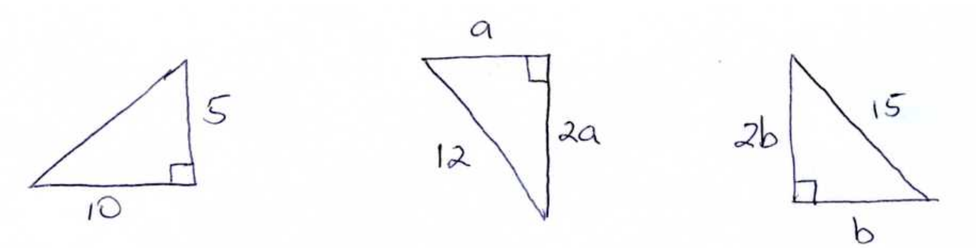
Questions

1. Solve $S = 4\pi r^2$ for r.

2. Solve $A = \frac{1}{2}r^2\theta$ for r.

3. Solve $A = P(1+r)^2$ for r.

4. Find the length of the sides in the following right angle triangles:



5. The brace for a bookshelf has the shape of a right triangle. It's hypotenuse is 10 inches long and the two legs are equal in length. How long are the legs of the triangle?

6. Knox college is creating a new rectangular parking lot. The length is 0.07 miles longer than the width and the area of the parking lot is 0.026 square miles. Find the length and width of the parking lot.

7. The formulas $A = P(1+r)^2$ gives the amount A in dollars that will be obtained in 2 years if P dollars are invested at an annual compound interest rate of r. If you invest $P = \$1400$ and it grows to $\$1514.24$ in 2 years, what is the annual interest rate r?

Solutions

1.

$$\begin{aligned} S &= 4\pi r^2 \\ \frac{S}{4\pi} &= r^2 \\ r^2 &= \frac{S}{4\pi} \\ r &= \pm\sqrt{\frac{S}{4\pi}} \\ r &= \pm\frac{1}{2}\sqrt{\frac{S}{\pi}} \end{aligned}$$

With formulas, typically the variables have some meaning. If r is a length, we would eliminate the negative solution and only keep $r = \frac{1}{2}\sqrt{\frac{S}{\pi}}$.

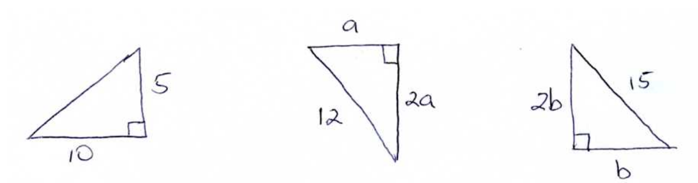
2.

$$\begin{aligned} A &= \frac{1}{2}r^2\theta \\ \frac{2A}{\theta} &= r^2 \\ r &= \pm\sqrt{\frac{2A}{\theta}} \end{aligned}$$

3.

$$\begin{aligned} A &= P(1+r)^2 \\ \frac{A}{P} &= (1+r)^2 \\ \pm\sqrt{\frac{A}{P}} &= 1+r \\ -1 \pm \sqrt{\frac{A}{P}} &= r \end{aligned}$$

4. Choose positive square roots since we are looking for lengths which are greater than zero.



First:

$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + 25 &= 100 \\ a &= \sqrt{75} = 5\sqrt{3} \end{aligned}$$

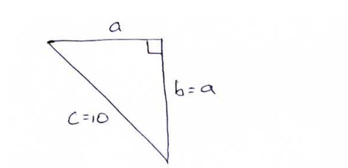
Second:

$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + (2a)^2 &= (12)^2 \\ 5a^2 &= 144 \Rightarrow a = \sqrt{\frac{144}{5}}; b = 2a = 2\sqrt{\frac{144}{5}} \end{aligned}$$

Third:

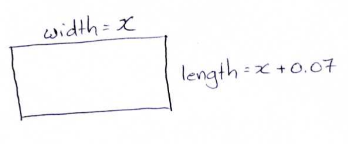
$$\begin{aligned} a^2 + b^2 &= c^2 \\ (2b)^2 + b^2 &= 15^2 \\ 5b^2 &= 225 \Rightarrow b = 3\sqrt{5}; a = 6\sqrt{5} \end{aligned}$$

5. Sketch:



$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + a^2 &= 10^2 \\ a^2 &= 50 \Rightarrow a = \sqrt{50} = 5\sqrt{2} \text{ inches.} \end{aligned}$$

6.Sketch:



$$\begin{aligned}
\text{Area} &= (\text{length})(\text{width}) \\
0.026 &= (x + 0.07)x \\
x^2 + 0.07x - 0.026 &= 0 \\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
x &= \frac{-0.07 \pm \sqrt{(0.07)^2 - 4(1)(-0.026)}}{2(1)} \\
x &= 0.13
\end{aligned}$$

The lot is $x = 0.13$ miles wide and $x + 0.07 = 0.2$ miles long.

7.

$$\begin{aligned}
A &= P(1 + r)^2 \\
1514.24 &= 1400(1 + r)^2 \\
1.0816 &= (1 + r)^2 \\
\pm\sqrt{1.0816} &= 1 + r \\
\pm 1.04 &= 1 + r \\
-1 \pm 1.04 &= r \\
r = 0.04 \text{ or } r &= -2.04
\end{aligned}$$

The interest rate is $0.04 = 4\%$ per year (the interest rate must be positive).

An alternate solution would be to expand $(1 + r)^2 = 1 + 2r + r^2$ and then use the quadratic formula.