I firmly believe the way to do variation problems is to determine the formula in terms of the constant of variation k, and then use given information to determine the value of the constant of variation. This process will allow you solve more complex problems than if you just try to do everything using ratios.

Questions

1. If y varies directly with x, and $y = 15$ when $x = 40$, find y when $x = 64$.

2. The distance a spring stretches varies directly with the weight of the object hung from the spring (this is Hooke's Law). If a 10 lb weight stretches the spring 6 inches, how far will a 35 lb weight stretch the spring?

3. When an object is dropped, the distance it falls in feet varies directly with the square of the duration of the fall in seconds. An apple that falls from a tree falls 1 ft in $\frac{1}{4}$ second. How far will it fall in 1 second? How far will it fall in 2 seconds?

4. The weight that can be safely supported by a 2 by 6 inch beam varies inversely with its length. An engineer finds that a beam 8 ft long will support 900 lbs. Find the weight that can be safely supported by a beam that is 18 ft long.

5. The strength of a rectangular beam varies jointly with its width and the square of its thickness. If a beam 5 inches wide and 2 inches thick supports 400 lbs, how much can a beam of the same material that is 4 inches wide and 3.5 inches thick support?

6. The *kinetic energy* of an object is the energy the object has due to its motion, and is directly proportional to the mass and directly proportional to the square of velocity. If an object of mass 10kg moving at a velocity of 8 m/s has kinetic energy 320 kg $\mathrm{m}^2/\mathrm{s}^2$ = 320 Newtons = 320 N, determine the formula for kinetic energy. Pay attention to what is happening with the units.

Solutions

1. Since y varies directly with x: $y = kx$.

Use given information to determine k: $15 = k(40) \Rightarrow k = \frac{3}{8}$.

The relationship between x and y now that we know k: $y = \frac{3}{8}x$.

Find y when $x = 64$: $y = \frac{3}{8}(64) = 24$.

2. Let d be the distance stretched, and w be the weight hung from the spring: $d = kw$.

Use given information to determine k: $6 \text{ inches} = k(10 \text{ lbs}) \Rightarrow k = \frac{3}{5}\frac{\text{inches}}{\text{lb}}$.

The relationship is given by: $d = \frac{3}{5}w$.

Find d when $w = 35$ lbs: $d = \frac{3}{5}(35) = 21$ inches.

3. Let d be the distance fallen in ft, and t be the duration of the fall in seconds: $d = kt^2$.

Use given information to determine k: $1 \text{ ft} = k(\frac{1}{4} \text{ sec})^2 \Rightarrow k = 16\frac{\text{ft}}{\text{sec}^2}$.

The relationship is given by: $d = 16t^2$.

Find d when $t = 1$ s: $d = 16(1)^2 = 16$ ft.

Find d when $t = 2$ s: $d = 16(2)^2 = 64$ ft.

4. Let w be the weight in lbs safely supported, and l be the length in ft of the beam: $w = \frac{k}{l}$.

Use given information to determine k: $900 \text{ lbs} = \frac{k}{8 \text{ ft}} \Rightarrow k = 7200 \text{ lbs} \cdot \text{ft}$.

The relationship is given by: $w = \frac{7200}{l}$.

Find w when $l = 18$ ft: $w = \frac{7200}{18} = 400$ lbs.

5. Let s be the weight in lbs safely supported, w be the width in inches of the beam, and t the thickness in inches of the beam: $s = kwt^2$.

Use given information to determine k: $400 \text{ lbs} = k(5 \text{ inches})(2 \text{ inches})^2 \Rightarrow k = 20\frac{\text{lbs}}{\text{inches}^3}$.

The relationship is given by: $s = 20wt^2$.

Find s when $w = 4$ inches and $t = 3.5$ inches: $w = 20(4)(3.5)^2 = 980$ lbs.

Notice that if we simply tip this beam over so the width is $w = 3.5$ inches and the thickness is $t = 4$ inches it becomes stronger: $w = 20(3.5)(4)^2 = 1120$ lbs.

It's obviously important in construction to understand in which direction the strength is required, and place your beam accordingly.

6. Let T be the kinetic energy in Newtons (N), m be the mass in kg, and v the velocity in m/s: $T = kmv^2$.

Use given information to determine k: $320 \text{ N} = k(10 \text{ kg})(8 \text{ m/s})^2 \Rightarrow k = \frac{1}{2}\frac{\text{N}}{kg \cdot (\text{m/s})^2} = \frac{1}{2}$.

Note: k has no units! It is dimensionless.

The formula for kinetic energy is given by: $T = \frac{1}{2}mv^2$.