Factoring polynomials is the distributive property done in reverse! To check your answers, use the distributive property to multiply our your final answer.

Sometimes when factoring by grouping you have to rearrange the terms to to get it to factor.

Questions

1. Factor by grouping $3x^2 - 6xy + 5x - 10y$.

2. Factor by grouping $4x + 8y - 3wx - 6wy$.

3. Factor by grouping $4u^2 + v + 4uv + u$.

4. Factor by grouping $6tx + r - 3t - 2rx$.

5. Factor by grouping $x^2 - 2x - xy + 2y$.

6. Tim was trying to factor $5x^2 - 3xy - 10x + 6y$. In his first step he wrote down $x(5x - 3y) + 2(-5x + 3y)$. Was he doing the problem correctly? What is the answer?

7. A train was traveling 73 miles per hour when the engineer spotted a stalled truck on the crossing ahead. He jammed on the brakes but was unable to stop the train before it collided with the truck. For every second the engineer applied the brakes, the train slowed down by 4 miles per hour. The accident reconstruction team found that the train was still traveling at 41 miles per hour at the time of impact. For how many second were the brakes applied?

Solutions

1. Identify common factors in pairs of terms. First two terms: Factor of $3x$. Last two terms: Factor of 5.

$$\begin{aligned}
3x^2 - 6xy + 5x - 10y &= 3x(x) - 3x(2y) + 5(x) - 5(2y) \\
&= 3x(x - 2y) + 5(x - 2y) \\
&= 3x(x - 2y) + 5(x - 2y) \text{ remove coloring, now factor this!} \\
&= 3x(x - 2y) + 5(x - 2y) \text{ Factor of } x - 2y \\
&= (3x + 5)(x - 2y)
\end{aligned}$$

Check:

$$\begin{aligned}
(3x + 5)(x - 2y) &= (3x + 5)(x) + (3x + 5)(-2y) \\
&= (3x)(x) + (5)(x) + (3x)(-2y) + (5)(-2y) \\
&= 3x^2 + 5x - 6xy - 10y, \text{ our answer is correct.}
\end{aligned}$$

2. Identify common factors in pairs of terms. First two terms: Factor of 4. Last two terms: Factor of $-3w$.

$$\begin{aligned}
4x + 8y - 3wx - 6wy &= 4(x) + 4(2y) + -3w(x) + -3w(2y) \\
&= 4(x + 2y) + -3w(x + 2y) \\
&= 4(x + 2y) + (-3w)(x + 2y) \\
&= (4 - 3w)(x + 2y)
\end{aligned}$$

3. Identify common factors in pairs of terms. Reorder to get factors. First two terms: Factor of u. Last two terms: Factor of v.

$$\begin{aligned}
4u^2 + v + 4uv + u &= 4u^2 + u + v + 4uv \text{ reorder to get a factor in first two terms} \\
&= u(4u) + u(1) + v(1) + v(4u) \text{ reorder to get a factor in first two terms} \\
&= u(4u + 1) + v(1 + 4u) \text{ factor} \\
&= u(4u + 1) + v(4u + 1) \text{ rearrange to clearly identify factor of } 4u + 1 \\
&= (4u + 1)(u + v) \text{ factor}
\end{aligned}$$

4.

$$\begin{aligned}
6tx + r - 3t - 2rx &= 6tx - 3t + r - 2rx \text{ reorder} \\
&= 3t(2x) - 3t(1) + r(1) + r(-2x) \text{ identify factors} \\
&= 3t(2x - 1) + r(1 - 2x) \text{ factor} \\
&= 3t(2x - 1) + (-r)(-1 + 2x) \text{ factor } -1 \text{ out of last term to get a common factor} \\
&= 3t(2x - 1) + (-r)(2x - 1) \text{ reorder to clearly identify factor} \\
&= (3t - r)(2x - 1) \text{ factor}
\end{aligned}$$

5.

$$\begin{aligned}
x^2 - 2x - xy + 2y &= x(x - 2) + y(-x + 2) \\
&= x(x - 2) - y(x - 2) \\
&= (x - y)(x - 2)
\end{aligned}$$

6. Damn straight Tim is on the right track! His approach is similar to what I did in the previous problem. Here's the complete solution:

$$
\begin{aligned}
5x^2 - 3xy - 10x + 6y &= x(5x - 3y) + 2(-5x + 3y) \\
&= x(5x - 3y) - 2(5x - 3y) \\
&= (x - 2)(5x - 3y)
\end{aligned}
$$

7. A nice work problem to chew on.

The train slowed by $73 - 41 = 32$ mph.

Brakes reduce the speed by $4 \ \frac{\text{mph}}{\text{sec}}$.

Time brakes were applied is $\dfrac{32\cancel{\text{mph}}}{4\cancel{\frac{\text{mph}}{\text{sec}}}} = 8$ sec.