Fractions

(see the Chapter Organizer on page 53 of the text for more details of each procedure)

Mathematics $\longrightarrow$ Fractions are preferred. For applied problems convert to decimal as the last step in your solution.

Science Courses $\longrightarrow$ Fractions for theoretical problems. For applied problems decimals with error analysis.

Prime Factorization

Technique: Find a number that divides evenly, and continue the process until you have a prime factorization:

Simplifying Fractions

Technique: Prime factor numerator and denominator, then cancel.

Finding the LCD (least common denominator) of two or more fractions

Technique: Look for the prime factorization of the two denominators. Collect common factors.

To ADD or SUBTRACT fractions you MUST use a common denominator.

To MULTIPLY fractions does NOT require an LCD, simply multiply numerators and multiply denominators.

To DIVIDE fractions does NOT require an LCD. You invert the fraction you are dividing by and multiply.

Adding and subtracting mixed numbers: change to improper fraction

Multiplying and dividing mixed numbers: change to improper fraction

Percents

The percent sign % can be thought of as "per one hundred", or $\% \leftrightarrow \frac{1}{100}$.

Examples

Example 0.2.78 Carl bought a 20 gallon aquarium. He put $17\frac{3}{4}$ gallons of water into the aquarium, but it looked too low so he added $1\frac{1}{4}$ gallons of water. Then he added plants and gravel, but the water was too high so he removed $2\frac{2}{3}$ gallons of water. How many gallons of water are now in the aquarium?

$$17\frac{3}{4}+1\frac{1}{4}-2\frac{2}{3}=\frac{71}{4}+\frac{5}{4}-\frac{8}{3}.$$

The LCD for these improper fractions is 12.

$$\frac{71}{4}+\frac{5}{4}-\frac{8}{3}=\frac{71\times 3}{4\times 3}+\frac{5\times 3}{4\times 3}-\frac{8\times 4}{3\times 4}=\frac{213}{12}+\frac{15}{12}-\frac{32}{12}=\frac{213+15-32}{12}=\frac{196}{12}=16\frac{1}{3}$$

Since 12 divides into 196 16 times with a remainder of $\frac{4}{12}=\frac{1}{3}$, there is $16\frac{1}{3}$ gallons of water in the tank.

Example 0.2.81 A country club maintains the putting greens with a grass height of $\frac{7}{8}$ inches. The grass on the fairways is maintained at $2\frac{1}{2}$ inches. How much must the mower blade be lowered if the person mowing the fairways uses the same machine to mow the greens?

$$2\frac{1}{2}-\frac{7}{8}=\frac{5}{2}-\frac{7}{8}=\frac{5\times 4}{2\times 4}-\frac{7}{8}=\frac{20}{8}-\frac{7}{8}=\frac{20-7}{8}=\frac{13}{8}=1\frac{5}{8}$$

The blade must be lowered $1\frac{5}{8}$ inches.

Example (based on 0.3) A denim shirt requires $2\frac{3}{4}$ yards of material to make. If you have 142 yards of material, how many shirts can you make (assume there is no wasted material).

$$\frac{142}{\left(2\frac{3}{4}\right)} = \frac{142}{\left(\frac{11}{4}\right)} = \frac{142}{1} \times \frac{4}{11} = \frac{568}{11} \sim 51.6$$

You only have enough material to make 51 shirts. Notice in this problem it made sense to convert to a decimal at the end.

Example 0.5.31 Dave took a multiple choice exam with 80 questions and answered 68 of them correctly. What was his percentage grade for the exam?

$$\frac{68}{80} = 0.85 = 85\%.$$

Example 0.5.34 Music cds have a failure rate of 1.8% (meaning 1.8% of them are defective). If 36,000 cds were manufactured in a week, how many of them are defective?

$1.8\% \times 36,000 = 0.018 \times 36,000 = 648$ were defective.

Example What percent by weight of carbon tetrachloride, CCl_4, is carbon, C? The atomic weights of these elements are (C, 12: Cl, 35).

Carbon tetrachloride is made up of 1 carbon and 4 chlorine.

The atomic weight of carbon tetrachloride is therefore $1 \times 12 + 4 \times 35 = 152$ atomic units.

The fraction of this weight that is carbon is $\frac{1 \times 12}{152} = \frac{3 \times \cancel{4}}{38 \times \cancel{4}} = \frac{3}{38}$.

Since $\frac{3}{38} \sim 0.07894 = 7.89\%$, carbon tetrachloride gets roughly 7.89% of its weight from carbon.

It made sense to convert to decimals at the end.

Example 0.5.64 Dave Bagley traveled 24,500 miles last year. He is a salesperson and 74% of his mileage is business related. He was planning to deduct 31 cents per business mile on his income tax return, but his accountant told him he can deduct 35 cents per business mile. How much would his deduction increase if he uses the new larger amount?

First, we need to determine how many miles are business related miles.

$24,500$ miles $\times$ 74% business related $= 24,500 \times 74\%$ business related miles

Let's drop the units and figure this out:

$24,500 \times 74\% = 24,500 \times 0.74 = 18,130.$

So Dave has 18,130 business miles.

His deduction at 31 cents per mile (or \$0.31 per mile) would be:

$18,130 \times \$0.31 = \$5620.30.$

His deduction at 35 cents per mile (or \$0.35 per mile) would be:

$18,130 \times \$0.35 = \$6345.50.$

Dave's accountant increased his deduction by

$\$6345.50 - \$5620.30 = \$725.20.$