Formulas

The Rule of 70 If a country's population continues to grow at a constant rate of $r\%$ per year, then it will double in size every $70/r$ years.

Static reserve $= S/U$ years.

Exponential reserve $= \dfrac{\ln\left(1+\frac{S}{U}r\right)}{\ln(1+r)}$ years.

Reproduction Curves

A reproduction curve is a mathematical model that gives the population size in the next year if the population size this year is known:

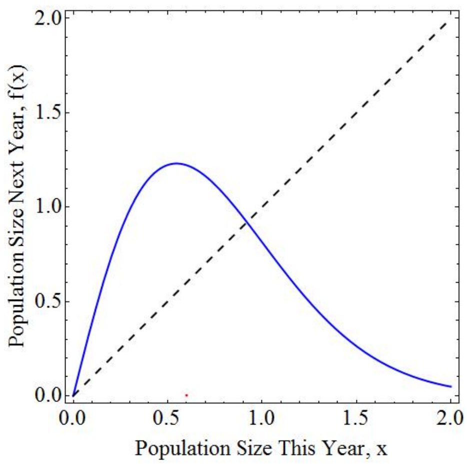


Figure 1: Population curve that relates the population in a current generation x to the population in the next generation by the formula $y = f(x)$. The dashed line is $y = x$ which represents no population change from year to year.

The basic shape of this curve is no accident. It rises on the left since there are enough resources for the population to increase, and falls on the right since the population is too large for the available resources and must decline. The details of the shape of the reproduction curve depend on the particular population being studied, but the basic shape is always the same.

The reproduction curve is usually used in the following manner. Vertical lines represent moving from one year to the next. The amount of the vertical line between the line $y = x$ and the reproduction curve represent the natural increase or natural decrease of the population from one year to the next. An equilibrium population is a population which does not change from year to year. An equilibrium population occurs where the reproduction curve intersects the line $y = x$.

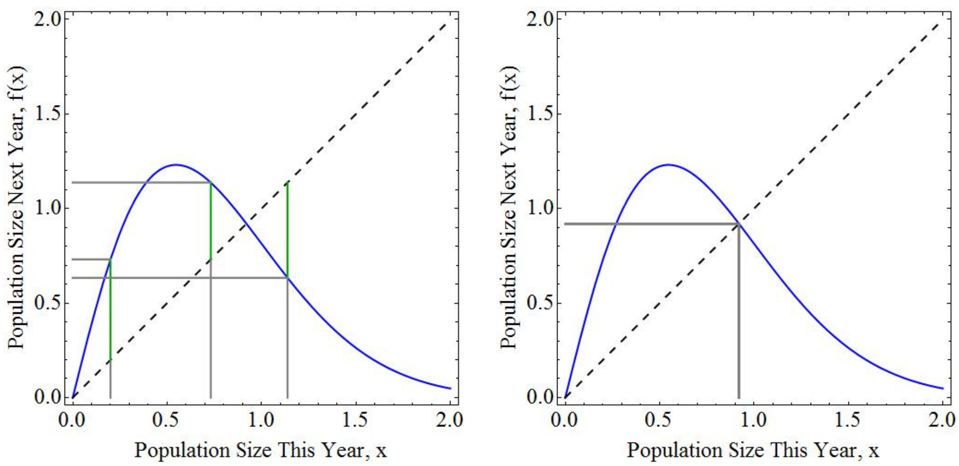


Figure 2: Left: Population starts at 0.2. In following years, the population is 0.725, 1.1.25, 0.625. The green lines represent population growth (above $y = x$) or decrease (below $y = x$) from year to year. Right: Equilibrium population.

Sustained Yield Harvesting of a Population

A simple model of harvesting a population would include the idea that greater effort would result in greater harvest. The harvest line could therefore be a straight line with steeper slope indicating greater harvest. It looks like the $y = x$ line we used, but it has a different slope to indicate different levels of harvesting.

A sustained yield harvesting policy is a policy that if continued indefinitely will maintain the same yield. We are most interested in the maximum sustainable yield, which would maintain the population at at a constant fixed amount, and harvest the maximum each year (if we harvest less than the maximum, the population would increase from year-to-year). This is described in Fig. 3.

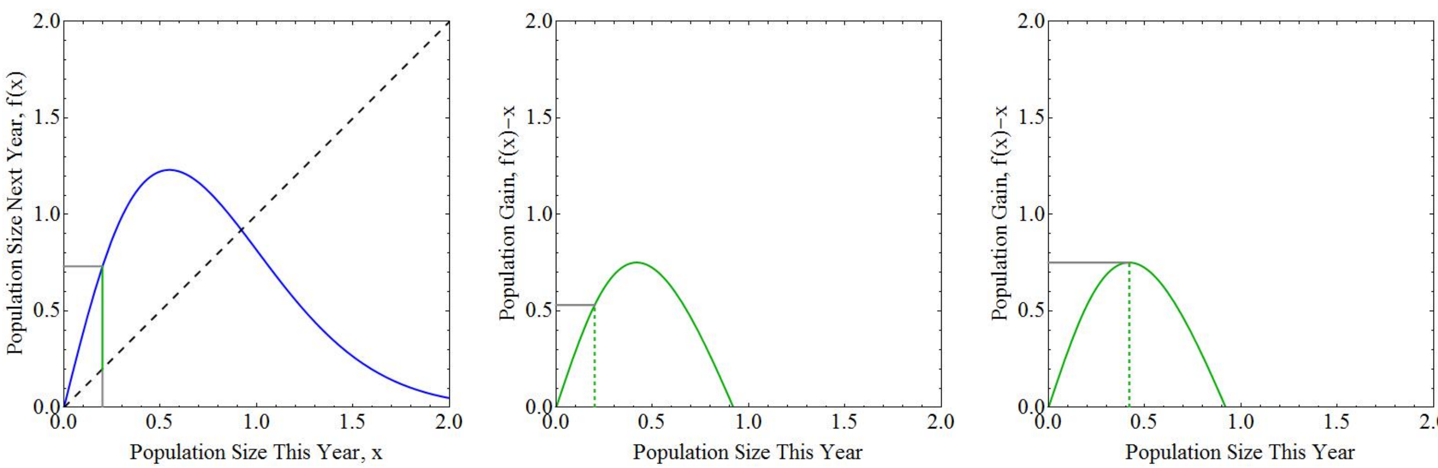


Figure 3: Left: Population curve. Center: Population Gain curve, $f(x) - x$. The dashed line shows the population gain going from 0.2 to 0.725, which is $0.725 - 0.2 = 0.525$. Right: The maximum sustainable yield would maintain the population at 0.42 year-to-year, with a maximum sustainable yield of 0.75.

An Alternate Way of Showing The Population Dynamics: Cobweb Diagrams

Another way of presenting this information is shown in Fig. 4. The horizontal lines that move towards the straight line $y = x$ are used to "reset" to the next year (and therefore represent the natural increase or natural decrease of the population, since they are the same length as the green lines on the left).

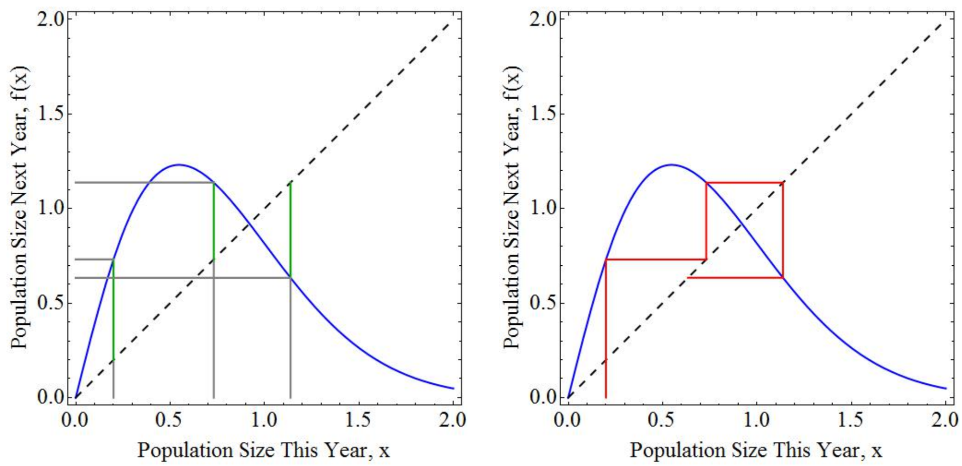
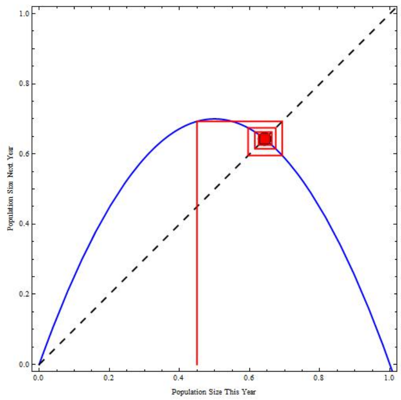
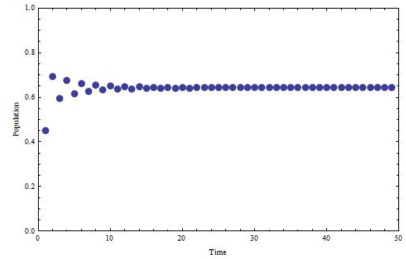


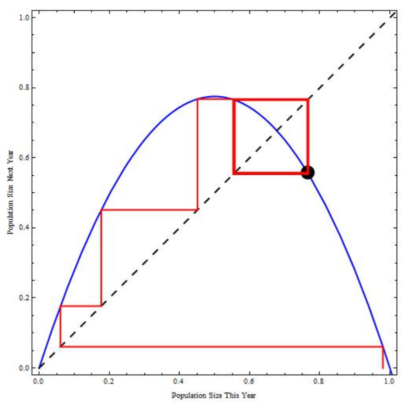
Figure 4: Left: Population curve. Right: Cobweb diagram.

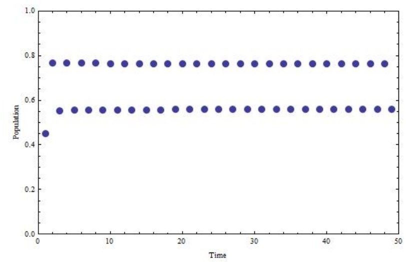
Chaos in Biological Populations

One definition of chaos is that knowing the current population does not allow you to predict the future population. Of course, mathematically we can predict future populations, so a better definition of chaos would be a system for which small changes in initial conditions lead to large differences later. This is really what predictability is all about–the predictions for a system without chaos would be roughly the same if we changed our starting position by a small amount. This isn't true for chaotic systems, and much of nature is chaotic.


Population Size Next Year
Population Size This Year


Population
Time


Population Size Next Year
Population Size This Year


Population
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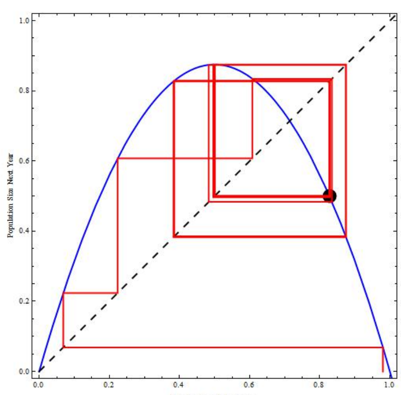
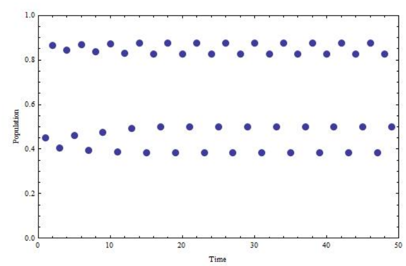

Population Size Next Year
Population Size This Year


Population
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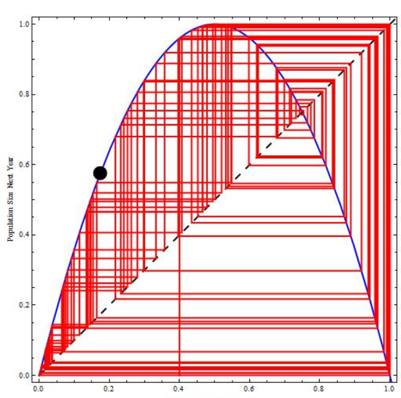
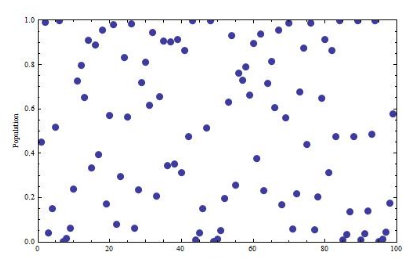

Population Size Next Year
Population Size This Year


Population
Time