Use the distributive property.

Questions

1. Simplify $(3\sqrt{3}+\sqrt{5})(\sqrt{3}-2\sqrt{5})$.

2. Simplify $(\sqrt{7}+4\sqrt{5x})(2\sqrt{7}+3\sqrt{5x})$.

3. Simplify $(3\sqrt{5}+\sqrt{3})(\sqrt{2}+2\sqrt{5})$.

4. Simplify $\sqrt{\frac{49}{25}}$.

5. Simplify $\sqrt{\frac{16}{36}}$.

6. Simplify $\sqrt{\frac{12x}{49y^6}}$.

7. Rationalize denominator in $\frac{3x}{\sqrt{10}-\sqrt{2}}$.

8. Rationalize denominator in $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$.

9. Rationalize numerator in $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$.

10. Rationalize denominator in $\frac{\sqrt{3x}-2\sqrt{y}}{\sqrt{3x}+\sqrt{y}}$.

11. Rationalize numerator in $\frac{\sqrt{3x}-2\sqrt{y}}{\sqrt{3x}+\sqrt{y}}$.

12. Rationalize numerator in $\frac{\sqrt{3}+2\sqrt{7}}{8}$.

Solutions

1. Distribute, Distribute, Distribute!

$$\begin{aligned}(3\sqrt{3}+\sqrt{5})(\sqrt{3}-2\sqrt{5}) &= (3\sqrt{3})(\sqrt{3}-2\sqrt{5})+(\sqrt{5})(\sqrt{3}-2\sqrt{5})\\ &= (3\sqrt{3})(\sqrt{3})-(3\sqrt{3})(2\sqrt{5})+(\sqrt{5})(\sqrt{3})-(\sqrt{5})(2\sqrt{5})\\ &= 3(\sqrt{3})^2-6\sqrt{3}\sqrt{5}+\sqrt{5}\sqrt{3}-2(\sqrt{5})^2\\ &= 3(3)-6\sqrt{3\cdot 5}+\sqrt{5\cdot 3}-2(5)\\ &= -1-5\sqrt{15}\end{aligned}$$

2.

$$\begin{aligned}(\sqrt{7}+4\sqrt{5x})(2\sqrt{7}+3\sqrt{5x}) &= (\sqrt{7})(2\sqrt{7}+3\sqrt{5x})+(4\sqrt{5x})(2\sqrt{7}+3\sqrt{5x})\\ &= (\sqrt{7})(2\sqrt{7})+(\sqrt{7})(3\sqrt{5x})+(4\sqrt{5x})(2\sqrt{7})+(4\sqrt{5x})(3\sqrt{5x})\\ &= 2(7)+3\sqrt{7\cdot 5x}+8\sqrt{5x\cdot 7}+12(5x)\\ &= 14+11\sqrt{35x}+60x\end{aligned}$$

3.

$$\begin{aligned}(3\sqrt{5}+\sqrt{3})(\sqrt{2}+2\sqrt{5}) &= (3\sqrt{5})(\sqrt{2}+2\sqrt{5})+(\sqrt{3})(\sqrt{2}+2\sqrt{5})\\ &= (3\sqrt{5})(\sqrt{2})+(3\sqrt{5})(2\sqrt{5})+(\sqrt{3})(\sqrt{2})+(\sqrt{3})(2\sqrt{5})\\ &= 3\sqrt{5\cdot 2}+(6)(5)+\sqrt{3\cdot 2}+2\sqrt{3\cdot 5}\\ &= 3\sqrt{10}+30+\sqrt{6}+2\sqrt{15}\end{aligned}$$

4. $\sqrt{\dfrac{49}{25}}=\dfrac{7}{5}$.

5. $\sqrt{\dfrac{16}{36}}=\dfrac{4}{6}=\dfrac{2}{3}$.

6. $\sqrt{\dfrac{12x}{49y^6}}=\sqrt{\dfrac{2^2\cdot 3x}{(7y^3)^2}}=\dfrac{\sqrt{2^2}\cdot\sqrt{3x}}{\sqrt{(7y^3)^2}}=\dfrac{2\sqrt{3x}}{7y^3}$.

7. Note the sign change when you rationalize! This is so the cross terms will cancel when you distribute.

$$\begin{aligned}\frac{3x}{\sqrt{10}-\sqrt{2}} &= \frac{(3x)(\sqrt{10}+\sqrt{2})}{(\sqrt{10}-\sqrt{2})(\sqrt{10}+\sqrt{2})}\\ &= \frac{3x(\sqrt{10}+\sqrt{2})}{10-2}\\ &= \frac{3x(\sqrt{10}+\sqrt{2})}{8}\end{aligned}$$

8.

$$\begin{aligned}\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} &= \frac{(\sqrt{5}+\sqrt{3})(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}\\ &= \frac{5+2\sqrt{15}+3}{5-3}\\ &= \frac{8+2\sqrt{15}}{2}\\ &= 4+\sqrt{15}\end{aligned}$$

9.

$$\begin{aligned}\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} &= \frac{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}-\sqrt{3})} \\ &= \frac{5-3}{5-2\sqrt{15}+3} \\ &= \frac{2}{8-2\sqrt{15}} \\ &= \frac{1}{4-\sqrt{15}}\end{aligned}$$

10.

$$\begin{aligned}\frac{\sqrt{3x}-2\sqrt{y}}{\sqrt{3x}+\sqrt{y}} &= \frac{(\sqrt{3x}-2\sqrt{y})(\sqrt{3x}-\sqrt{y})}{(\sqrt{3x}+\sqrt{y})(\sqrt{3x}-\sqrt{y})} \\ &= \frac{3x-2\sqrt{3x}\sqrt{y}-\sqrt{3x}\sqrt{y}+2(y)}{3x-y} \\ &= \frac{3x-3\sqrt{3xy}+2y}{3x-y}\end{aligned}$$

11.

$$\begin{aligned}\frac{\sqrt{3x}-2\sqrt{y}}{\sqrt{3x}+\sqrt{y}} &= \frac{(\sqrt{3x}-2\sqrt{y})(\sqrt{3x}+2\sqrt{y})}{(\sqrt{3x}+\sqrt{y})(\sqrt{3x}+2\sqrt{y})} \\ &= \frac{3x-4y}{3x+2\sqrt{3x}\sqrt{y}+\sqrt{3x}\sqrt{y}+2y} \\ &= \frac{3x-4y}{3x+3\sqrt{3xy}+2y}\end{aligned}$$

12.

$$\begin{aligned}\frac{\sqrt{3}+2\sqrt{7}}{8} &= \frac{(\sqrt{3}+2\sqrt{7})(\sqrt{3}-2\sqrt{7})}{8(\sqrt{3}-2\sqrt{7})} \\ &= \frac{3-4(7)}{8(\sqrt{3}-2\sqrt{7})} \\ &= \frac{-25}{8(\sqrt{3}-2\sqrt{7})}\end{aligned}$$