These problems can be done using the grouping method (except for the cubes), but try to focus on identifying them as one of the special cases and use the formulas. You will be expected to know these formulas in future math classes.

• Difference of Squares (*Note there is no Sum of Squares formula!*)

$$a^2 - b^2 = (a - b)(a + b).$$

• Perfect Square (sum and difference)

$$a^2 + 2ab + b^2 = (a + b)^2,$$
$$a^2 - 2ab + b^2 = (a - b)^2.$$

• Sum of Cubes, and Difference of Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2),$$
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2).$$

Remember that for the cubes, you will not be able to factor the resulting quadratic using the techniques of this unit.

Remember to check your answers by multiplying out.

**Questions**

**1.** Factor $25a^2 - 16$.

**2.** Factor $9x^2 - 49y^2$.

**3.** Factor $25x^2 + 10x + 1$.

**4.** Factor $y^2 - 12y + 36$.

**5.** Factor $16x^2 - 72x + 81$.

**6.** Factor $49x^2 - 42x + 9$.

**7.** Factor $169a^2 + 26ab + b^2$.

**8.** Factor $9x^2 - 25$.

**9.** Factor $r^3 - 1$.

**10.** Factor $8x^3 + 27$.

**11.** Jerome says he can find two values of $b$ so that $100x^2 + bx - 9$ will be a perfect square. Kesha says there is only one that fits, and Larry says there are none. Who is correct and why?

**12.** Factor $128x^2 + 96x + 18$.

**Solutions**

**1.** Difference of squares, with $5a$ and 4 being squared.

$$25a^2 - 16 = (5a+4)(5a-4)$$

Check:

$$\begin{aligned}(5a+4)(5a-4) &= (5a+4)(5a) + (5a+4)(-4)\\ &= (5a)(5a) + (4)(5a) + (5a)(-4) + (4)(-4)\\ &= 25a^2 + 20a - 20a - 16 = 25a^2 - 16\end{aligned}$$

**2.** Difference of squares, with $3x$ and $7y$ being squared.

$$9x^2 - 49y^2 = (3x-7y)(3x+7y)$$

Check:

$$\begin{aligned}(3x-7y)(3x+7y) &= (3x)(3x+7y) + (-7y)(3x+7y)\\ &= (3x)(3x) + (3x)(7y) + (-7y)(3x) + (-7y)(7y)\\ &= 9x^2 + 21xy - 21xy - 49y^2 = 9x^2 - 49y^2\end{aligned}$$

**3.** With $5x$ and 1 being squared, cross term should be $2 \cdot 5x \cdot 1 = 10x$, which it is. Since the cross term is positive, this is a Perfect Square (sum).

$$25x^2 + 10x + 1 = (5x+1)^2$$

Check:

$$\begin{aligned}(5x+1)^2 &= (5x+1)(5x+1)\\ &= (5x)(5x+1) + (1)(5x+1)\\ &= (5x)(5x) + (5x)(1) + 5x + 1\\ &= 25x^2 + 10x + 1\end{aligned}$$

**4.** With $y$ and 6 being squared, cross term should be $2 \cdot y \cdot 6 = 12y$, which it is. Since the cross term is negative, this is a Perfect Square (difference).

$$y^2 - 12y + 36 = (y-6)^2$$

Check:

$$\begin{aligned}(y-6)^2 &= (y-6)(y-6)\\ &= (y-6)(y) + (y-6)(-6)\\ &= (y)(y) + (-6)(y) + (y)(-6) + (-6)(-6)\\ &= y^2 - 12y + 36\end{aligned}$$

**5.** With $4x$ and 9 being squared, cross term should be $2 \cdot 4x \cdot 9 = 72x$, which it is. Since the cross term is negative, this is a Perfect Square (difference).

$$16x^2 - 72x + 81 = (4x-9)^2$$

Check:

$$\begin{aligned}(4x-9)^2 &= (4x-9)(4x-9)\\ &= (4x-9)(4x) + (4x-9)(-9)\\ &= (4x)(4x) + (-9)(4x) + (4x)(-9) + (-9)(-9)\\ &= 16x^2 - 72x + 81\end{aligned}$$

**6.** With $7x$ and 3 being squared, cross term should be $2 \cdot 7x \cdot 3 = 42x$, which it is. Since the cross term is negative, this is a Perfect Square (difference).

$$49x^2 - 42x + 9 = (7x-3)^2$$

Check:

$$\begin{aligned}(7x-3)^2 = (7x-3)(7x-3) &= (7x-3)(7x-3)\\ &= (7x)(7x-3) + (-3)(7x-3)\\ &= (7x)(7x) + (7x)(-3) + (-3)(7x) + (-3)(-3)\\ &= 49x^2 - 42x + 9\end{aligned}$$

**7.** With $13a$ and $b$ being squared, cross term should be $2 \cdot 13a \cdot b = 26ab$, which it is. Since the cross term is positive, this is a Perfect Square (sum).

$$169a^2 + 26ab + b^2 = (13a+b)^2$$

Check:

$$\begin{aligned}(5x+1)^2 &= (13a+b)(13a+b)\\ &= (13a+b)(13a) + (13a+b)(b)\\ &= (13a)(13a) + (b)(13a) + (13a)(b) + (b)(b)\\ &= 169a^2 + 26ab + b^2\end{aligned}$$

**8.** Difference of squares, with $3x$ and 5 being squared.

$$9x^2 - 25 = (3x-5)(3x+5)$$

Check:

$$\begin{aligned}(3x-5)(3x+5) &= (3x)(3x+5) + (-5)(3x+5)\\ &= (3x)(3x) + (3x)(5) + (-5)(3x) + (-5)(5)\\ &= 9x^2 - 25\end{aligned}$$

**9.** Difference of cubes, where $r$ and 1 are being cubed.

$$r^3 - 1 = (r-1)(r^2+r+1)$$

Check:

$$\begin{aligned}(r-1)(r^2+r+1) &= (r-1)(r^2) + (r-1)(r) + (r-1)(1)\\ &= r^3 - r^2 + r^2 - r + r - 1 = r^3 - 1\end{aligned}$$

**10.** Sum of cubes, where $2x$ and 3 are being cubed.

$$8x^3 + 27 = (2x-3)((2x)^2 + (2x)(3) + (3)^2) = (2x-3)(4x^2+6x+9)$$

Check:

$$\begin{aligned}(2x-3)(4x^2+6x+9) &= (2x-3)(4x^2) + (2x-3)(6x) + (2x-3)(9)\\ &= 8x^3 - 12x^2 + 12x^2 - 18x + 18x - 27 = 8x^3 - 27\end{aligned}$$

**11.** Larry is right. In both the perfect square formulas, the last term is positive ($a^2 \pm 2ab\underline{+b^2}$), and since $100x^2 + bx - 9 = (10x)^2 + bx - (3)^2$ has the last term negative, there is no way to make this into a perfect square.

**12.** Start by removing a common factor, so $128x^2 + 96x + 18 = 2(64x^2 + 48x + 9)$. Now factor $64x^2 + 48x + 9$.

With $8x$ and 3 being squared, cross term should be $2 \cdot 8x \cdot 3 = 48x$, which it is. Since the cross term is positive, this is a Perfect Square (sum).

$$\begin{aligned} 64x^2 + 48x + 9 &= (8x + 3)^2 \\ 128x^2 + 96x + 18 &= 2(8x + 3)^2 \end{aligned}$$

Check:

$$\begin{aligned} 2(8x + 3)^2 &= 2(8x + 3)(8x + 3) \\ &= (16x + 6)(8x + 3) \\ &= (16x + 6)(8x) + (16x + 6)(3) \\ &= (16x)(8x) + (6)(8x) + (16x)(3) + (6)(3) \\ &= 128x^2 + 96x + 18 \end{aligned}$$