is 90 F?
(c) If a person recorded 48 cricket chirps in a minute, what would the temperature be?

Gather Facts	Assign Variables	Basic Formula or Equation	Key Points
	Let the number of cricket chirps in a minute be be x. Let the temperature be T (in degrees Fahrenheit).	The temperature decreased by forty is $T - 40$. One-forth the number of chirps in a minute is $\frac{1}{4}x$. These are equal: $T - 40 = \frac{1}{4}x$.	We are looking for a relationship between the number of cricket chirps and the temperature.

(a) The relationship between the number of cricket chirps per minute and the temperature is

$$T - 40 = \frac{1}{4}x$$

(b) If the temperature is 90 F, the number of chirps per minute will satisfy $90 - 40 = \frac{1}{4}x \Rightarrow 200 = x$. There should be 200 chirps per minute.

(c) If the number of chirps is 148, the temperature will satisfy $T - 40 = \frac{1}{4}(148) \Rightarrow T = 77$. The temperature is 77 F.