When canceling quantities like $\frac{\not{a}}{\not{a}} = 1$, to avoid the division by zero case we usually specify that $a \neq 0$.

Remember that you can only cancel factors, not terms.

Correct math canceling of factors: $\frac{x^2\cancel{(y+1)}}{z^3\cancel{(y+1)}} = \frac{x^2}{z^3}$, and $y + 1 \neq 0$

Incorrect math canceling of terms: $\frac{y+\cancel{1}}{x+\cancel{1}} \neq \frac{y}{x}$

Factoring plays an important role in simplifying rational expressions.

Questions

1. Simplify $\frac{5x+2y}{35x+14y}$.

2. Simplify $\frac{9ab^2}{6a^2b^2(b+3a)}$.

3. Simplify $\frac{3x^2+7x-6}{x^2+7x+12}$.

4. Simplify $\frac{3x^2-8x+5}{4x^2-5x+1}$.

5. Simplify $\frac{10-2x}{4x^2-20x}$.

6. Simplify $\frac{6-2ab}{ab^2-3b}$.

7. Simplify $\frac{16x^2-24xy+9y^2}{16x^2-9y^2}$.

8. Simplify $\frac{bxy+bx^2-axy-ax^2}{ay^2+axy+2by^2+2bxy}$.

Solutions

1.

$$\frac{5x+2y}{35x+14y} = \frac{\cancel{5x+2y}}{7\cancel{(5x+2y)}} = \frac{1}{7} \text{ and } 5x+2y \neq 0$$

2.

$$\frac{9ab^2}{6a^2b^2(b+3a)} = \frac{3\cdot\cancel{3ab^2}}{2a\cdot\cancel{3ab^2}(b+3a)} = \frac{3}{2a(b+3a)} \text{ and } 3ab^2 \neq 0$$

3. Factor numerator using grouping method, look for two numbers whose product is -18 and sum is 7: 9,-2.

$$\begin{aligned} 3x^2+7x-6 &= 3x^2+9x-2x-6 \\ &= 3x(x+3)-2(x+3) \\ &= (3x-2)(x+3) \end{aligned}$$

Factor denominator: look for two numbers whose product is 12 and sum is 7: 3,4.

$$x^2+7x+12 = (x+3)(x+4)$$

Putting this into our rational expression:

$$\begin{aligned} \frac{3x^2+7x-6}{x^2+7x+12} &= \frac{(3x-2)\cancel{(x+3)}}{\cancel{(x+3)}(x+4)} \\ &= \frac{3x-2}{x+4} \text{ and } x+3 \neq 0 \end{aligned}$$

4. Factor numerator using grouping method, look for two numbers whose product is 15 and sum is -8: -5,-3.

$$\begin{aligned} 3x^2-8x+5 &= 3x^2-5x-3x+5 \\ &= x(3x-5)-1(3x-5) \\ &= (x-1)(3x-5) \end{aligned}$$

Factor denominator using grouping method, look for two numbers whose product is 4 and sum is -5: -1,-4.

$$\begin{aligned} 4x^2-5x+1 &= 4x^2-x-4x+1 \\ &= x(4x-1)-1(4x-1) \\ &= (x-1)(4x-1) \end{aligned}$$

Putting this into our rational expression:

$$\begin{aligned} \frac{3x^2-8x+5}{4x^2-5x+1} &= \frac{(3x-5)\cancel{(x-1)}}{\cancel{(x-1)}(4x-1)} \\ &= \frac{3x-5}{4x-1} \text{ and } x-1 \neq 0 \end{aligned}$$

5.

$$\begin{aligned} \frac{10-2x}{4x^2-20x} &= \frac{2(5-x)}{4x(x-5)} \\ &= \frac{-2\cancel{(x-5)}}{4x\cancel{(x-5)}} \\ &= \frac{-2}{4x} = -\frac{1}{2x} \text{ and } x-5 \neq 0 \end{aligned}$$

6.

$$\begin{aligned}\frac{6-2ab}{ab^2-3b} &= \frac{2(3-ab)}{b(ab-3)}\\ &= \frac{-2\cancel{(ab-3)}}{b\cancel{(ab-3)}}\\ &= \frac{-2}{b} \text{ and } ab-3 \neq 0\end{aligned}$$

7. Numerator is a perfect square; denominator is a difference of squares.

$$\begin{aligned}\frac{16x^2-24xy+9y^2}{16x^2-9y^2} &= \frac{(4x-3y)^2}{(4x-3y)(4x+3y)}\\ &= \frac{\cancel{(4x-3y)}(4x-3y)}{\cancel{(4x-3y)}(4x+3y)}\\ &= \frac{4x-3y}{4x+3y} \text{ and } 4x-3y \neq 0\end{aligned}$$

8. This requires removing a greatest common factor from numerator and denominator, followed by factoring by grouping.

$$\begin{aligned}\frac{bxy+bx^2-axy-ax^2}{ay^2+axy+2by^2+2bxy} &= \frac{x(by+bx-ay-ax)}{y(ay+ax+2by+2bx)}\\ &= \frac{x(b[y+x]-a[y+x])}{y(a[y+x]+2b[y+x])}\\ &= \frac{x([b-a][y+x])}{y([a+2b][y+x])}\\ &= \frac{x(b-a)\cancel{(y+x)}}{y(a+2b)\cancel{(y+x)}} \text{ rewrite now that everything is factored}\\ &= \frac{x(b-a)}{y(a+2b)} \text{ and } y+x \neq 0\end{aligned}$$