Questions

1. Solve $\frac{8}{x} + \frac{2}{5} = -\frac{2}{x}$.

2. Solve $\frac{x+1}{2x} = \frac{2}{3}$.

3. Solve $\frac{2}{2x+5} = \frac{4}{x-4}$.

4. Solve $\frac{3}{x+5} = \frac{3}{3x-2}$.

5. Solve $7 - \frac{x}{x+5} = \frac{5}{x+5}$.

6. Solve $\frac{8x}{4x^2-1} = \frac{3}{2x+1} + \frac{3}{2x-1}$.

7. Solve $\frac{6}{x-5} + \frac{3x+1}{x^2-2x-15} = \frac{5}{x+3}$.

8. Solve $\frac{6}{x-3} = \frac{-5}{x-2} - \frac{5}{x^2-5x+6}$.

Solutions

1. Lowest common denominator is $5x$.

$$\left(\frac{8}{x}\right)5x + \left(\frac{2}{5}\right)5x = \left(-\frac{2}{x}\right)5x$$
$$40 + 2x = -10$$
$$2x = -10 - 40$$
$$x = -\frac{50}{2} = -25$$

Check:

$$\frac{8}{(-25)} + \frac{2}{5} = -\frac{2}{(-25)}$$
$$-\frac{8}{25} + \frac{10}{25} = \frac{2}{25}$$
$$\frac{2}{25} = \frac{2}{25} \text{ it's a solution}$$

2. LCD is $6x$.

$$\left(\frac{x+1}{2x}\right)6x = \left(\frac{2}{3}\right)6x$$
$$3x + 3 = 4x$$
$$3 = 4x - 3x$$
$$3 = x$$

Check:

$$\frac{(3)+1}{2(3)} = \frac{2}{3}$$
$$\frac{4}{6} = \frac{2}{3}$$
$$\frac{2}{3} = \frac{2}{3} \text{ it's a solution!}$$

3. LCD is $(2x+5)(x-4)$.

$$\left(\frac{2}{\cancel{2x+5}}\right)\cancel{(2x+5)}(x-4) = \left(\frac{4}{\cancel{x-4}}\right)(2x+5)\cancel{(x-4)}$$
$$2(x-4) = 4(2x+5)$$
$$2x - 8 = 8x + 20$$
$$-6x = 28$$
$$x = \frac{28}{-6} = -\frac{14}{3}$$

Check:

$$\frac{2}{2(-14/3)+5} = \frac{4}{(-14/3)-4}$$
$$\frac{2}{-28/3+15/3} = \frac{4}{(-14/6)-12/3}$$
$$\frac{2}{-13/3} = \frac{4}{-26/3}$$
$$\frac{6}{-13} = \frac{12}{-26}$$
$$\frac{6}{-13} = \frac{6}{-13} \text{ it's a solution}$$

4. LCD is $(x+5)(3x-2)$.

$$\left(\frac{3}{\cancel{x+5}}\right)\cancel{(x+5)}(3x-2) = \left(\frac{3}{\cancel{3x-2}}\right)(x+5)\cancel{(3x-2)}$$
$$3(3x-2) = 3(x+5)$$
$$3x - 2 = \frac{\cancel{3}}{\cancel{3}}(x+5)$$
$$3x - 2 = x + 5$$
$$2x = 7$$
$$x = \frac{7}{2}$$

Check:

$$\frac{3}{(7/2)+5} = \frac{3}{3(7/2)-2}$$
$$\frac{3}{7/2+10/2} = \frac{3}{21/2-4/2}$$
$$\frac{3}{17/2} = \frac{3}{17/2} \text{ it's a solution}$$

5. LCD is $x+5$.

$$(7)(x+5) - \left(\frac{x}{\cancel{x+5}}\right)\cancel{(x+5)} = \left(\frac{5}{\cancel{x+5}}\right)\cancel{(x+5)}$$
$$7x + 35 - x = 5$$
$$6x = -30$$
$$x = -5$$

As soon as you try to check this in the original equation you will get a division by zero. Therefore $x = -5$ is not a solution. Therefore, the original equation has no solution.

6. Factor polynomials.

$4x^2 - 1 = (2x - 1)(2x + 1)$ difference of squares

Looking at the equation, we now see the LCD is $(2x - 1)(2x + 1)$.

$$\left(\frac{8x}{\cancel{(2x-1)}\cancel{(2x+1)}}\right)\cancel{(2x-1)}\cancel{(2x+1)} = \left(\frac{3}{\cancel{2x+1}}\right)(2x-1)\cancel{(2x+1)} + \left(\frac{3}{\cancel{2x-1}}\right)\cancel{(2x-1)}(2x+1)$$

$$\begin{aligned} 8x &= 3(2x-1) + 3(2x+1) \\ 8x &= 6x - 3 + 6x + 3 \\ 8x &= 12x \\ -4x &= 0 \\ x &= \frac{0}{-4} = 0 \end{aligned}$$

Check:

$$\begin{aligned} \frac{8(0)}{4(0)^2 - 1} &= \frac{3}{2(0)+1} + \frac{3}{2(0)-1} \\ 0 &= 3 - 3 \text{ it's a solution} \end{aligned}$$

7. Factor polynomials.

$x^2 - 2x - 15 = (x + 3)(x - 5)$ Need two numbers whose product is -15 sum is -2: $3, -5$

Looking at the equation, we now see the LCD is $(x + 3)(x - 5)$.

$$\left(\frac{6}{\cancel{x-5}}\right)(x+3)\cancel{(x-5)} + \left(\frac{3x+1}{\cancel{(x+3)}\cancel{(x-5)}}\right)\cancel{(x+3)}\cancel{(x-5)} = \left(\frac{5}{\cancel{x+3}}\right)\cancel{(x+3)}(x-5)$$

$$\begin{aligned} 6(x+3) + 3x + 1 &= 5(x-5) \\ 6x + 18 + 3x + 1 &= 5x - 25 \\ 4x &= -44 \\ x &= -11 \end{aligned}$$

Check:

$$\begin{aligned} \frac{6}{(-11)-5} + \frac{3(-11)+1}{(-11)^2 - 2(-11) - 15} &= \frac{5}{(-11)+3} \\ \frac{6}{-16} + \frac{-32}{128} &= \frac{5}{-8} \\ \frac{3}{-8} + \frac{-1}{4} &= \frac{5}{-8} \\ -\frac{3}{8} - \frac{2}{8} &= -\frac{5}{8} \\ -\frac{5}{8} &= -\frac{5}{8} \text{ it's a solution} \end{aligned}$$

8. Factor polynomials.

$x^2 - 5x + 6 = (x-3)(x-2)$ Need two numbers whose product is 6 sum is -5: $-2, -3$

Looking at the equation, we now see the LCD is $(x-3)(x-2)$.

$$\begin{aligned}
\frac{6}{x-3} &= \frac{-5}{x-2} - \frac{5}{(x-3)(x-2)} \\
\left(\frac{6}{\cancel{x-3}}\right)\cancel{(x-3)}(x-2) &= \left(\frac{-5}{\cancel{x-2}}\right)(x-3)\cancel{(x-2)} - \left(\frac{5}{\cancel{(x-3)}\cancel{(x-2)}}\right)\cancel{(x-3)}\cancel{(x-2)} \\
6(x-2) &= -5(x-3) - 5 \\
6x - 12 &= -5x + 15 - 5 \\
11x &= 22 \\
x &= 2
\end{aligned}$$

As soon as you try to check this in the original equation you will get a division by zero. Therefore $x = 2$ is not a solution. Therefore, the original equation has no solution.