The mathematical concept of a function is used here in the text, but a more detailed study of functions is seen in Math 1011 Precalculus: Functions.

At this time, we are only concerned with the idea of domain, which essentially means the set of x values we can put into an expression and get a real value out. We use the shorthand notation $f(x)$ to refer to the expression.

Questions

1. Simplify $-\sqrt{\frac{1}{9}}$

2. Simplify $\sqrt{0.04}$.

3. Find the value of the function $f(x) = \sqrt{10x + 5}$ at $x = 0$, $x = 1$, $x = 2$, and $x = 3$. What is the domain of the function $f(x)$?

4. Find the value of the function $f(x) = \sqrt{1.5x - 4}$ at $x = 4$, $x = 6$, $x = 8$, and $x = 14$. What is the domain of the function $f(x)$?

5. Simplify $\sqrt[3]{-\frac{8}{27}}$.

6. Replace the radicals with rational exponents in $\sqrt[5]{2x}$.

7. Replace the radicals with rational exponents in $\sqrt[4]{3y}$.

8. Replace the radicals with rational exponents in $\sqrt[7]{(a+b)^3}$.

9. Simplify $\sqrt[3]{-125x^{30}}$.

10. Simplify $\sqrt[3]{-27a^6}$.

11. Simplify $\sqrt[4]{a^{12}b^4}$.

12. Simplify $\sqrt[4]{a^4b^{20}}$.

Solutions

1. $-\sqrt{\frac{1}{9}} = -\sqrt{\left(\frac{1}{3}\right)^2} = -\frac{1}{3}$.

2. $\sqrt{0.04} = \sqrt{(0.2)^2} = 0.2$.

3. Use a calculator to approximate some of the square roots.

$$\begin{aligned} f(x) &= \sqrt{10x+5} \\ f(0) &= \sqrt{10(0)+5} = \sqrt{5} \sim 2.2 \\ f(1) &= \sqrt{10(1)+5} = \sqrt{15} \sim 3.9 \\ f(2) &= \sqrt{10(2)+5} = \sqrt{25} = 5 \\ f(3) &= \sqrt{10(3)+5} = \sqrt{35} \sim 5.9 \end{aligned}$$

For the domain, we know that we can only get a real number out of a square root if we put in a number greater than or equal to zero, so for this expression the domain is

$$\begin{aligned} 10x+5 &\geq 0 \\ 10x &\geq -5 \\ x &\geq -\frac{5}{10} \\ x &\geq -\frac{1}{2} \end{aligned}$$

The domain is $x \geq -1/2$.

4. Use a calculator to approximate the square roots.

$$\begin{aligned} f(x) &= \sqrt{1.5x-4} \\ f(4) &= \sqrt{1.5(4)-4} = \sqrt{2} \sim 1.4 \\ f(6) &= \sqrt{1.5(6)-4} = \sqrt{5} \sim 2.2 \\ f(8) &= \sqrt{1.5(8)-4} = \sqrt{8} \sim 2.8 \\ f(14) &= \sqrt{1.5(14)-4} = \sqrt{17} \sim 4.1 \end{aligned}$$

For the domain, we know that we can only get a real number out of a square root if we put in a number greater than or equal to zero, so for this expression the domain is

$$\begin{aligned} 1.5x-4 &\geq 0 \\ 1.5x &\geq 4 \\ x &\geq \frac{4}{1.5} \\ x &\geq \frac{4}{3/2} \\ x &\geq \frac{8}{3} \end{aligned}$$

The domain is $x \geq 8/3$.

5.

$$\begin{aligned}\sqrt[3]{-\frac{8}{27}} &= \left(-\frac{8}{27}\right)^{1/3} \\ &= \left(-\frac{2^3}{3^3}\right)^{1/3} \\ &= \left(\left(-\frac{2}{3}\right)^3\right)^{1/3} \\ &= -\frac{2}{3}\end{aligned}$$

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6. $\sqrt[5]{2x} = (2x)^{1/5}$.

7. $\sqrt[4]{3y} = (3y)^{1/4}$.

8. $\sqrt[7]{(a+b)^3} = ((a+b)^3)^{1/7} = (a+b)^{3/7}$.

9.

$$\begin{aligned}\sqrt[3]{-125x^{30}} &= (-125x^{30})^{1/3} \\ &= (-125)^{1/3}(x^{30})^{1/3} \\ &= ((-5)^3)^{1/3}x^{10} \\ &= (-5)x^{10} \\ &= -5x^{10}\end{aligned}$$

10.

$$\begin{aligned}\sqrt[3]{-27a^6} &= \left(-27a^6\right)^{1/3} \\ &= (-27)^{1/3}\left(a^6\right)^{1/3} \\ &= \left((-3)^3\right)^{1/3}a^2 \\ &= (-3)a^2 = -3a^2\end{aligned}$$

11.

$$\begin{aligned}\sqrt[4]{a^{12}b^4} &= \left(a^{12}b^4\right)^{1/4} \\ &= \left(a^{12}\right)^{1/4}\left(b^4\right)^{1/4} \\ &= \left((a^3)^4\right)^{1/4}|b| \text{ Note: Need to use rule that } (x^n)^{1/n} = |x| \text{ if } n \text{ is even.} \\ &= |a^3||b| = |a^3b|\end{aligned}$$

12.

$$\begin{aligned}\sqrt[4]{a^4b^{20}} &= (a^4b^{20})^{1/4} \\ &= (a^4)^{1/4}(b^{20})^{1/4} \\ &= |a||b^5| = |ab^5|\end{aligned}$$