Questions

1. Ninety-eight passengers rode in a train from Boston to Denver. Tickets for regular coach seats cost $120, and sleeper car seats cost $290. The receipts from the trip totaled $19,750. How many passengers purchased each type of ticket?

2. Ventex makes auto radar detectors. Ventex has found that its basic model requires 3 hours of manufacturing for the inside components and 2 hours for the housing and controls. Its advanced model requires 5 hours to manufacture the inside components and 3 hours for the housing and controls. This week, the production division has available 1050 hours for producing inside components and 660 hours for housing and controls. How many detectors of each type can be made?

3. Against the wind a small plane flew 210 miles in 1 hour and 10 minutes. The return trip took only 50 minutes. What was the speed of teh wind? What was the speed of the plane in still air?

4. Tim Duncun scored 32 points in an NBA basketball game without scoring any 3-point shots. He scored 21 times. He made several free throws worth 1 point each and several regular shots from the floor, which were worth 2 points each. How many free throws did he make? How many 2-point shots did he make?

Solutions

1. Let x be the number of coach seats sold, and y the number of sleeper car seats sold.

$$\begin{aligned} \text{Total Receipts} &= \$120x + \$290y = \$19750 \\ \text{Number of Passengers} &= x + y = 98 \end{aligned}$$

System of linear equations in two unknowns. Solve by substitution, so second equation tells us $x = 98 - y$. Put this into the first equation:

$$\begin{aligned} 120x + 290y &= 19750 \\ 120(98 - y) + 290y &= 19750 \\ 120(98 - y) + 290y &= 19750 \\ 11760 - 120y + 290y &= 19750 \\ 170y &= 7990 \\ y &= 47 \end{aligned}$$

Now, use any earlier equation to solve for x when $y = 47$:

$$\begin{aligned} x &= 98 - y \\ x &= 98 - 47 \\ x &= 51 \end{aligned}$$

So there were 47 sleeper seats sold and 51 coach seats sold.

2. Let x be the number of basic models, and y the number of advanced models.

$$\begin{aligned} \text{Time for inside components} &= 1050 = 3x + 5y \\ \text{Time for housing controls} &= 660 = 2x + 3y \end{aligned}$$

System of linear equations in two unknowns. Solve by substitution, so second equation tells us $x = 330 - \frac{3}{2}y$. Put this into the first equation:

$$\begin{aligned} 1050 &= 3x + 5y \\ 1050 &= 3(330 - \frac{3}{2}y) + 5y \text{ solve for } y \\ 1050 &= 990 - \frac{9}{2}y + 5y \\ 1050 - 990 &= \frac{1}{2}y \\ 60 &= \frac{1}{2}y \\ 120 &= y \end{aligned}$$

Now, use any earlier equation to solve for x when $y = 120$:

$$\begin{aligned} x &= 330 - \frac{3}{2}y \\ x &= 330 - \frac{3}{2}(120) \\ x &= 330 - 180 = 150 \end{aligned}$$

So they can make 150 basic models and 120 advanced models.

3. Let speed of plane in still air be x. Let speed of wind be y.

Use the formula speed $= \dfrac{\text{distance}}{\text{time}}$.

Units (my choice) will be miles and hours, so 70 minutes = 7/6 hours, and 50 minutes = 5/6 hours.

$$\text{Speed for plane flying against wind } = x - y = \frac{210 \text{ miles}}{7/6 \text{ hours}} = 180 \text{ mph}$$
$$\text{Speed for plane flying with the wind } = x + y = \frac{210 \text{ miles}}{5/6 \text{ hours}} = 252 \text{ mph}$$

These reduce to:

$$x - y = 180$$
$$x + y = 252$$

System of linear equations in two unknowns. Solve by substitution, so second equation tells us $x = 252 - y$. Put this into the first equation:

$$\begin{aligned} x - y &= 180 \\ 252 - y - y &= 180 \text{ solve for } y \\ -2y &= -72 \\ y &= 36 \end{aligned}$$

Now, use any earlier equation to solve for x when $y = 36$:

$$x = 252 - y$$
$$x = 252 - 36 = 216$$

So the plane had speed in still air of 216 mph, and the wind speed was 36 mph.

4. Let x be the number of free throws, and y be the number of regular shots.

$$\text{Number of times Duncun scored } = 21 = x + y$$
$$\text{Total points Duncun scored } = 32 = x + 2y$$

System of linear equations in two unknowns. Solve by substitution, so second equation tells us $x = 32 - 2y$. Put this into the first equation:

$$\begin{aligned} 21 &= x + y \\ 21 &= 32 - 2y + y \\ 21 - 32 &= -y \\ -11 &= -y \\ 11 &= y \end{aligned}$$

Now, use any earlier equation to solve for x when $y = 11$:

$$x = 32 - 2y$$
$$x = 32 - 2(11) = 10$$

So Duncun scored 11 regular shots and 10 free throws.