

Although these are called special cases, using the distributive property is all you need to do these multiplications correctly. However, it will help if you can recognize these special cases when they occur—these cases are given special attention since they will become special factoring formulas when we look at factoring.

We will call $(a + b)(a - b) = a^2 - b^2$ the difference of squares formula when we do factoring, so I will use that nomenclature here.

Using the distributive property to multiply quantities like $(3x^2 + x - 1)(x + 2)$ will serve you well in the future, so I will use that instead of vertical multiplication.

Questions

1. Multiply $(x + 6)(x - 6)$.
2. Multiply $(4x - 9)(4x + 9)$.
3. Multiply $\left(5x - \frac{1}{5}\right)\left(5x + \frac{1}{5}\right)$.
4. Multiply $(6x + 5)^2$.
5. Multiply $(8x - 3)^2$.
6. Multiply $\left(\frac{3}{5}x - \frac{1}{3}\right)\left(\frac{3}{5}x + \frac{1}{3}\right)$.
7. Multiply $\left(\frac{3}{5}x - \frac{1}{3}\right)^2$.
8. Multiply $(a^2 - 3a + 2)(a^2 + 4a - 3)$.
9. Multiply $(x^2 + 4x - 5)(x^2 - 3x + 4)$.
10. Multiply $(x + 7)(3x - 2)(x - 7)$.

Solutions

1. Difference of squares, $(a + b)(a - b) = a^2 - b^2$.

$$(x + 6)(x - 6) = x^2 - 36$$

2. Difference of squares, $(a + b)(a - b) = a^2 - b^2$.

$$(4x - 9)(4x + 9) = 16x^2 - 81$$

3. Difference of squares, $(a + b)(a - b) = a^2 - b^2$.

$$\left(5x - \frac{1}{5}\right)\left(5x + \frac{1}{5}\right) = 25x^2 - \frac{1}{25}$$

4. A binomial squared with addition, $(a + b)^2 = a^2 + 2ab + b^2$.

$$(6x + 5)^2 = 36x^2 + 60x + 25$$

5. A binomial squared with subtraction, $(a - b)^2 = a^2 - 2ab + b^2$.

$$(8x - 3)^2 = 64x^2 - 48x + 9$$

6. Difference of squares, $(a + b)(a - b) = a^2 - b^2$.

$$\left(\frac{3}{5}x - \frac{1}{3}\right)\left(\frac{3}{5}x + \frac{1}{3}\right) = \frac{9}{25}x^2 - \frac{1}{9}$$

7. A binomial squared with subtraction, $(a - b)^2 = a^2 - 2ab + b^2$.

$$\left(\frac{3}{5}x - \frac{1}{3}\right)^2 = \frac{9}{25}x^2 - \frac{2}{5}x + \frac{1}{9}$$

8. Use distributive property.

$$\begin{aligned} (a^2 - 3a + 2)(a^2 + 4a - 3) &= (a^2 - 3a + 2)(a^2) + (a^2 - 3a + 2)(4a) + (a^2 - 3a + 2)(-3) \\ &= (a^2)(a^2) - 3a(a^2) + 2(a^2) + a^2(4a) - 3a(4a) + 2(4a) + a^2(-3) - 3a(-3) + 2(-3) \\ &= a^4 - 3a^3 + 2a^2 + 4a^3 - 12a^2 + 8a - 3a^2 + 9a - 6 \\ &= a^4 + a^3 - 13a^2 + 17a - 6 \end{aligned}$$

9. Use distributive property.

$$\begin{aligned} (x^2 + 4x - 5)(x^2 - 3x + 4) &= (x^2 + 4x - 5)(x^2) + (x^2 + 4x - 5)(-3x) + (x^2 + 4x - 5)(4) \\ &= x^2(x^2) + 4x(x^2) - 5(x^2) + x^2(-3x) + 4x(-3x) - 5(-3x) + x^2(4) + 4x(4) - 5(4) \\ &= x^4 + 4x^3 - 5x^2 - 3x^3 - 12x^2 + 15x + 4x^2 + 16x - 20 \\ &= x^4 + x^3 - 13x^2 + 31x - 20 \end{aligned}$$

10. Use distributive property.

$$\begin{aligned}(x+7)(3x-2)(x-7) &= \left[\overbrace{(x+7)}^{\curvearrowright} \overbrace{(3x-2)}^{\curvearrowright} + \overbrace{(x+7)}^{\curvearrowright} \overbrace{(-2)}^{\curvearrowright} \right] (x-7) \\ &= \left[x(3x) + 7(3x) + x(-2) + 7(-2) \right] (x-7) \\ &= \left[3x^2 + 21x - 2x - 14 \right] (x-7) \\ &= \left[3x^2 + 19x - 14 \right] (x-7) \\ &= (3x^2 + 19x - 14)(x) + (3x^2 + 19x - 14)(-7) \\ &= \underbrace{3x^3 + 19x^2 - 14x} - \underbrace{21x^2 - 133x + 98} \\ &= 3x^3 - 2x^2 - 147x + 98\end{aligned}$$