In many of your science (and math) classes you can often solve a quadratic equation by using the quadratic formula. This replaces having to go through the process of completing the square (remember, competing the square in general is what leads to the quadratic formula).

You must memorize the quadratic formula.

Questions

Include complex solutions in your answers. Some of these equations can be solved easily using techniques other than the quadratic formula.

1. Write down the quadratic formula that solves $ax^2 + bx + c = 0$.

2. Solve $x^2 = \dfrac{2}{3}x$.

3. Solve $7x^2 + 4x - 3 = 0$.

4. Solve $4x^2 + 3x - 2 = 0$.

5. Solve $\dfrac{1}{4} + \dfrac{6}{y+2} = \dfrac{6}{y}$.

6. Solve $2x^2 + 15 = 0$.

7. Solve $5x^2 = -3$.

8. Use the discriminant to find what type of solution $9x^2 + 4 = 12x$ has.

9. Write a quadratic equation which has the solutions $1 - 4i$ and $1 + 4i$.

Solutions

1. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

2.

$$x^2 - \frac{2}{3}x = 0 \text{ So } a = 1, b = -2/3, \text{ and } c = 0.$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2/3) \pm \sqrt{(-2/3)^2 - 4(1)(0)}}{2(1)}$$

$$x = \frac{2/3 \pm 2/3}{2}$$

$$x = 1/3 \pm 1/3$$

$$x = 1/3 + 1/3 \text{ or } 1/3 - 1/3$$

$$x = 2/3 \text{ or } 0$$

Since $c = 0$, this could have been solved by factoring:

$$x^2 - \frac{2}{3}x = 0$$

$$x\left(x - \frac{2}{3}\right) = 0$$

$$x = 0 \text{ or } x - \frac{2}{3} = 0$$

$$x = 0 \text{ or } x = \frac{2}{3}$$

3.

$$7x^2 + 4x - 3 = 0 \text{ So } a = 7, b = 4, \text{ and } c = -3.$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(7)(-3)}}{2(7)}$$

$$x = \frac{-4 \pm \sqrt{100}}{14}$$

$$x = \frac{-4 \pm 10}{14}$$

$$x = \frac{-4 + 10}{14} \text{ or } \frac{-4 - 10}{14}$$

$$x = \frac{3}{7} \text{ or } -1$$

This could also have been done using factoring by grouping.

4.

$$4x^2 + 3x - 2 = 0 \text{ So } a = 4, b = 3, \text{ and } c = -2.$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(4)(-2)}}{2(4)}$$

$$x = \frac{-3 \pm \sqrt{41}}{8}$$

Because the roots are complicated, the only way to do this one was using quadratic formula or completing the square.

5.

$$\frac{1}{\cancel{4}} \cdot \cancel{4}(y+2)(y) + \frac{6}{\cancel{y+2}} \cdot 4\cancel{(y+2)}(y) = \frac{6}{\cancel{y}} \cdot 4(y+2)\cancel{(y)} \text{ Multiply by LCD.}$$
$$(y+2)(y) + 6 \cdot 4y = 6 \cdot 4(y+2)$$
$$y^2 + 2y + 24y - 24y - 48 = 0$$
$$y^2 + 2y - 48 = 0 \text{ So } a = 1, \ b = 2, \text{ and } c = -48.$$
$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$y = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-48)}}{2(1)}$$
$$y = \frac{-2 \pm \sqrt{196}}{2}$$
$$y = \frac{-2 \pm 14}{2}$$
$$y = -1 \pm 7$$
$$y = -1 + 7 \text{ or } -1 - 7$$
$$y = 6 \text{ or } -8$$

Recall than when you multiply by an LCD to solve an equation, you must check for extraneous solutions.

$$\text{Check } y = 6: \quad \frac{1}{4} + \frac{6}{(6) + 2} = \frac{6}{(6)} \Rightarrow 1 = 1 \text{ True}$$
$$\text{Check } y = -8: \quad \frac{1}{4} + \frac{6}{(-8) + 2} = \frac{6}{(-8)} \Rightarrow -\frac{3}{4} = -\frac{3}{4} \text{ True}$$

6. Solve using the property $w^2 = a \Rightarrow w = \pm\sqrt{a}$.

$$2x^2 + 15 = 0$$
$$2x^2 = -15$$
$$x^2 = -\frac{15}{2}$$
$$x = \pm\sqrt{-\frac{15}{2}}$$
$$x = \pm\sqrt{\frac{15}{2}}i$$

7. Solve using quadratic formula.

$5x^2 + 3 = 0$ So $a = 5$, $b = 0$, $c = 3$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(0) \pm \sqrt{(0)^2 - 4(5)(3)}}{2(5)}$$

$$x = \frac{\pm\sqrt{-60}}{10}$$

$$x = \pm\frac{\sqrt{60}}{10}i$$

$$x = \pm\frac{\sqrt{2^2 \cdot 15}}{10}i$$

$$x = \pm\frac{2\sqrt{15}}{10}i$$

$$x = \pm\frac{\sqrt{15}}{5}i$$

8. The discriminant is $b^2 - 4ac = (-12)^2 - 4(9)(4) = 0$. This means there will be one rational root.

9. If the quadratic has solution r, then it has a factor $(x - r)$.

$$(x - (1 - 4i))(x - (1 + 4i)) = 0 \text{ Now, carefully multiply everything out to get the quadratic.}$$

$$(x)(x - (1 + 4i)) + (-(1 - 4i))(x - (1 + 4i)) = 0$$

$$x^2 - x - 4xi + (-(1 - 4i))(x) - (-(1 - 4i))(1 + 4i) = 0$$

$$x^2 - x - 4xi + (-x + 4xi) + (1 - 4i)(1 + 4i) = 0$$

$$x^2 - x - 4xi - x + 4xi + 1 + 4i - 4i - 16i^2 = 0$$

$$x^2 - 2x + 1 - 16(-1) = 0$$

$$x^2 - 2x + 17 = 0$$