Some of these problems are challenging. You should be able to figure out a way to check if you have the correct solution somehow–I am not providing solutions to these problems.

Proportion

Problem 1. The volume of a sphere is proportional to the cube of its radius. If a sphere of radius 10 cm has a volume of 4000 $\pi/3$ cm^3, determine the formula for the volume of a sphere. Then, solve for the radius in terms of the volume. Pay attention to what is happening with the units.

Problem 2. The distance an object falls (neglecting wind resistance) is proportional to the square of the time it falls. If an object falls 176.4 m in 6 s, determine the formula for the distance an object falls. Then, solve the formula for time. Pay attention to what is happening with the units.

Problem 3. The kinetic energy of an object (energy of motion) is directly proportional to the mass and directly proportional to the square of velocity. If an object of mass 10kg moving at a velocity of 8 m/s has kinetic energy 320 kg m^2/s^2 = 320 N, determine the formula for kinetic energy. Then, solve the formula for the velocity. Pay attention to what is happening with the units.

Rationalizing

Problem 1. Rationalize the numerator in $\dfrac{\sqrt{y}+x}{\sqrt{x}-6}$.

Problem 2. Rationalize the denominator in $\dfrac{\sqrt{14}+x}{\sqrt{x}-6}$.

Problem 3. Rationalize the denominator in $\dfrac{x+y}{\sqrt{x}+\sqrt{yx}}$. Try to factor your final answer.

Problem 4. Rationalize the numerator in $\sqrt{34x}+\sqrt{x^2+y^2}$.

Problem 5. Rationalize the denominator in $\dfrac{12x-\sqrt{12x}}{12x+\sqrt{12x}}$. Try to factor your final answer.

Solving Equations

Problem 1. Solve the van der Waals equation (used to model fluid compression in chemistry) $\left(p+\dfrac{n^2a}{V^2}\right)(v-nb)=nRT$ for p, and then solve for a.

Problem 2. Solve the equation (used to model resistance in electrical systems that have a temperature dependence) $R=\dfrac{L}{A}\rho_0\Big(\alpha(T-T_0)+1\Big)$ for ρ_0 and then solve for α. The quantities ρ_0 and T_0 are labeled with subscripts in this equation.

Problem 3. Solve Graham's law of effusion (used in molecular chemistry) $\dfrac{\rho_1}{\rho_2}=\sqrt{\dfrac{M_2}{M_1}}$ for M_2, then solve for M_1.

Problem 4. Solve Archie's law (used in geology to measure conductivity of sedimentary rock) $C_t = C_w\phi^m S_w^n$ for ϕ. Here, the quantities C_t, C_w and S_w are labeled with subscripts. If you have a brine saturated interval, then you can say $S_w = 1$. What is the expression for ϕ in this case?

Problem 5. Solve Fitt's law (used in human computer interaction models) $2^{(T-a)/b}=\dfrac{D}{W}+1$ for D, and then solve for W.

Sketching

Problem 1. Sketch $14x - 7y + 1 = 56$.

Problem 2. Sketch $x^2 - 6x + y = 1$.

Problem 3. Sketch the region that satisfies the inequalities $x > y$, $y \geq 3x^2 - 3x - 8$ and $y < -4x + 3$. Don't worry about determining points of intersection–although you certainly could!

Problem 4. Sketch the line that is perpendicular to the line $y = 3x + 4$ that passes through the point $(1, 2)$.

Rules of Exponents and Radicals

Problem 1. Simplify so there are no negative exponents: $\left(\dfrac{4xz^{-2}}{5z}\right)^{-3}$.

Problem 2. Simplify so there are no negative exponents: $\left(\dfrac{y^2x}{xzy^{-2}}\right)^{9}$.

Problem 3. Simplify so there are no negative exponents: $\left(2^{-2}x^3y^{-2}x^3y^3\right)^9$.

Problem 4. Simplify so there are no negative exponents (write your final answer using exponents): $\left(2^{-2}\sqrt{x}y^{1/3}x^3y^{3/2}\right)^2$.

Problem 5. Simplify by distributing and write the final answer using exponents instead of radicals: $(\sqrt{x} - x)(\sqrt[3]{x} - x^2)$.

Polynomials and Equations

Problem 1. Combine to be a single fraction: $\dfrac{x}{3x^2 - 4x - 4} - \dfrac{x - 1}{5x^2 - 8x - 4}$.

Problem 2. Solve for x if $\dfrac{x - 1}{3x - 2} = \dfrac{1 - x}{2}$.

Problem 3. Solve for x if $\dfrac{x - 1}{3x - 2} - \dfrac{1 - x}{2} = \dfrac{1}{3x - 2}$.

Problem 4. Solve for x if $x + 4\sqrt{x} - 2 = 0$.

Problem 5. Divide the polynomial $-20x^3 + 33x^2 - 8x - 21$ by $4x + 3$.

Problem 6. What is the domain of $f(x) = \sqrt{x^2 - 5x - 2}$?

Problem 7. Solve for x if $\log_2(5x + 1) - \log_2(x - 2) = 1$.

Problem 8. What is the domain of $f(x) = \sqrt{3x - 2}$?

Problem 9. Solve for x if $\left|x^2 - 3x\right| = -4$.

Problem 10. Solve for x if $\sqrt{x - 2} + \sqrt{x} = 4$.

Problem 11. Solve the following equation for x: $x^4 + 4x^2 - 1 = 0$.

Problem 12. Solve the following for y: $y^{-2} + 6y^{-1} + 4 = 0$.

Problem 13. Solve the following for x: $(\sqrt{x} - 2)^4 + (\sqrt{x} - 2)^2 - 2 = 0$.

Other Problems

Problem 1. Write down the quadratic formula.

Problem 2. Derive the quadratic formula.

Problem 3. A right triangle has hypotenuse of length 3/4 cm and one side of length 7/8 cm. What is the length of the other side?

Problem 4. Factor $x^4 - 1$.

Problem 5. Divide $a^3 + b^3$ by $a + b$ using long division of polynomials, which gives you the *sum of cubes* factoring rule: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$.

Problem 6. What is the distance between the two points $(8, -7)$ and $(-2, 1/4)$ in the xy-plane?

Problem 7. What is the solution to the system of equations $4x - 5y = 10$ and $5x - 7y = 9$.

Problem 8. What is the solution to $12(x - 3) - 11(x - 18) = x - 9$?