

When adding or subtracting rational expressions you might have to do a lot of work. In general, you might need to

- factor any polynomials in the expressions
- get a common denominator for the rational expressions (**the critical step!**)
- add or subtract using  $\frac{a}{c} \pm \frac{b}{c} = \frac{a \pm b}{c}$
- simplify the numerator (this could even involve another factoring!)
- simplify further by canceling any common terms in the numerator and denominator

Be careful, show all your work, and make sure minus signs get distributed correctly; for example,  $-3x(x + 4)$  is equal to  $-3x^2 - 12x$  NOT  $-3x^2 + 12x$ .

### Questions

1. Simplify  $\frac{8x + 3}{5x + 7} - \frac{6x + 10}{5x + 7}$ .
2. Find the lowest common denominator for  $\frac{1}{x^2 - 9}$  and  $\frac{1}{x + 3}$ .
3. Find the lowest common denominator for  $\frac{1}{2x^2 - 9x - 35}$  and  $\frac{1}{4x^2 + 20x + 25}$ .
4. Simplify  $\frac{8}{cd} + \frac{9}{d}$ .
5. Simplify  $\frac{2}{y - 1} + \frac{2}{y + 1}$ .
6. Simplify  $\frac{2}{3xy} + \frac{1}{6yz}$ .
7. Simplify  $\frac{6}{3x - 4} - \frac{5}{4x - 3}$ .
8. Simplify  $\frac{x}{x^2 + 2x - 3} - \frac{x}{x^2 - 5x + 4}$ .
9. Simplify  $\frac{3x + 5}{x^2 + 4x + 3} + \frac{-x + 5}{x^2 + 2x - 3}$ .
10. Simplify  $\frac{2x}{x^2 + 5x + 6} - \frac{x + 1}{x^2 + 2x - 3}$ .

### Solutions

1. The denominators are the same, so we can subtract immediately.

$$\begin{aligned} \frac{8x+3}{5x+7} - \frac{6x+10}{5x+7} &= \frac{(8x+3) - (6x+10)}{5x+7} \text{ subtract rational expressions with common denominators} \\ &= \frac{8x+3-6x-10}{5x+7} \\ &= \frac{2x-7}{5x+7} \end{aligned}$$

2. To find lowest common denominator we need to factor.

$$\begin{aligned} x^2 - 9 &= (x+3)(x-3) \text{ difference of squares} \\ x+3 &= (x+3) \end{aligned}$$

The lowest common denominator is  $(x+3)(x-3)$ . I've highlighted the overlap in red.

3. Factor everything first.

$$\begin{aligned} 2x^2 - 9x - 35 &= 2x^2 - 14x + 5x - 35 \text{ need two numbers whose product is } -70 \text{ and sum is } -9: -14, 5 \\ &= 2x(x-7) + 5(x-7) \text{ factor by grouping} \\ &= (2x+5)(x-7) \\ 4x^2 + 20x + 25 &= 4x^2 + 10x + 10x + 25 \text{ need two numbers whose product is } 100 \text{ and sum is } 20: 10, 10 \\ &= 2x(2x+5) + 5(2x+5) \text{ factor by grouping} \\ &= (2x+5)(2x+5) \text{ this was a perfect square} \end{aligned}$$

$$\begin{aligned} 2x^2 - 9x - 35 &= (2x+5)(x-7) \\ 4x^2 + 20x + 25 &= (2x+5)(2x+5) \end{aligned}$$

$$\text{LCD is } (2x+5)(2x+5)(x-7)$$

4. Nothing needs to be factored.

$$\begin{aligned} \frac{8}{cd} + \frac{9}{d} &= \frac{8}{cd} + \frac{9 \cdot c}{d \cdot c} \text{ multiply by appropriate quantities to make the denominators the same.} \\ &= \frac{8}{cd} + \frac{9c}{cd} \\ &= \frac{8+9c}{cd} \text{ add rational expressions with common denominators} \end{aligned}$$

5. Nothing needs to be factored.

$$\begin{aligned} \frac{2}{y-1} + \frac{2}{y+1} &= \frac{2(y+1)}{(y-1)(y+1)} + \frac{2(y-1)}{(y+1)(y-1)} \text{ multiply by appropriate quantities to make the denominators the same.} \\ &= \frac{2(y+1) + 2(y-1)}{(y-1)(y+1)} \text{ add rational expressions with common denominators} \\ &= \frac{2y+2+2y-2}{(y-1)(y+1)} \text{ simplify numerator} \\ &= \frac{4y}{(y-1)(y+1)} \end{aligned}$$

6. Nothing needs to be factored.

$$\begin{aligned}\frac{2}{3xy} + \frac{1}{6yz} &= \frac{2(2z)}{3xy(2z)} + \frac{1(x)}{6yz(x)} \\ &= \frac{4z}{6xyz} + \frac{x}{6xyz} \\ &= \frac{4z + x}{6xyz}\end{aligned}$$

7. Nothing needs to be factored.

$$\begin{aligned}\frac{6}{3x-4} - \frac{5}{4x-3} &= \frac{6(4x-3)}{(3x-4)(4x-3)} - \frac{5(3x-4)}{(4x-3)(3x-4)} \\ &= \frac{6(4x-3) - 5(3x-4)}{(3x-4)(4x-3)} \\ &= \frac{24x - 18 - 15x + 20}{(3x-4)(4x-3)} \\ &= \frac{9x + 2}{(3x-4)(4x-3)}\end{aligned}$$

8. We need to factor here.

$$x^2 + 2x - 3 = (x + 3)(x - 1) \text{ two numbers whose product is } -3 \text{ sum is } 2: 3, -1$$

$$x^2 - 5x + 4 = (x - 4)(x - 1) \text{ two numbers whose product is } 4 \text{ sum is } -5: -4, -1$$

$$\begin{aligned}\frac{x}{x^2 + 2x - 3} - \frac{x}{x^2 - 5x + 4} &= \frac{x}{(x + 3)(x - 1)} - \frac{x}{(x - 4)(x - 1)} \\ &= \frac{x(x - 4)}{(x + 3)(x - 1)(x - 4)} - \frac{x(x + 3)}{(x - 4)(x - 1)(x + 3)} \text{ get common denominator} \\ &= \frac{x(x - 4) - x(x + 3)}{(x + 3)(x - 1)(x - 4)} \text{ subtract now that we have common denominator} \\ &= \frac{x^2 - 4x - x^2 - 3x}{(x + 3)(x - 1)(x - 4)} \text{ simplify} \\ &= \frac{-7x}{(x + 3)(x - 1)(x - 4)}\end{aligned}$$

9. We need to factor here.

$$x^2 + 4x + 3 = (x + 3)(x + 1) \text{ two numbers whose product is 3 sum is 4: } 3, 1$$

$$x^2 + 2x - 3 = (x + 3)(x - 1) \text{ two numbers whose product is } -3 \text{ sum is 2: } 3, -1$$

$$\begin{aligned} \frac{3x + 5}{x^2 + 4x + 3} + \frac{-x + 5}{x^2 + 2x - 3} &= \frac{3x + 5}{(x + 3)(x + 1)} + \frac{5 - x}{(x + 3)(x - 1)} \text{ factor} \\ &= \frac{(3x + 5)(x - 1)}{(x + 3)(x + 1)(x - 1)} + \frac{(5 - x)(x + 1)}{(x + 3)(x - 1)(x + 1)} \\ &= \frac{(3x + 5)(x - 1)}{(x + 3)(x + 1)(x - 1)} + \frac{(5 - x)(x + 1)}{(x + 3)(x - 1)(x + 1)} \text{ get common denominator} \\ &= \frac{(3x + 5)(x - 1) + (5 - x)(x + 1)}{(x + 3)(x + 1)(x - 1)} \text{ add} \\ &= \frac{3x^2 + 2x - 5 - x^2 + 4x + 5}{(x + 3)(x + 1)(x - 1)} \text{ simplify numerator: distribute} \\ &= \frac{2x^2 + 6x}{(x + 3)(x + 1)(x - 1)} \text{ simplify numerator: collect like terms} \\ &= \frac{2x(x + 3)}{(x + 3)(x + 1)(x - 1)} \text{ simplify: factor numerator} \\ &= \frac{2x}{(x + 1)(x - 1)} \text{ and } x + 3 \neq 0 \end{aligned}$$

10. We need to factor here.

$$x^2 + 5x + 6 = (x + 3)(x + 2) \text{ two numbers whose product is 6 sum is 5: } 3, 2$$

$$x^2 + 2x - 3 = (x + 3)(x - 1) \text{ two numbers whose product is } -3 \text{ sum is 2: } 3, -1$$

$$\begin{aligned} \frac{2x}{x^2 + 5x + 6} - \frac{x + 1}{x^2 + 2x - 3} &= \frac{2x}{(x + 3)(x + 2)} - \frac{x + 1}{(x + 3)(x - 1)} \text{ factor} \\ &= \frac{2x(x - 1)}{(x + 3)(x + 2)(x - 1)} - \frac{(x + 1)(x + 2)}{(x + 3)(x - 1)(x + 2)} \text{ get common denominator} \\ &= \frac{2x(x - 1) - (x + 1)(x + 2)}{(x + 3)(x + 2)(x - 1)} \text{ subtract} \\ &= \frac{2x^2 - 2x - x^2 - 3x - 2}{(x + 3)(x + 2)(x - 1)} \text{ simplify numerator: distribute} \\ &= \frac{x^2 - 5x - 2}{(x + 3)(x + 2)(x - 1)} \text{ simplify numerator: collect like terms} \end{aligned}$$

The numerator is prime. If we could factor it, we would.