

Questions

1. Solve $x^2 - x - 20 = 0$.
2. Solve $x^2 + 11x + 18 = 0$.
3. Solve $8x^2 = 72$.
4. Solve $(x - 5)(x + 4) = 2(x - 5)$.
5. Solve $\frac{x^2 + 5x}{6} = 4$.
6. Solve $\frac{12x^2 - 4x}{5} = 8$.
7. The area of a rectangular garden is 140 square meters. The width is 3 meters longer than one-half of the length. Find the length and width of the garden.
8. Jules is standing on a platform 6 meters high and throws a ball straight up as high as he can at a velocity of 13 meters per second. At what time t will the ball hit the ground? How far from the ground is the ball 2 seconds after Jules threw the ball (assume the ball is 6 meters from the ground when it leaves Jules' hand).

Solutions

1.

$$\begin{aligned}x^2 - x - 20 &= 0 \text{ Find two numbers product is } -20 \text{ and sum is } -1: -5, 4. \\(x - 5)(x + 4) &= 0 \text{ Use Zero Factor Property.} \\(x - 5) = 0 \text{ or } (x + 4) = 0 &\text{ Solve each linear equation.} \\x = 5 \text{ or } x = -4 &\end{aligned}$$

Check:

$$\begin{aligned}(5)^2 - (5) - 20 &= 25 - 25 = 0 \\(-4)^2 - (-4) - 20 &= 16 - 16 = 0\end{aligned}$$

2.

$$\begin{aligned}x^2 + 11x + 18 &= 0 \text{ Find two numbers product is } 18 \text{ and sum is } 11: 2, 9. \\(x + 2)(x + 9) &= 0 \\(x + 2) = 0 \text{ or } (x + 9) = 0 & \\x = -2 \text{ or } x = -9 &\end{aligned}$$

Check:

$$\begin{aligned}(-2)^2 + 11(-2) + 18 &= 4 - 22 + 18 = 0 \\(-9)^2 + 11(-9) + 18 &= 81 - 99 + 18 = 0\end{aligned}$$

3.

$$\begin{aligned}8x^2 - 72 &= 0 \text{ Factor.} \\8(x^2 - 9) &= 0 \text{ Factor.} \\x^2 - 9 &= 0 \text{ Divide by 8. Difference of Squares.} \\(x + 3)(x - 3) &= 0 \\(x + 3) = 0 \text{ or } (x - 3) = 0 & \\x = -3 \text{ or } x = 3 &\end{aligned}$$

Check:

$$8(-3)^2 = 8(9) = 72$$

$$8(3)^2 = 8(9) = 72$$

Alternate solution, which only works because there was no x term:

$$8x^2 = 72$$

$$x^2 = 9$$

$\sqrt{x^2} = \pm\sqrt{9}$ when taking square root of both sides of equation, one side can be \pm .

$$x = \pm 3$$

4. Start by multiplying everything to get in form $ax^2 + bx + c = 0$.

$$(x - 5)(x + 4) = 2(x - 5)$$

$$x^2 - x - 20 = 2x - 10$$

$$x^2 - x - 20 - 2x + 10 = 0$$

$$x^2 - 3x - 10 = 0 \text{ Find two numbers product is } -10 \text{ and sum is } -3: -5, 2.$$

$$(x - 5)(x + 2) = 0$$

$$(x - 5) = 0 \text{ or } (x + 2) = 0$$

$$x = 5 \text{ or } x = -2$$

Check:

$$((5) - 5)((5) + 4) - 2((5) - 5) = 0$$

$$((-2) - 5)((-2) + 4) - 2((-2) - 5) = -14 + 14 = 0$$

5. Start by multiplying everything to get in form $ax^2 + bx + c = 0$.

$$\frac{x^2 + 5x}{6} = 4$$

$$x^2 + 5x = 24$$

$$x^2 + 5x - 24 = 0 \text{ Find two numbers product is } -24 \text{ and sum is } 5: 8, -3.$$

$$(x + 8)(x - 3) = 0$$

$$(x + 8) = 0 \text{ or } (x - 3) = 0$$

$$x = -8 \text{ or } x = 3$$

Check:

$$\frac{(-8)^2 + 5(-8)}{6} = \frac{64 - 40}{6} = \frac{24}{6} = 4$$

$$\frac{(3)^2 + 5(3)}{6} = \frac{9 + 15}{6} = \frac{24}{6} = 4$$

6. Start by multiplying everything to get in form $ax^2 + bx + c = 0$.

$$\frac{12x^2 - 4x}{5} = 8$$

$$12x^2 - 4x = 40$$

$$12x^2 - 4x - 40 = 0$$

$$3x^2 - x - 10 = 0 \text{ Grouping Method: Find two numbers product is } -30 \text{ and sum is } -1: -6, 5.$$

$$\underline{3x^2 - 6x} + \underline{5x - 10} = 0 \text{ Factor by grouping.}$$

$$3x(x - 2) + 5(x - 2) = 0$$

$$(3x + 5)(x - 2) = 0$$

$$(3x + 5) = 0 \text{ or } (x - 2) = 0$$

$$x = -\frac{5}{3} \text{ or } x = 2$$

Check:

$$\frac{12(-5/3)^2 - 4(-5/3)}{5} = \frac{12(25/9) + 20/3}{5} = \frac{100/3 + 20/3}{5} = \frac{120/3}{5} = \frac{40}{5} = 8$$

$$\frac{12(2)^2 - 4(2)}{5} = \frac{48 - 8}{5} = \frac{40}{5} = 8$$

7. Let x be the length (in meters). Then the width is $\frac{x}{2} + 3$ meters. Area is 140 m^2 .

$$\text{Area} = (\text{length})(\text{width})$$

$$140 = x \left(\frac{x}{2} + 3 \right)$$

$$140 = \frac{x^2}{2} + 3x \text{ write in form } ax^2 + bx + c = 0.$$

$$280 = x^2 + 6x$$

$$0 = x^2 + 6x - 280$$

$$x^2 + 6x - 280 = 0 \text{ Find two numbers product is } 6 \text{ and sum is } -280: -14, 20.$$

$$(x - 14)(x + 20) = 0$$

$$(x - 14) = 0 \text{ or } (x + 20) = 0$$

$$x = 14 \text{ or } x = -20$$

Exclude the $x = -20$ as unphysical (can't have negative length). So The length is $x = 14$ meters. Width is 10 meters.

8. Set $h = 6$ and $v = 13$ in our model equation $S = -5t^2 + vt + h$ (see handout).

$$-5t^2 + 13t + 6 = 0 \text{ Ball hits ground when } S = 0. \text{ Use Grouping Method to factor.}$$

$$-5t^2 + 13t + 6 = 0 \text{ Find two numbers product is } -30 \text{ and sum is } 13: 15, -2.$$

$$-5t^2 + 15t - 2t + 6 = 0$$

$$-5t(t - 3) - 2(t - 3) = 0$$

$$(-5t - 2)(t - 3) = 0$$

$$(-5t - 2) = 0 \text{ or } (t - 3) = 0$$

$$t = -2/5 \text{ or } t = 3$$

Exclude the $t = -5/3$ as unphysical, so the ball hits the ground after 3 seconds.

Two second after throwing the ball, it it $S = -5(2)^2 + 13(2) + 6 = -20 + 26 + 6 = 12$ meters above the ground.