Try to do these without a calculator. Remember you can check your answers by multiplying out.

Common factors in terms

Usually you want the greatest common factor so you can work with smaller numbers:

$$48x^2 + 96x + 36 = 12(4x^2 + 8x + 3).$$

Factoring by Grouping

This is sort of like using the distribution property in the other direction:

$$(3y - 8)(2x - 7) = 3y(2x - 7) - 8(2x - 7) \quad \text{(distribution property)}$$
$$3y(2x - 7) - 8(2x - 7) = (3y - 8)(2x - 7) \quad \text{(factoring by grouping)}$$

Factoring trinomials of form $x^2 + bx + c$

$x^2 + bx + c = (x + m)(x + n)$ where m and n are two numbers whose product is c and sum is b.

Factoring trinomials of form $ax^2 + bx + c$

The Grouping Method to factor trinomials of form $ax^2 + bx + c$:

1. Determine the grouping number ac.

2. Find two numbers whose product is ac and sum is b.

3. Use these numbers to write bx as the sum of two terms.

4. Factor by grouping.

5. Check your answer by multiplying out.

Difference of Squares

$$a^2 - b^2 = (a - b)(a + b).$$

Perfect Square (sum and difference)

$$a^2 + 2ab + b^2 = (a + b)^2,$$
$$a^2 - 2ab + b^2 = (a - b)^2.$$

Sum of Cubes, and Difference of Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2),$$
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2).$$

Remember that for the cubes, you will not be able to factor the resulting quadratic using the techniques of this unit.

My solutions for these questions are brief, the minimum you would need to show to get the correct answer. I will leave it to your to check your answers by multiplying out.

Questions

1. Factor $12x^2 - 2x - 18x^3$.

2. Factor $4x^2 - 28x - 72$.

3. Factor $7x^2 + 3x - 2$.

4. Factor $14x^2 - x^3 + 32x$.

5. Factor $30x^3 - 25x^2y - 30xy^2$.

6. Factor $27x^4 - 64x$.

Solutions

1.

$$
\begin{aligned}
12x^2 - 2x - 18x^3 &= 2x(6x - 1 - 9x^2) \text{ Factor } 2x \\
&= -2x(9x^2 - 6x + 1) \text{ Reorder and factor } -1 \\
&= -2x(3x - 1)^2 \text{ Perfect square (difference), } 3x \text{ and } 1
\end{aligned}
$$

2.

$$
\begin{aligned}
4x^2 - 28x - 72 &= 4(x^2 - 7x - 18) \text{ Factor 4. Need two numbers: sum is } -7, \text{ product is } -18: -9, 2 \\
&= -2x(x - 9)(x - 2)
\end{aligned}
$$

3. $7x^2 + 3x - 2$ is a prime polynomial. You cannot find two integers whose sum is 3 and product is -14.

4.

$$
\begin{aligned}
14x^2 - x^3 + 32x &= -x(-14x + x^2 - 32) \text{ Factor } x. \\
&= -x(x^2 - 14x - 32) \text{ Reorder. Need two numbers: sum is } -14, \text{ product is } -32: -16, 2 \\
&= -x(x - 16)(x + 2)
\end{aligned}
$$

5.

$$
\begin{aligned}
30x^3 - 25x^2y - 30xy^2 &= 5x(6x^2 - 5xy - 6y^2) \text{ Factor } 5x. \text{ Grouping Method is next, let } y \text{ follow along with constant.} \\
&= 5x(6x^2 - 5xy - 6y^2) \text{ Need two numbers: sum is } -5y, \text{ product is } -36y^2: -9y, 4y \\
&= 5x\left[\underline{6x^2 - 9yx} + \underaccent{\sim}{4yx - 6y^2}\right] \text{ find greatest common factor in first two terms and last two terms.} \\
&= 5x\left[3x(2x - 3y) + 2y(2x - 3y)\right] \\
&= 5x\left[(3x + 2y)(2x - 3y)\right] = 5x(3x + 2y)(2x - 3y)
\end{aligned}
$$

6.

$$
\begin{aligned}
27x^5 - 64x^2 &= x^2(27x^3 - 64) \text{ Factor } x^2. \text{ Difference of cubes with } (3x)^3 = 27x^3 \text{ and } 4^3 = 64. \\
&= x^2(3x - 4)((3x)^2 + (3x)(4) + 4^2) \\
&= x^2(3x - 4)(9x^2 + 12x + 16)
\end{aligned}
$$