Questions

1. Factor $9x^2 + 9x + 2$.

2. Factor $4x^2 + 11x + 6$.

3. Factor $15x^2 - 34x + 15$.

4. Factor $3a^2 - 10a - 8$.

5. Factor $12x^2 + 28x + 15$.

6. Factor $4x^4 - 11x^2 - 3$.

7. Factor $8x^2 + 16x - 10$.

8. Factor $16x^2 + 36x - 10$.

Note: My solutions contain both *trial and error* and *grouping method* (which is why they are so long–you don't need to do both).

Remember, your solution can be different in detail from mine and still be completely correct. You can always check your factoring by multiplying out.

Solutions

1. Factor $9x^2 + 9x + 2$.

Since the coefficient of x^2 is not 1, and there are no common factors we try *trial and error* or *the grouping method.*

Trial and Error

Factors of 9:	9 and 1
	3 and 3
Factors of 2:	1 and 2

Possible Factors	Middle Term	Correct?
$(9x+1)(x+2)$	$19x$	No
$(9x+2)(x+1)$	$11x$	No
$(3x+1)(3x+2)$	$9x$	Yes

Check: $(3x+1)(3x+2) = 9x^2 + 3x + 6x + 2 = 9x^2 + 9x + 2$.

Grouping Method

$9x^2 + 9x + 2$ has grouping number $9 \times 2 = 18$.

Find two numbers whose product is 18 and whose sum is 9: 3 and 6.

Now write the $9x$ term as two terms based on the numbers you found.

$$\begin{aligned} 9x^2 + 9x + 2 &= 9x^2 + 3x + 6x + 2 \\ &\text{(red terms have a factor of } 3x\text{)} \\ &\text{(blue terms have a factor of 2)} \\ &= 3x(3x+1) + 2(3x+1) \\ &\text{(both terms have a factor of } 3x+1\text{)} \\ &= (3x+2)(3x+1) \end{aligned}$$

Check: $(3x+1)(3x+2) = 9x^2 + 3x + 6x + 2 = 9x^2 + 9x + 2$.

You might also have written the following, which is entirely correct.

$$\begin{aligned} 9x^2 + 9x + 2 &= 9x^2 + 6x + 3x + 2 \\ &\text{(red terms have a factor of } 3x\text{)} \\ &\text{(blue terms have no factor (it appears))} \\ &= 3x(3x+2) + (3x+2) \\ &= 3x(3x+2) + 1(3x+2) \text{ (those blue terms actually have a factor of 1, so put it in)} \\ &\text{(both terms have a factor of } 3x+2\text{)} \\ &= (3x+1)(3x+2) \end{aligned}$$

2. Factor $4x^2 + 11x + 6$.

Since the coefficient of x^2 is not 1, and there are no common factors we try *trial and error* or *the grouping method.*

Trial and Error

Factors of 4:	4 and 1
	2 and 2
Factors of 6:	1 and 6
	2 and 3

Possible Factors	Middle Term	Correct?
$(4x + 1)(x + 6)$	$25x$	No
$(4x + 2)(x + 3)$	$14x$	No
$(2x + 1)(2x + 6)$	$14x$	No
$(2x + 2)(2x + 3)$	$10x$	No
$(4x + 6)(x + 1)$	$20x$	No
$(4x + 3)(x + 2)$	$11x$	Yes

Check: $(4x + 3)(x + 2) = 4x^2 + 3x + 8x + 6 = 4x^2 + 11x + 6$.

Grouping Method

$4x^2 + 11x + 6$ has grouping number $4 \times 6 = 24$.

Find two numbers whose product is 24 and whose sum is 11: 3 and 8.

Now write the $11x$ term as two terms based on the numbers you found.

$$
\begin{aligned}
4x^2 + 11x + 6 &= 4x^2 + 3x + 8x + 6 \\
&\text{(red terms have a factor of } x) \\
&\text{(blue terms have a factor of 2)} \\
&= x(4x + 3) + 2(4x + 3) \\
&\text{(both terms have a factor of } 4x + 3) \\
&= (x + 2)(4x + 3)
\end{aligned}
$$

Check: $(4x + 3)(x + 2) = 4x^2 + 3x + 8x + 6 = 4x^2 + 11x + 6$.

3. Factor $15x^2 - 34x + 15$.

Since the coefficient of x^2 is not 1, and there are no common factors we try *trial and error* or *the grouping method.*

Trial and Error

Factors of 15:	15 and 1 3 and 5	Signs must be negative since the middle term is negative $-34x$.

Possible Factors	Middle Term	Correct?
$(15x - 15)(1x - 1)$	$-30x$	No
$(15x - 3)(1x - 5)$	$-78x$	No
$(3x - 15)(5x - 1)$	$-78x$	No
$(3x - 3)(5x - 5)$	$-30x$	No
$(15x - 1)(1x - 15)$	$-226x$	No
$(15x - 5)(1x - 3)$	$-50x$	No
$(3x - 1)(5x - 15)$	$-50x$	No
$(3x - 5)(5x - 3)$	$-34x$	Yes (finally!)

Check: $(3x - 5)(5x - 3) = 15x^2 - 25x - 9x + 15 = 15x^2 - 34x + 15$.

Grouping Method

$15x^2 - 34x + 15$ has grouping number $15 \times 15 = 225$.

Find two numbers whose product is 225 and whose sum is -34: -9 and -25.

Hint: Look for numbers "in the middle" rather than on the edges (this would help in the trial and error as well). What I mean is, don't start with $-1 \times (-225)$ since that does equal 225, but obviously won't have a sum of -34. This will just speed things up, you can always examine all the factors of 225.

Now write the $-34x$ term as two terms based on the numbers you found.

$$\begin{aligned}
15x^2 - 34x + 15 &= 15x^2 - 9x - 25x + 15 \\
&\text{(red terms have a factor of } 3x) \\
&\text{(blue terms have a factor of } 5) \\
&= 3x(5x - 3) + 5(-5x + 3) \\
&= 3x(5x - 3) - 5(5x - 3) \text{ (factor a } -1 \text{ out of second term to get common factor in each term)} \\
&\text{(both terms have a factor of } 5x - 3) \\
&= (3x - 5)(5x - 3)
\end{aligned}$$

Check: $(3x - 5)(5x - 3) = 15x^2 - 25x - 9x + 15 = 15x^2 - 34x + 15$.

4. Factor $3a^2 - 10a - 8$.

Since the coefficient of a^2 is not 1, and there are no common factors we try *trial and error* or *the grouping method.*

Trial and Error

Factors of 3:	3 and 1	
Factors of 8:	2 and 4	Signs must be opposite since the last term is negative (-8).
	1 and 8	

Possible Factors	Middle Term	Correct?
$(3a - 2)(1a + 4)$	$+10x$	No, but only out by sign, so switch them
$(3a + 2)(1a - 4)$	$-10x$	Yes

Check: $(3a + 2)(a - 4) = 3a^2 - 12a + 2s - 8 = 3a^2 - 10a - 8$.

Grouping Method

$3a^2 - 10a - 8$ has grouping number $3 \times (-8) = -24$.

Find two numbers whose product is -24 and whose sum is -10: -12 and 2.

Now write the $-10a$ term as two terms based on the numbers you found.

$$\begin{aligned}
3a^2 - 10a - 8 &= 3a^2 - 12a + 2a - 8 \\
&\text{(red terms have a factor of } 3a) \\
&\text{(blue terms have a factor of } 2) \\
&= 3a(a - 4) + 2(a - 4) \\
&\text{(both terms have a factor of } a - 4) \\
&= (3a + 2)(a - 4)
\end{aligned}$$

Check: $(3a + 2)(a - 4) = 3a^2 - 12a + 2s - 8 = 3a^2 - 10a - 8$.

5. Factor $12x^2 + 28x + 15$.

Since the coefficient of x^2 is not 1, and there are no common factors we try *trial and error* or *the grouping method.*

Trial and Error

Factors of 12:	12 and 1
	6 and 2
	3 and 4
Factors of 15:	15 and 1
	5 and 3

Signs must be the same since all terms are positive.

Possible Factors	Middle Term	Correct?
$(12x + 15)(1x + 1)$	$27x$	No
$(6x + 5)(2x + 3)$	$28x$	Yes

Check: $(6x + 5)(2x + 3) = 12x^2 + 10x + 18x + 15 = 12x^2 + 28x + 15$.

Grouping Method

$12x^2 + 28x + 15$ has grouping number $12 \times (15) = 180$.

Find two numbers whose product is 180 and whose sum is 28: 10 and 18.

Now write the $28x$ term as two terms based on the numbers you found.

$$
\begin{aligned}
12x^2 + 28x + 15 &= 12x^2 + 10x + 18x + 15 \\
&\text{(red terms have a factor of } 2x\text{)} \\
&\text{(blue terms have a factor of 3)} \\
&= 2x(6x + 5) + 3(6x + 5) \\
&\text{(both terms have a factor of } 6x + 5\text{)} \\
&= (2x + 3)(6x + 5)
\end{aligned}
$$

Check: $(2x + 3)(6x + 5) = 12x^2 + 10x + 18x + 15 = 12x^2 + 28x + 15$.

6. Factor $4x^4 - 11x^2 - 3$.

Note: We can work with $z = x^2$ in this problem. The problem has been cooked so $4z^2 - 11z - 3$ is one we can solve with our current techniques.

Since the coefficient of z^2 is not 1, and there are no common factors we try *trial and error* or *the grouping method.*

Trial and Error

Factors of 4:	4 and 1
	2 and 2
Factors of 3:	3 and 1

Signs must be the opposite since the last term is negative.

Possible Factors	Middle Term	Correct?
$(4z - 3)(1z + 1)$	z	No
$(2z - 3)(2z + 1)$	$-4z$	No
$(4z - 1)(1z + 3)$	$11z$	No, but only differs by sign, so switch signs.
$(4z + 1)(1z - 3)$	$-11z$	Yes.

Check: $(4z + 1)(1z - 3) = 4z^2 - 11z - 3$, or $(4x^2 + 1)(x^2 - 3) = 4x^4 - 11x^2 - 3$.

Grouping Method

$4x^4 - 11x^2 - 3$ has grouping number $4 \times (-3) = -12$.

Find two numbers whose product is -12 and whose sum is -11: -12 and 1.

Now write the $-11x^2$ term as two terms based on the numbers you found.

$$
\begin{aligned}
4x^4 - 11x^2 - 3 &= 4x^4 - 12x^2 + x^2 - 3 \\
&\text{(red terms have a factor of } 4x^2\text{)} \\
&\text{(blue terms have a factor of 1)} \\
&= 4x^2(x^2 - 3) + (x^2 - 3) \\
&\text{(both terms have a factor of } x^2 - 3\text{)} \\
&= (4x^2 + 1)(x^2 - 3)
\end{aligned}
$$

Check: $(4x^2+1)(x^2-3) = 4x^4 + x^2 - 12x^2 - 3 = 4x^4 - 11x^2 - 3$.

Note that in the grouping method, you didn't have to introduce $z = x^2$.

7. Factor $8x^2 + 16x - 10$.

There is a common factor: $8x^2 + 16x - 10 = 2(4x^2 + 8x - 5)$.

Since the coefficient of x^2 is not 1, and there are no common factors we try *trial and error* or *the grouping method.*

Trial and Error

Factors of 4:	2 and 2
	4 and 1
Factors of 5:	5 and 1

Signs must be opposite since the last term is negative.

Possible Factors	Middle Term	Correct?
$(2x+5)(2x-1)$	$8x$	Yes

Check: $2(2x+5)(2x-1) = 2(4x^2 + 10x - 2x - 5) = 2(4x^2 + 8x - 5) = 8x^2 + 16x - 10$.

Grouping Method

$8x^2 + 16x - 10$ has grouping number $8 \times (-10) = -80$.

Find two numbers whose product is -80 and whose sum is 16: -4 and 20.

Now write the $16x$ term as two terms based on the numbers you found.

$$\begin{aligned}
8x^2 + 16x - 10 &= 8x^2 + 20x - 4x - 10 \\
&\text{(red terms have a factor of } 4x\text{)} \\
&\text{(blue terms have a factor of } -2\text{)} \\
&= 4x(2x+5) + (-2)(2x+5) \\
&\text{(both terms have a factor of } 2x+5\text{)} \\
&= (4x-2)(2x+5) \\
&= 2(2x-1)(2x+5)
\end{aligned}$$

Check: $2(2x+5)(2x-1) = 2(4x^2 + 10x - 2x - 5) = 2(4x^2 + 8x - 5) = 8x^2 + 16x - 10$.

Note that in the grouping method, you didn't have to factor the 2 out at the beginning!

8. Factor $16x^2 + 36x - 10$.

Since the coefficient of x^2 is not 1, we try *trial and error* or *the grouping method.* Let's see what happens with the trial and error method if we don't factor out the common factor of 2 at the beginning.

Trial and Error

Factors of 16:	8 and 2
	4 and 4
	16 and 1
Factors of 10:	2 and 5
	10 and 1

Signs must be opposite since the last term is negative.

Possible Factors	Middle Term	Correct?
$(8x+2)(2x-5)$	$-36x$	No, but only differs in sign, so change the signs
$(8x-2)(2x+5)$	$36x$	Yes

Check: $(8x - 2)(2x + 5) = 2(4x - 1)(2x + 5) = 2(8x^2 - 2x + 20x - 5) = 2(8x^2 + 18x - 5) = 16x^2 + 36x - 10$.

$\boxed{\text{Note that in the trial and error method, you didn't have to factor the 2 out at the beginning!}}$

Factoring out the common factor at the beginning just makes the problem easier since you are working with smaller numbers.

Grouping Method

$16x^2 + 36x - 10$ has grouping number $16 \times (-10) = -160$.

Find two numbers whose product is -160 and whose sum is 36: 40 and -4.

Now write the $36x$ term as two terms based on the numbers you found.

$$
\begin{aligned}
16x^2 + 36x - 10 &= 16x^2 + 40x - 4x - 10 \\
&\quad \text{(red terms have a factor of } 8x\text{)} \\
&\quad \text{(blue terms have a factor of } -2\text{)} \\
&= 8x(2x + 5) + (-2)(2x + 5) \\
&\quad \text{(both terms have a factor of } 2x + 5\text{)} \\
&= (8x - 2)(2x + 5) \\
&= 2(4x - 1)(2x + 5)
\end{aligned}
$$

Check: $(8x - 2)(2x + 5) = 2(4x - 1)(2x + 5) = 2(8x^2 - 2x + 20x - 5) = 2(8x^2 + 18x - 5) = 16x^2 + 36x - 10$.