Questions

Note: The negative exponent rule is extremely useful, but it isn't discussed in the test until Section 5.2.

- Negative Exponent: $x^{-n} = \dfrac{1}{x^n}$, if $x \neq 0$.

Knowing the negative exponent rule means you don't need to mess around with different cases when dealing with the quotient rule $\dfrac{x^a}{x^b} = x^{a-b}$. Feel free to use it here.

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1. To use the rules of exponents (except for product raised to a power and quotient raised to a power), what must be true of the bases? Write down the rules of exponents.

2. Evaluate $3x^0$ and $(3x)^0$. Why are the results different?

3. Multiply $(7^4)(7^6)$.

4. Multiply $w^{12} \cdot w^{20}$.

5. Multiply $(-16x^2y^4)(-5xy^3)$.

6. Multiply $(-12x^4y)(-7x^5y^3)$.

7. Multiply $(-8x^3y^2)(3xy^5)$.

8. Divide. Assume all variables in denominators are nonzero. $\dfrac{48x^5y^3}{24xy^3}$.

9. Divide. Assume all variables in denominators are nonzero. $\dfrac{45a^4b^3}{15a^4b^2}$.

10. Divide. Assume all variables in denominators are nonzero. $\dfrac{16x^5y}{-32x^2y^3}$.

11. What expression can be multiplied by $-3x^3yz$ to obtain $81x^8y^2z^4$?

12. Simplify $\left(\dfrac{8}{y^5}\right)^2$.

13. Simplify $\left(\dfrac{a^3b}{c^5d}\right)^5$.

Solutions

1. The bases must be the same. The rules of exponents are:

- $x^0 = 1$ if $x \neq 0$ (0^0 is indeterminant and is dealt with in calculus).
- Product Rule: $x^a \cdot x^b = x^{a+b}$.
- Quotient Rule: $\dfrac{x^a}{x^b} = x^{a-b}$.
- Power Rule: $(x^a)^b = x^{ab}$.
- Product Raised to Power Rule: $(xy)^a = x^a y^a$.
- Quotient Raised to a Power Rule: $\left(\dfrac{x}{y}\right)^a = \dfrac{x^a}{y^a}$ if $y \neq 0$.
- Negative Exponent: $x^{-n} = \dfrac{1}{x^n}$, if $x \neq 0$.

2. The results are different because the bases are different (one has base x, the other base $3x$).

$$3x^0 = 3(1) = 1$$
$$(3x)^0 = 1$$

3. $(7^4)(7^6) = 7^{4+6} = 7^{10}$

4. $w^{12} \cdot w^{20} = w^{12+20} = w^{32}$

5. $(-16x^2y^4)(-5xy^3) = 80x^{2+1}y^{4+3} = 80x^3y^7$

6. $(-12x^4y)(-7x^5y^3) = 84x^{4+5}y^{1+3} = 84x^9y^4$

7. $(-8x^3y^2)(3xy^5) = -24x^{3+1}y^{2+5} = -24x^4y^7$

8. $\dfrac{48x^5y^3}{24xy^3} = 2x^{5-1}y^{3-3} = 2x^4y^0 = 2x^4$

9. $\dfrac{45a^4b^3}{15a^4b^2} = 3a^{4-4}b^{3-2} = 3a^0b^1 = 3b$

10. $\dfrac{16x^5y}{-32x^2y^3} = -\dfrac{1}{2}x^{5-2}y^{1-3} = -\dfrac{1}{2}x^3y^{-2} = -\dfrac{x^3}{2y^2}$

11.

$$(\text{something})(-3x^3yz) = 81x^8y^2z^4$$
$$\text{something} = \frac{81x^8y^2z^4}{-3x^3yz} = -27x^{8-3}y^{2-1}z^{4-1} = -27x^5yz^3$$

12. $\left(\dfrac{8}{y^5}\right)^2 = \dfrac{8^2}{(y^5)^2} = \dfrac{64}{y^{5\cdot 2}} = \dfrac{64}{y^{10}}$

13. $\left(\dfrac{a^3b}{c^5d}\right)^5 = \dfrac{(a^3b)^5}{(c^5d)^5} = \dfrac{(a^3)^5(b)^5}{(c^5)^5(d)^5} = \dfrac{a^{15}b^5}{c^{25}d^5}$