

Procedure for Solving radical equations

1. Algebraically isolate one radical by itself on one side of equal sign.
2. Raise each side of the equation to an appropriate power to remove the radical.
3. Simplify.
4. If the equation still contains a radical, repeat steps 1 through 3.
5. Once all the radicals are removed, solve the equation.
6. Check all solutions, and exclude any that do not satisfy the original equation. These excluded solutions are called extraneous solutions.

Example Solve $2\sqrt{4x+1} + 5 = x + 9$.

$$2\sqrt{4x+1} + 5 = x + 9 \text{ (isolate the radical } \sqrt{4x+1})$$

$$(\sqrt{4x+1})^2 = \left(\frac{x+4}{2}\right)^2 \text{ (now square each side to remove radical)}$$

$$4x+1 = \frac{(x+4)^2}{4} \text{ (simplify)}$$

$$16x+4 = (x+4)^2 \text{ (quadratic-set it up for factoring)}$$

$$16x+4 = x^2 + 8x + 16$$

$$x^2 - 8x + 12 = 0$$

$$(x-6)(x-2) = 0$$

$$(x-6) = 0 \text{ or } (x-2) = 0$$

$$x = 6 \text{ or } x = 2$$

We are not done until we have checked that these really are solutions:

$$2\sqrt{4(6)+1} + 5 \stackrel{?}{=} (6) + 9 \text{ (check } x = 6)$$

$$2\sqrt{25} + 5 \stackrel{?}{=} 15$$

$$15 \stackrel{?}{=} 15 \text{ (True! } x = 6 \text{ is a solution)}$$

$$2\sqrt{4(2)+1} + 5 \stackrel{?}{=} (2) + 9 \text{ (check } x = 2)$$

$$2\sqrt{9} + 5 \stackrel{?}{=} 11$$

$$11 \stackrel{?}{=} 11 \text{ (True! } x = 2 \text{ is a solution)}$$

Example Solve $\sqrt{3x+4} + \sqrt{x+5} = \sqrt{7-2x}$.

$$\sqrt{3x+4} + \sqrt{x+5} = \sqrt{7-2x} \text{ (we've got a radical isolated, so square both sides)}$$

$$(\sqrt{3x+4} + \sqrt{x+5})^2 = (\sqrt{7-2x})^2 \text{ (simplify)}$$

$$3x+4+x+5+2\sqrt{3x+4}\sqrt{x+5} = 7-2x$$

$$\sqrt{(3x+4)(x+5)} = -1-3x \text{ (isolate the remaining radical)}$$

$$(\sqrt{(3x+4)(x+5)})^2 = (-1-3x)^2 \text{ (square both sides to remove the radical)}$$

$$(3x+4)(x+5) = 1+6x+9x^2 \text{ (simplify)}$$

$$3x^2+19x+20 = 1+6x+9x^2 \text{ (we have a quadratic, so set it up for factoring)}$$

$$6x^2-13x-19 = 0 \text{ (factor by grouping: two numbers whose product is } -114 \text{ sum is } -13: -19, 6)$$

$$6x^2-19x+6x-19 = 0$$

$$x(6x-19)+1(6x-19) = 0$$

$$(x+1)(6x-19) = 0$$

$$(x+1) = 0 \text{ or } (6x-19) = 0$$

$$x = -1 \text{ or } x = \frac{19}{6} \text{ (but we aren't done yet!)}$$

We aren't done until we see if these solve the original equation:

$$\sqrt{3(-1)+4} + \sqrt{(-1)+5} \stackrel{?}{=} \sqrt{7-2(-1)} \text{ (check } x = -1)$$

$$\sqrt{1} + \sqrt{4} \stackrel{?}{=} \sqrt{9}$$

$$1+2 \stackrel{?}{=} 3 \text{ (True! So } x = -1 \text{ is a solution)}$$

$$\sqrt{3(19/6)+4} + \sqrt{(19/6)+5} \stackrel{?}{=} \sqrt{7-2(19/6)} \text{ (check } x = 19/6)$$

$$\sqrt{27/2} + \sqrt{49/6} \stackrel{?}{=} \sqrt{2/3} \text{ (False! So } x = 19/6 \text{ is not a solution)}$$

The only solution to the original radical equation is $x = -1$.

Complex numbers

$a + bi$ where a is the real part and b is the imaginary part.

- $i^2 = -1$ or $i = \sqrt{-1}$ and i is called an imaginary number.
- For all $a \geq 0$, $\sqrt{-a} = i\sqrt{a}$.
- $(a + bi) + (c + di) = (a + c) + (b + d)i$ (real part goes with real part, imaginary part goes with imaginary part).
- The complex conjugate of $a + bi$ is equal to $a - bi$.
- To divide two complex numbers, multiply the numerator and denominator by the complex conjugate of the denominator.

The phrase “imaginary number” is a bit misleading—the number exists, it is just something other than a real number! Complex numbers are incredibly useful in a variety of areas of mathematics. For now, you should see that the real parts are collected together and the imaginary parts are collected together when you are working with complex numbers. The basic fact that $i^2 = -1$ allows you to simplify complex numbers.

Example Divide the two complex numbers: $\frac{7 + 14i}{6 - 3i}$.

Use complex conjugate of the denominator:

$$\begin{aligned}\frac{7 + 14i}{6 - 3i} &= \frac{7 + 14i}{6 - 3i} \cdot \frac{6 + 3i}{6 + 3i} \\ &= \frac{(7 + 14i)(6 + 3i)}{(6 - 3i)(6 + 3i)} \\ &= \frac{42 + 84i + 21i + 42i^2}{36 - 9i^2} \\ &= \frac{42 + 105i + 42(-1)}{36 - 9(-1)} \quad (\text{use } i^2 = -1) \\ &= \frac{105i}{45} = \frac{7}{3}i \quad (\text{simplify})\end{aligned}$$

Variation

- **Direct variation** between x and y means $y = kx$, where k is the constant of variation.
- **Inverse variation** between x and y means $y = \frac{k}{x}$, where k is the constant of variation.
- In either case, use the given information to determine the value of k .
- Then, use the equation you have created to determine the unknown quantity.
- **Joint variation** just means a quantity depends on the product of more than one quantity.

Example Police officers can use variation to detect speeding. The speed of a car varies inversely with the time it takes to cover a certain fixed distance. Between two points on a highway, a car travels 45 mph in 6 seconds. What is the speed of a car that travels the same distance in 9 seconds?

We are told there is an inverse variation between speed and time: $v = \frac{k}{t}$.

where t is time in seconds and v is velocity in mph.

Use the given information (car traveling at 45 mph covers the distance in 6 seconds) to determine the value of the constant of variation k :

$$v = \frac{k}{t} \Rightarrow 45 = \frac{k}{6} \Rightarrow k = 270.$$

Now we can answer the question asked about the speed of a car that covers the distance in 9 seconds:

$$v = \frac{270}{t} \Rightarrow v = \frac{270}{9} \Rightarrow v = 30\text{mph.}$$

Another interesting question to ask is: If the speed limit is 65 mph, what time will a speeding car cover the distance?

$$v = \frac{270}{t} \Rightarrow 65 = \frac{270}{t} \Rightarrow t = 4.15\text{seconds.}$$

Any car that covers the distance faster than 4.15 seconds is speeding.

Solving Absolute Value Equalities and Inequalities: Three Cases

Case1 For equalities of the form $|ax + b| = |cx + d|$, the solution is

$$ax + b = cx + d \quad \text{or} \quad ax + b = -(cx + d).$$

The solution will be two distinct numbers.

Case2 For inequalities of the form $|ax + b| < c$, where $c > 0$ the solution is

$$-c < ax + b < c.$$

NOTE: it is important that the $-c$ is on the left and the c is on the right. If this isn't the case, you will get the wrong solution.

The solution will be a set of points between two numbers.

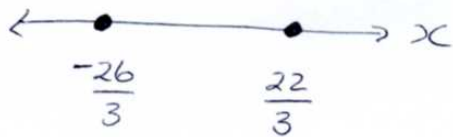
Case3 For inequalities of the form $|ax + b| > c$, where $c > 0$ the solution is

$$ax + b < -c \quad \text{or} \quad ax + b > c.$$

The solution will be a set of numbers less than one number or greater than another number.

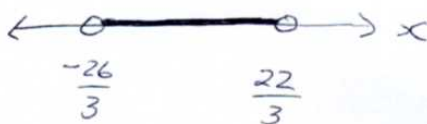
Example Solve the inequality $|3x + 2| = 24$ and sketch the solution on a number line.

$$\begin{aligned} |3x + 2| &= 24 \\ 3x + 2 &= 24 \quad \text{or} \quad 3x + 2 = -24 \\ 3x &= 22 \quad \text{or} \quad 3x = -26 \\ x &= \frac{22}{3} \quad \text{or} \quad x = -\frac{26}{3} \end{aligned}$$



Example Solve the inequality $|3x + 2| < 24$ and sketch the solution on a number line.

$$\begin{aligned} |3x + 2| &< 24 \\ -24 &< 3x + 2 < 24 \\ -26 &< 3x < 22 \\ -\frac{26}{3} &< x < \frac{22}{3} \end{aligned}$$



Example Solve the inequality $|3x + 2| > 24$ and sketch the solution on a number line.

$$|3x + 2| > 24$$

$$3x + 2 > 24 \text{ or } 3x + 2 < -24$$

$$3x > 22 \text{ or } 3x < -26$$

$$x > \frac{22}{3} \text{ or } x < -\frac{26}{3}$$

