Note: all the previous quadratics we have solved we were able to solve by *factoring*. If you can factor the quadratic, you can still solve it using those techniques. *Completing the square* is the technique you can use when you cannot factor the quadratic.

We study completing the square because it is the mathematical technique that leads to the *quadratic formula*, and also because it can be used in the future to help you sketch shapes like ellipses, hyperbolas, and circles. It will show up occasionally in future math classes.

Questions

Include complex solutions in your answers.

1. Solve $(x+9)^2 = 21$.

2. Solve $(4x-3)^2 = 36$.

3. Solve $(5x-2)^2 - 25 = 0$.

4. Solve by completing the square $x^2 + 6x + 2 = 0$.

5. Solve by completing the square $x^2 - 14x = -48$.

6. Solve by completing the square $\dfrac{x^2}{3} - \dfrac{x}{3} = 3$.

7. Solve by completing the square $2y^2 - y = 15$.

8. Solve $x^2 - 2x = -7$.

9. Solve $3x^2 + 8x + 3 = 2$.

Solutions

1. Use the square root property, $w^2 = a \Rightarrow w = \pm\sqrt{a}$.

$$\begin{aligned}(x+9)^2 &= 21\\ x+9 &= \pm\sqrt{21}\\ x &= -9 \pm \sqrt{21}\end{aligned}$$

2. Use the square root property.

$$\begin{aligned}(4x-3)^2 &= 36\\ 4x-3 &= \pm\sqrt{36}\\ 4x &= 3 \pm 6\\ x &= \frac{3 \pm 6}{4} = \frac{3+6}{4} \text{ or } \frac{3-6}{4}\\ x &= \frac{3+6}{4} \text{ or } \frac{3-6}{4}\\ x &= \frac{9}{4} \text{ or } -\frac{3}{4}\end{aligned}$$

3. Use the square root property.

$$\begin{aligned}(5x-2)^2 - 25 &= 0\\ (5x-2)^2 &= 25\\ 5x-2 &= \pm\sqrt{25}\\ x &= \frac{2 \pm 5}{5}\\ x &= \frac{2+5}{5} \text{ or } \frac{2-5}{5}\\ x &= \frac{7}{5} \text{ or } -\frac{3}{5}\end{aligned}$$

4.

$$\begin{aligned}x^2 + 6x + 2 &= 0 \text{ To complete the square: } \left(\frac{6}{2}\right)^2 = 9.\\ \underline{x^2 + 6x + 9} - 9 + 2 &= 0 \text{ underlined piece is a perfect square}\\ \underline{(x+3)^2} - 7 &= 0\\ (x+3)^2 &= 7\\ x+3 &= \pm\sqrt{7}\\ x &= -3 \pm \sqrt{7}\end{aligned}$$

5.

$$x^2 - 14x = -48 \text{ To complete the square: } \left(\frac{14}{2}\right)^2 = 49.$$

$$\underbrace{x^2 - 14x + 49} - 49 = -48 \text{ underlined piece is a perfect square}$$

$$\underbrace{(x-7)^2} = 1$$

$$x - 7 = \pm\sqrt{1}$$

$$x = 7 \pm 1$$

$$x = 7 + 1 \text{ or } 7 - 1$$

$$x = 8 \text{ or } 6$$

6.

$$\frac{x^2}{3} - \frac{x}{3} = 3 \text{ We MUST have a coefficient of 1 in front of the } x^2 \text{ before we complete the square.}$$

$$x^2 - x = 9$$

$$x^2 - 1x = 9 \text{ To complete the square: } \left(\frac{1}{2}\right)^2 = \frac{1}{4}.$$

$$\underbrace{x^2 - x + \frac{1}{4}} - \frac{1}{4} = 9 \text{ underlined piece is a perfect square}$$

$$\underbrace{\left(x - \frac{1}{2}\right)^2} = 9 + \frac{1}{4}$$

$$\left(x - \frac{1}{2}\right)^2 = \frac{37}{4}$$

$$x - \frac{1}{2} = \pm\sqrt{\frac{37}{4}}$$

$$x = \frac{1}{2} \pm \frac{\sqrt{37}}{2}$$

7.

$$
\begin{aligned}
2y^2 - y &= 15 \\
y^2 - {\color{red}\frac{1}{2}}y &= \frac{15}{2} \text{ To complete the square: } \left(\frac{1}{4}\right)^2 = \frac{1}{16}. \\
\underbrace{y^2 - \frac{1}{2}y + \frac{1}{16}} - \frac{1}{16} &= \frac{15}{2} \\
\underbrace{\left(y - \frac{1}{4}\right)^2} - \frac{1}{16} &= \frac{15}{2} \\
\left(y - \frac{1}{4}\right)^2 &= \frac{1}{16} + \frac{120}{16} \\
y - \frac{1}{4} &= \pm\sqrt{\frac{121}{16}} \\
y &= \frac{1}{4} \pm \frac{11}{4} \\
y &= \frac{1}{4} + \frac{11}{4} \text{ or } \frac{1}{4} - \frac{11}{4} \\
y &= 3 \text{ or } -\frac{5}{2}
\end{aligned}
$$

8. Solve by completing the square.

$$
\begin{aligned}
x^2 - {\color{red}2}x &= -7 \text{ To complete the square: } \left(\frac{2}{2}\right)^2 = 1. \\
\underbrace{x^2 - 2x + 1} - 1 &= -7 \\
\underbrace{(x-1)^2} - 1 &= -7 \\
x - 1 &= \pm\sqrt{-6} \\
x - 1 &= \pm\sqrt{6}i \\
x &= 1 \pm \sqrt{6}i
\end{aligned}
$$

9. Solve by completing the square.

$$
\begin{aligned}
3x^2 + 8x + 3 &= 2 \\
x^2 + \frac{8}{3}x + 1 &= \frac{2}{3} \\
x^2 + \frac{8}{3}x &= -\frac{1}{3} \text{ To complete the square: } \left(\frac{1}{2}\cdot\frac{8}{3}\right)^2 = \frac{16}{9}. \\
x^2 + \frac{8}{3}x + \frac{16}{9} - \frac{16}{9} &= -\frac{1}{3} \\
\left(x + \frac{4}{3}\right)^2 &= \frac{16}{9} - \frac{1}{3} \\
\left(x + \frac{4}{3}\right)^2 &= \frac{13}{9} \\
x + \frac{4}{3} &= \pm\sqrt{\frac{13}{9}} \\
x &= -\frac{4}{3} \pm \frac{\sqrt{13}}{3}
\end{aligned}
$$