Questions

1. Simplify $\dfrac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{xy}}$.

2. Simplify $\dfrac{\frac{8}{x}-\frac{2}{3x}}{\frac{2}{3}+\frac{5}{x}}$.

3. Simplify $\dfrac{1-\frac{36}{x^2}}{1-\frac{6}{x}}$.

4. Simplify $\dfrac{x+\frac{4}{x}}{\frac{x^2+3}{4x}}$.

5. Simplify $\dfrac{\frac{y}{y+1}+1}{\frac{2y+1}{y-1}}$.

6. For what values of x is $\dfrac{\frac{5}{x-2}}{\frac{6}{x}+1}$ not defined?

Solutions

1.

$$\begin{aligned}
\frac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{xy}} &= \frac{\frac{y}{xy}+\frac{x}{xy}}{\frac{1}{xy}} \text{ common denominator}\\
&= \frac{\left(\frac{y+x}{xy}\right)}{\left(\frac{1}{xy}\right)}\\
&= \left(\frac{y+x}{xy}\right)\left(\frac{xy}{1}\right) \text{ invert and multiply}\\
&= \frac{(y+x)(xy)}{(xy)(1)} \text{ simplify}\\
&= \frac{(y+x)\cancel{(xy)}}{\cancel{(xy)}(1)} \text{ simplify}\\
&= y+x, \text{ and } xy \neq 0
\end{aligned}$$

2.

$$\begin{aligned}
\frac{\frac{8}{x}-\frac{2}{3x}}{\frac{2}{3}+\frac{5}{x}} &= \frac{\frac{24}{3x}-\frac{2}{3x}}{\frac{2x}{3x}+\frac{15}{3x}} \text{ common denominators}\\
&= \frac{\frac{24-2}{3x}}{\frac{2x+15}{3x}}\\
&= \frac{\left(\frac{22}{3x}\right)}{\left(\frac{2x+15}{3x}\right)}\\
&= \frac{22}{3x}\cdot\frac{3x}{2x+15}\\
&= \frac{(22)\cancel{(3x)}}{\cancel{(3x)}(2x+15)}\\
&= \frac{22}{2x+15} \text{ and } 3x \neq 0
\end{aligned}$$

3.

$$\begin{aligned}
\frac{1-\frac{36}{x^2}}{1-\frac{6}{x}} &= \frac{\frac{x^2}{x^2}-\frac{36}{x^2}}{\frac{x}{x}-\frac{6}{x}}\\
&= \frac{\left(\frac{x^2-36}{x^2}\right)}{\left(\frac{x-6}{x}\right)}\\
&= \frac{x^2-36}{x^2}\cdot\frac{x}{x-6}\\
&= \frac{(x^2-36)(x)}{(x^2)(x-6)}\\
&= \frac{\cancel{(x-6)}(x+6)\cancel{(x)}}{(x^{\cancel{2}})\cancel{(x-6)}}\\
&= \frac{x+6}{x} \text{ and } x+6 \neq 0, x \neq 0
\end{aligned}$$

4.

$$\begin{aligned}
\frac{x + \frac{4}{x}}{\frac{x^2+3}{4x}} &= \frac{\frac{x^2}{x} + \frac{4}{x}}{\frac{x^2+3}{4x}} \\
&= \frac{\left(\frac{x^2+4}{x}\right)}{\left(\frac{x^2+3}{4x}\right)} \\
&= \left(\frac{x^2+4}{x}\right) \cdot \left(\frac{4x}{x^2+3}\right) \\
&= \frac{(x^2+4)(4\cancel{x})}{\cancel{x}(x^2+3)} \\
&= \frac{4(x^2+4)}{x^2+3} \text{ and } x \neq 0
\end{aligned}$$

5.

$$\begin{aligned}
\frac{\frac{y}{y+1} + 1}{\frac{2y+1}{y-1}} &= \frac{\frac{y}{y+1} + \frac{y+1}{y+1}}{\frac{2y+1}{y-1}} \\
&= \frac{\left(\frac{y+y+1}{y+1}\right)}{\left(\frac{2y+1}{y-1}\right)} \\
&= \left(\frac{2y+1}{y+1}\right) \cdot \left(\frac{y-1}{2y+1}\right) \\
&= \frac{\cancel{(2y+1)}(y-1)}{(y+1)\cancel{(2y+1)}} \\
&= \frac{y-1}{y+1} \text{ and } 2y+1 \neq 0
\end{aligned}$$

6. No denominator in the expression can equal zero. There are three denominators in the expression.

$$\frac{\frac{5}{\color{red}{x-2}}}{\frac{6}{x}+1} \Rightarrow x - 2 \neq 0 \Rightarrow x \neq 2$$

$$\frac{\frac{5}{x-2}}{\frac{6}{\color{red}{x}}+1} \Rightarrow x \neq 0$$

$$\frac{\frac{5}{x-2}}{\color{red}{\frac{6}{x}+1}} \Rightarrow \frac{6}{x} + 1 \neq 0 \Rightarrow x \neq -6$$