**Questions**

**1.** Sketch $f(x) = 2x^2 + 2x - 4$.

**2.** Sketch $w(x) = x^2 - 6x + 8$.

**3.** Sketch $g(x) = x^2 - 2x - 8$.

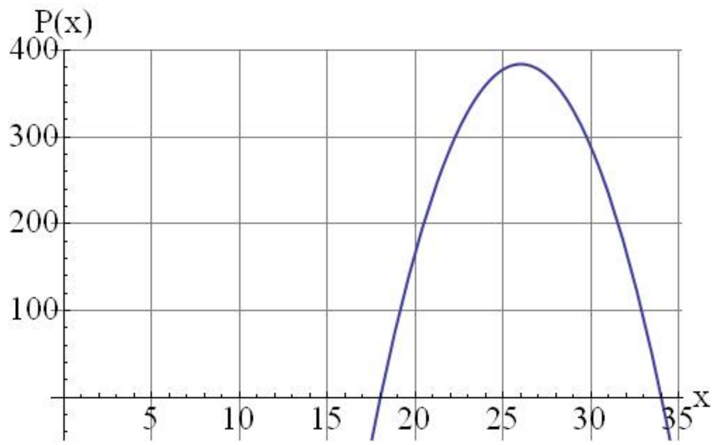
**4.** Sketch $r(x) = -3x^2 + 6x - 4$.

**5.** Given the sketch below for $P(x) = -6x^2 + 312x - 3672$, which represents the profit $P$ in dollars where $x$ is the number of widgets manufactured by a company each day.

What is the maximum profit? How many widgets are produced each day to get the maximum profit?

How any widgets are made each day if the company has a daily profit of zero dollars?

How many widgets are made per day if the company has a profit of \$288 per day?



**Solutions**

**1.** Sketch $f(x) = 2x^2 + 2x - 4$.

Since $a = 2 > 0$, quadratic opens up

$$\text{Vertex: } x = -\frac{b}{2a} = -\frac{2}{2(2)} = -\frac{1}{2}$$

$$y = f(-1/2) = 2(-1/2)^2 + 2(-1/2) - 4 = -\frac{9}{2}$$

$$x\text{-intercepts: } 2x^2 + 2x - 4 = 0$$

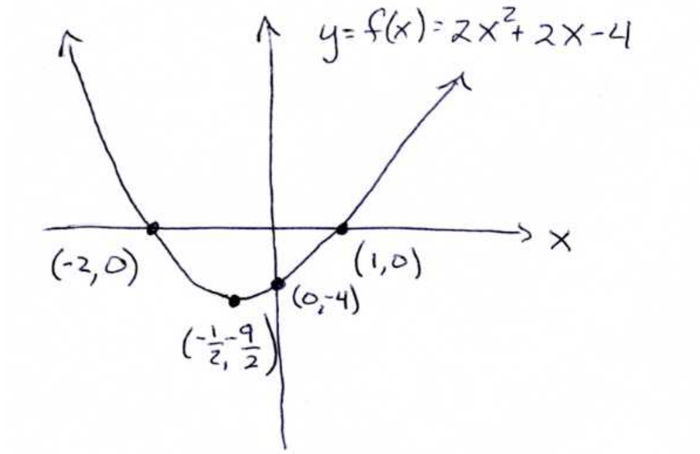
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(2)(-4)}}{2(2)}$$

$$x = \frac{-2 \pm 6}{4}$$

$$x = 1 \text{ or } x = -2$$

$$y\text{-intercept: } f(0) = 2(0)^2 + 2(0) - 4 = -4$$

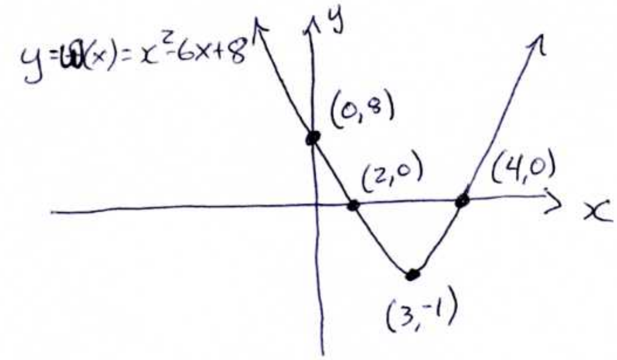


**2.** Sketch $w(x) = x^2 - 6x + 8$.

Since $a = 1 > 0$, quadratic opens up

$$\text{Vertex: } x = -\frac{b}{2a} = -\frac{(-6)}{2(1)} = 3$$
$$y = f(3) = (3)^2 - 6(3) + 8 = -1$$

$$x\text{-intercepts: } x^2 - 6x + 8 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(8)}}{2(1)}$$
$$x = \frac{6 \pm 2}{2}$$
$$x = 4 \text{ or } x = 2$$
$$y\text{-intercept: } f(0) = (0)^2 - 6(0) + 8 = +8$$



**3.** Sketch $g(x) = x^2 - 2x - 8$.

Since $a = 1 > 0$, quadratic opens up

$$\text{Vertex: } x = -\frac{b}{2a} = -\frac{(-2)}{2(1)} = -1$$

$$y = f(-1) = (-1)^2 - 2(-1) - 8 = -9$$

$$x\text{-intercepts: } x^2 - 2x - 8 = 0$$

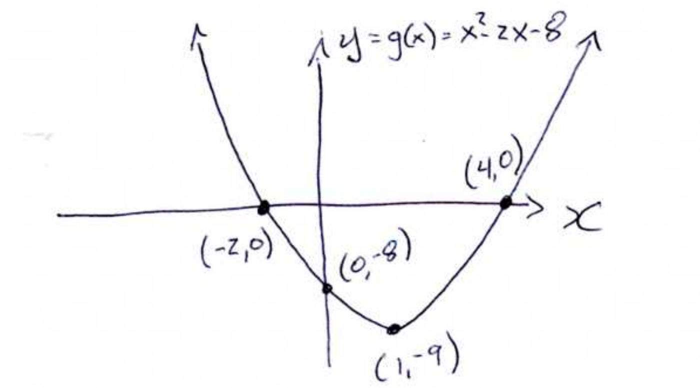
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-8)}}{2(1)}$$

$$x = \frac{2 \pm 6}{2}$$

$$x = 4 \text{ or } x = -2$$

$$y\text{-intercept: } f(0) = (0)^2 - 2(0) - 8 = -8$$



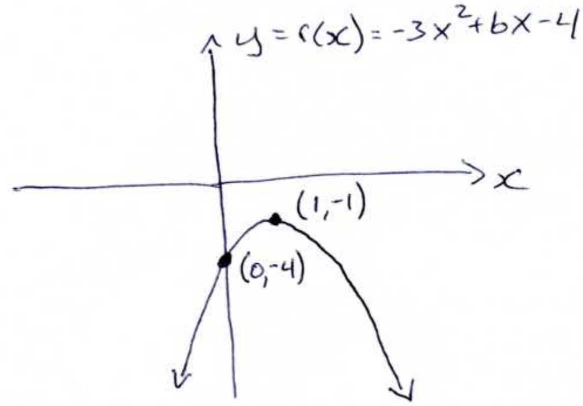
**4.** Sketch $r(x) = -3x^2 + 6x - 4$.

Since $a = -3 < 0$, quadratic opens down

$$\text{Vertex: } x = -\frac{b}{2a} = -\frac{6}{2(-3)} = 1$$
$$y = f(1) = -3(1)^2 + 6(1) - 4 = -1$$

$$x\text{-intercepts: } -3x^2 + 6x - 4 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-6 \pm \sqrt{6^2 - 4(-3)(-4)}}{2(-3)}$$
since $b^2 - 4ac = -12 < 0$, there are no $x$-intercepts

$$y\text{-intercept: } f(0) = -3(0)^2 + 6(0) - 4 = -4$$



**5.** We don't actually need the sketch to answer these questions.

Let's analyze $P(x) = -6x^2 + 312x - 3672$ to answer the questions.

Maximum profit occurs at the vertex.

$$\text{Vertex: } x = -\frac{b}{2a} = -\frac{312}{2(-6)} = 26 \text{(number of widgets to get max profit)}$$
$$y = f(26) = -6(26)^2 + 312(26) - 3672 = 384 \text{ (maximum profit)}$$

Daily profit will be zero at the $x$-intercepts.

$$x\text{-intercepts: } -6x^2 + 312x - 3672 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-312 \pm \sqrt{(312)^2 - 4(-6)(-3672)}}{2(-6)}$$
$$x = \frac{-312 \pm 96}{-12}$$
$$x = 18 \text{ or } x = 34$$

Profit is zero if they produce 18 or 34 widgets.

For a profit of \$288, we must solve the equation $P(x) = 288$.

$$-6x^2 + 312x - 3672 = 288$$
$$-6x^2 + 312x - 3960 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-312 \pm \sqrt{(312)^2 - 4(-6)(-3960)}}{2(-6)}$$
$$x = \frac{-312 \pm 48}{-12}$$
$$x = 22 \text{ or } x = 30$$

Daily profit is \$288 if they produce 22 or 30 widgets.