You must remember to check your solutions and eliminate any extraneous solutions!

Questions

1. Solve $12 + \sqrt{4x + 5} = 7$.

2. Solve $y - \sqrt{y - 3} = 5$.

3. Solve $\sqrt{2y - 4} + 2 = y$.

4. Solve $\sqrt[3]{3 - 5x} = 2$.

5. Solve $\sqrt{8x + 17} = \sqrt{2x + 8} + 3$.

6. Solve $\sqrt{2x + 9} - \sqrt{x + 1} = 2$.

7. In geology, the water depth d near a mid-ocean spreading ridge depends on the square root of the distance x from the ridge axis according to the relation $d = d_0 + a\sqrt{x}$, where d_0 is the depth of the ridge axis and a is some constant. Solve the equation $d = d_0 + a\sqrt{x}$ for x.

8. Solve Graham's law of effusion (used in molecular chemistry) $\frac{\rho_1}{\rho_2} = \sqrt{\frac{M_2}{M_1}}$ for M_2, then solve for M_1.

Solutions

Technique: Isolate a radical expression; square both sides of equation; solve for unknown; eliminate extraneous solutions.

1.

$$
\begin{aligned}
12 + \sqrt{4x+5} &= 7 \\
(\sqrt{4x+5})^2 &= (-5)^2 \\
4x + 5 &= 25 \\
4x &= 25 - 5 \\
4x &= 20 \\
x &= 5
\end{aligned}
$$

Check for Extraneous Solutions:

$$x = 5: \qquad 12 + \sqrt{4(5)+5} = 12 + \sqrt{25} = 12 + 5 = 17 \neq 7$$

So $x = 5$ is extraneous, and the original equation has no solution.

2.

$$
\begin{aligned}
y - \sqrt{y-3} &= 5 \\
(y-5)^2 &= (\sqrt{y-3})^2 \\
y^2 - 10y + 25 &= y - 3 \\
y^2 - 11y + 28 &= 0 \text{ Factor: Two numbers whose sum is } -11 \text{ product is 28: } -7, -4 \\
(y-7)(y-4) &= 0 \\
y - 7 = 0 \text{ or } y - 4 &= 0 \\
y = 7 \text{ or } y &= 4
\end{aligned}
$$

Check for Extraneous Solutions:

$$
\begin{aligned}
y = 7: &\qquad (7) - \sqrt{(7)-3} = 7 - \sqrt{4} = 7 - 2 = 5 \\
y = 4: &\qquad (4) - \sqrt{(4)-3} = 4 - \sqrt{1} = 3 \neq 5
\end{aligned}
$$

So $y = 7$ is the only solution to the original equation.

3.

$$
\begin{aligned}
\sqrt{2y-4} + 2 &= y \\
(\sqrt{2y-4})^2 &= (y-2)^2 \\
2y - 4 &= (y-2)^2 \\
2(y-2) &= (y-2)^2 \text{ Need to factor.} \\
2(y-2) - (y-2)^2 &= 0 \\
(y-2)(2 - (y-2)) &= 0 \\
(y-2)(2 - y + 2) &= 0 \\
(y-2)(4-y) &= 0 \\
y - 2 = 0 \text{ or } 4 - y &= 0 \\
y = 2 \text{ or } y &= 4
\end{aligned}
$$

Check for Extraneous Solutions:

$$
\begin{aligned}
y = 2: &\qquad \sqrt{2(2)-4} + 2 = 2 \\
y = 4: &\qquad \sqrt{2(4)-4} + 2 = \sqrt{4} + 2 = 4
\end{aligned}
$$

So both $y = 2$ and $y = 4$ are solutions.

4. Since we have a cube root, we cube both sides of the equation here.

$$(\sqrt[3]{3-5x})^3 = (2)^3$$
$$3-5x = 8$$
$$-5x = 5 \Rightarrow x = -1$$

Check for Extraneous Solutions:

$$x=-1: \qquad \sqrt[3]{3-5(-1)} = \sqrt[3]{8} = 2$$

So $x=-1$ is a solution.

5.

$$(\sqrt{8x+17})^2 = (\sqrt{2x+8}+3)^2$$
$$8x+17 = 2x+8+9+6\sqrt{2x+8}$$
$$6x = 6\sqrt{2x+8}$$
$$x = \sqrt{2x+8}$$
$$(x)^2 = (\sqrt{2x+8})^2$$
$$x^2 = 2x+8$$
$$x^2-2x-8 = 0$$
$$(x-4)(x+2) = 0$$
$$x-4=0 \text{ or } x+2=0$$
$$x=4 \text{ or } x=-2$$

Check for Extraneous Solutions:

$$x=4: \qquad \sqrt{8(4)+17} = \sqrt{2(4)+8}+3 \Rightarrow \sqrt{49} = \sqrt{16}+3 \Rightarrow 7=7 \text{ True}$$
$$x=-2: \qquad \sqrt{8(-2)+17} = \sqrt{2(-2)+8}+3 \Rightarrow \sqrt{1} = \sqrt{4}+3 \Rightarrow 1=5 \text{ False}$$

So $x=4$ is the only solution.

6.

$$\sqrt{2x+9}-\sqrt{x+1} = 2$$
$$(\sqrt{2x+9})^2 = (2+\sqrt{x+1})^2$$
$$2x+9 = 4+4\sqrt{x+1}+(x+1)$$
$$2x+9 = 5+x+4\sqrt{x+1}$$
$$x+4 = 4\sqrt{x+1}$$
$$(x+4)^2 = (4\sqrt{x+1})^2$$
$$x^2+8x+16 = 16(x+1)$$
$$x^2+8x+16 = 16x+16$$
$$x^2-8x = 0$$
$$x(x-8) = 0$$
$$x=0 \text{ or } x-8=0$$
$$x=0 \text{ or } x=8$$

Check for Extraneous Solutions:

$$x=0: \qquad \sqrt{2(0)+9}-\sqrt{(0)+1} = 3-1 = 2 \text{ True}$$
$$x=8: \qquad \sqrt{2(8)+9}-\sqrt{(8)+1} = 5-3 = 2 \text{ True}$$

So both $x=0$ and $x=8$ are solutions.

7.

$$\begin{aligned} d &= d_0 + a\sqrt{x} \\ d - d_0 &= a\sqrt{x} \\ \frac{d - d_0}{a} &= \sqrt{x} \\ \left(\frac{d - d_0}{a}\right)^2 &= (\sqrt{x})^2 \\ \left(\frac{d - d_0}{a}\right)^2 &= x \end{aligned}$$

8.

$$\begin{aligned} \left(\frac{\rho_1}{\rho_2}\right)^2 &= \left(\sqrt{\frac{M_2}{M_1}}\right)^2 \\ \left(\frac{\rho_1}{\rho_2}\right)^2 &= \frac{M_2}{M_1} \\ \left(\frac{\rho_1}{\rho_2}\right)^2 M_1 &= M_2 \Rightarrow M_2 = \frac{M_1\rho_1^2}{\rho_2^2} \end{aligned}$$

$$\begin{aligned} \left(\frac{\rho_1}{\rho_2}\right)^2 &= \left(\sqrt{\frac{M_2}{M_1}}\right)^2 \\ \frac{\rho_1^2}{\rho_2^2} &= \frac{M_2}{M_1} \\ \frac{\rho_2^2}{\rho_1^2} &= \frac{M_1}{M_2} \\ \frac{\rho_2^2}{\rho_1^2} M_2 &= M_1 \Rightarrow M_1 = \frac{M_2\rho_2^2}{\rho_1^2} \end{aligned}$$