

When multiplying polynomials you make use of The Distributive Property. You may want to review Section 1.6 The Distributive Property.

You will also frequently use the Product Rule of Exponents, $x^a \cdot x^b = x^{a+b}$.

Pay close attention to minus signs! It's often easiest to write subtraction as addition of a negative to ensure you get the sign correct. For example

$$\begin{aligned}(3x - 2)(x - 1) &= (3x + (-2))(x + (-1)) \text{ write subtraction as adding negative} \\ &= (3x + (-2))(x) + (3x + (-2))(-1) \text{ distribute} \\ &= 3x(x) + (-2)(x) + (3x)(-1) + (-2)(-1) \text{ distribute} \\ &= 3x^2 - 2x - 3x + 2 \text{ simplify} \\ &= 3x^2 - 5x + 2 \text{ collect like terms}\end{aligned}$$

This is not absolutely necessary, but useful if you find you make sign errors frequently.

Questions

1. Multiply $-2x(6x^3 - x)$.
2. Multiply $\frac{1}{2}(2x + 3x^2 + 5x^3)$.
3. Multiply $(2b^3 - 5b^2 + 3ab)(-3b^2)$.
4. Multiply $(-4x^3 + 6x^2 - 5x)(-7xy^2)$.
5. Multiply $(3x + 4)(5x - y)$.
6. Multiply $(4y + 1)(5y - 3)$.
7. Multiply $(5y + 1)(6y - 5)$.
8. What is wrong with this multiplication: $(x - 2)(-3) = 3x - 6$?
9. What is wrong with this multiplication: $-(3x - 7) = -3x - 7$?
10. Multiply $\left(\frac{1}{3}x + \frac{1}{5}\right)\left(\frac{1}{3}x - \frac{1}{2}\right)$.

Solutions

$$1. -2x(6x^3 - x) = -2x(6x^3) - 2x(-x) = -12x^4 + 2x^2$$

$$2. \frac{1}{2}(2x + 3x^2 + 5x^3) = \frac{1}{2}(2x) + \frac{1}{2}(3x^2) + \frac{1}{2}(5x^3) = x + \frac{3}{2}x^2 + \frac{5}{2}x^3$$

3.

$$\begin{aligned}(2b^3 - 5b^2 + 3ab)(-3b^2) &= (2b^3)(-3b^2) - (5b^2)(-3b^2) + (3ab)(-3b^2) \text{ distribute} \\ &= -6b^{3+2} + 15b^{2+2} - 9ab^{1+2} \text{ product rule of exponents} \\ &= -6b^5 + 15b^4 - 9ab^3 \text{ simplify}\end{aligned}$$

4.

$$\begin{aligned}(-4x^3 + 6x^2 - 5x)(-7xy^2) &= (-4x^3)(-7xy^2) + (6x^2)(-7xy^2) - (5x)(-7xy^2) \text{ distribute} \\ &= 28x^{3+1}y^2 - 42x^{2+1}y^2 + 35x^{1+1}y^2 \text{ product rule of exponents} \\ &= 28x^4y^2 - 42x^3y^2 + 35x^2y^2 \text{ simplify}\end{aligned}$$

5.

$$\begin{aligned}(3x + 4)(5x - y) &= (3x)(5x - y) + (4)(5x - y) \text{ distribute the } 5x - y \\ &= (3x)(5x) - (3x)(y) + (4)(5x) - (4)(y) \text{ distribute the } 3x \text{ and } 4 \\ &= 15x^2 - 3xy + 20x - 4y \text{ simplify}\end{aligned}$$

Sometimes it helps to use arrows to indicate how you are distributing, to make sure you get all the terms when you distribute. Notice that the order you distribute in does not matter:

$$\begin{aligned}\overbrace{(3x+4)}^{\curvearrowright} \overbrace{(5x-y)}^{\curvearrowleft} &= \underbrace{3x(5x-y)}_{\curvearrowright} + \underbrace{4(5x-y)}_{\curvearrowleft} \\ &= (3x)(5x) - 3x(y) + 4(5x) - 4y \\ &= 15x^2 - 3xy + 20x - 4y\end{aligned}$$

$$\begin{aligned}\overbrace{(3x+4)}^{\curvearrowright} \overbrace{(5x-y)}^{\curvearrowleft} &\stackrel{OR}{=} \underbrace{(3x+4)5x}_{\curvearrowright} - \underbrace{(3x+4)y}_{\curvearrowleft} \\ &= (3x)(5x) + 4(5x) - (3x)y - 4y \\ &= 15x^2 + 20x - 3xy - 4y\end{aligned}$$

6.

$$\begin{aligned}\overbrace{(4y+1)}^{\curvearrowright} \overbrace{(5y-3)}^{\curvearrowleft} &= \underbrace{(4y+1)5y}_{\curvearrowright} - \underbrace{(4y+1)3}_{\curvearrowleft} && \text{distribute} \\ &= (4y)(5y) + (1)(5y) - (4y)(3) - (1)(3) && \text{distribute} \\ &= 20y^2 + 5y - 12y - 3 && \text{simplify} \\ &= 20y^2 - 7y - 3 && \text{collect like terms}\end{aligned}$$

7.

$$\begin{aligned}
 (5y+1)(6y-5) &= (5y+1)(6y) + (5y+1)(-5) && \text{distribute} \\
 &= 5y(6y) + 1(6y) + 5y(-5) + 1(-5) && \text{distribute} \\
 &= 30y^2 + 6y - 25y - 5 && \text{simplify} \\
 &= 30y^2 - 19y - 5 && \text{collect like terms}
 \end{aligned}$$

8. Let's do the multiplication and then compare to find the error.

$$\begin{aligned}
 (x-2)(-3) &= (x)(-3) - (2)(-3) \\
 &= -3x - (-6) = -3x + 6
 \end{aligned}$$

Signs are incorrect.

9. Let's do the multiplication and then compare to find the error.

$$\begin{aligned}
 -(3x-7) &= -1(3x-7) \\
 &= -1(3x) + (-1)(-7) \\
 &= -3x + 7
 \end{aligned}$$

The minus sign was not distributed to the last term.

10.

$$\begin{aligned}
 \left(\frac{1}{3}x + \frac{1}{5}\right) \left(\frac{1}{3}x - \frac{1}{2}\right) &= \left(\frac{1}{3}x + \frac{1}{5}\right) \left(\frac{1}{3}x\right) + \left(\frac{1}{3}x + \frac{1}{5}\right) \left(-\frac{1}{2}\right) && \text{distribute} \\
 &= \left(\frac{1}{3}x\right) \left(\frac{1}{3}x\right) + \left(\frac{1}{5}\right) \left(\frac{1}{3}x\right) + \left(\frac{1}{3}x\right) \left(-\frac{1}{2}\right) + \left(\frac{1}{5}\right) \left(-\frac{1}{2}\right) && \text{distribute} \\
 &= \left(\frac{1}{9}x^2\right) + \left(\frac{1}{15}x\right) + \left(-\frac{1}{6}x\right) + \left(-\frac{1}{10}\right) && \text{simplify} \\
 &= \frac{1}{9}x^2 + \frac{2}{30}x - \frac{5}{30}x - \frac{1}{10} && \text{collect like terms: add fractions} \\
 &= \frac{1}{9}x^2 - \frac{3}{30}x - \frac{1}{10} && \text{collect like terms: add fractions} \\
 &= \frac{1}{9}x^2 - \frac{1}{10}x - \frac{1}{10} && \text{simplify}
 \end{aligned}$$