

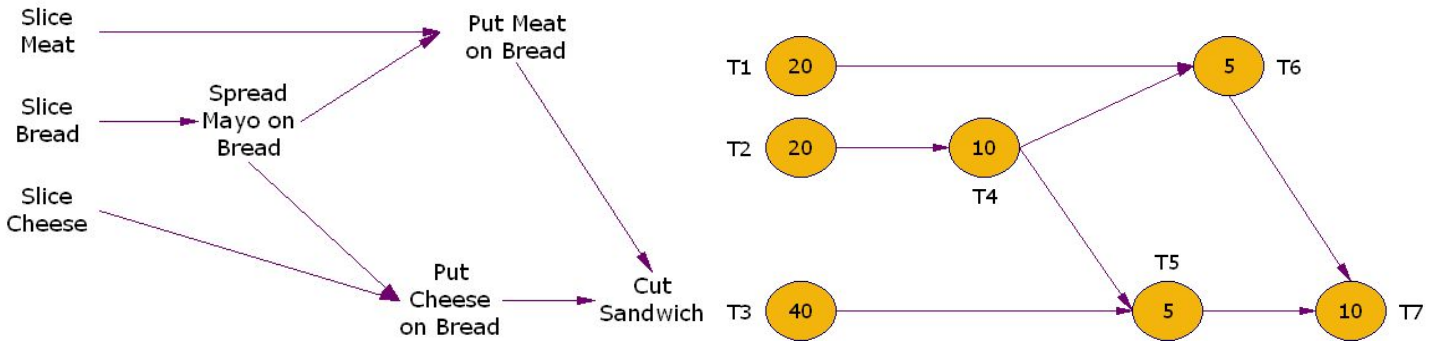
### Ordered Requirement Digraph

Job: Building a sandwich.

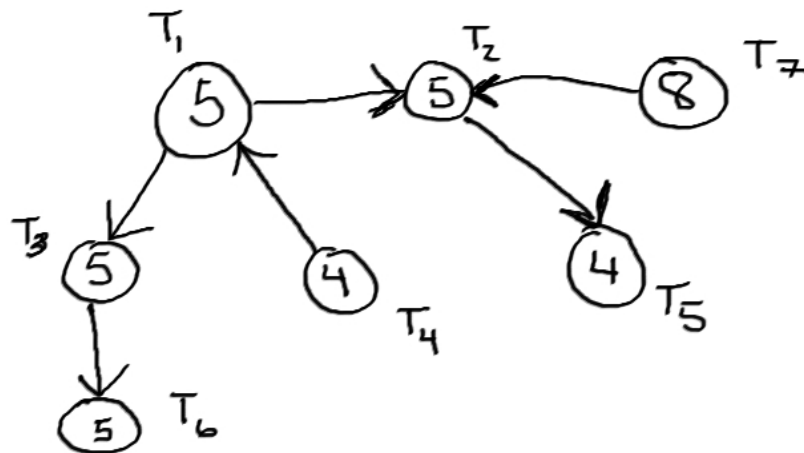
Tasks:

- slice bread (20 seconds)
- put meat on bread (5 seconds)
- put cheese on bread (5 seconds)
- spread mayo (10 seconds)
- slice meat (20 seconds)
- slice cheese (40 seconds)
- cut sandwich in half (10 seconds)

Here is my graph of the tasks on the right (the figure on the left just helped me create the graph):



### Example with Video Solution online



1. What is the quickest completion time this job can be completed?
2. Use critical path scheduling to create a priority list for this job.
3. Use the priority list you created to schedule the job on 2 processors using the List Processing Algorithm.
4. How long will the job take on 2 processors? Could the job be sped up on 3 processors?

## Bin Packing

Bin packing is related to the idea of scheduling independent tasks on a processor. We will assume that the shape of the object doesn't matter, just that it has some volume. If a bin has a volume of  $12 \text{ cm}^3$  left in it, then we can fit an object of  $12 \text{ cm}^3$  in the bin.

Bin packing is used to:

- buy wood for construction of shelving,
- photocopy multiple jobs on a given number of copiers,
- schedule commercials in time slots during TV shows,
- store computer files on different sectors of a hard drive.

For small problems, we could try different combinations of the items to be packed until we completely filled a bin, then move on to the next box. As you might guess, this procedure, although easy to describe, takes a tremendously long time for larger problems. In this way, it is similar to the travelling salesman problem in that we need to find heuristic algorithms to use on bin packing problems.

**Next Fit Algorithm** Put items into the first bin until the next item doesn't fit. Then, close up the bin and move on to a new bin. Keep doing this until all the items are packed.

**First Fit Algorithm** Put items into the first bin until the next item doesn't fit. Then open a new bin, but leave the first bin open. Put each item from the list into the lowest numbered bin in which it will fit, opening new bins when an item won't fit in any of the open bins.

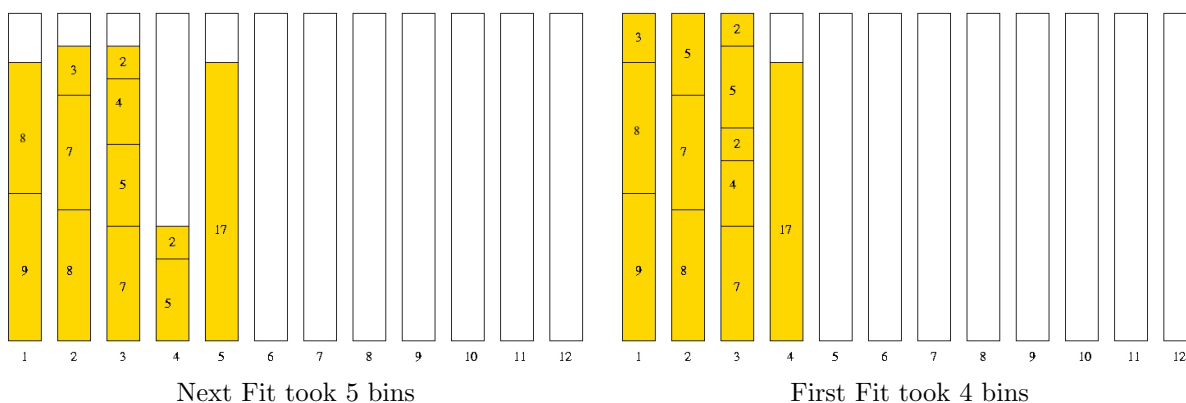
**Worst Fit Algorithm** Put items into the first bin until the next item doesn't fit. Then open a new bin, but leave the first bin open. Put the next item into the bin which has the most room left in it. Open a new bin if an item won't fit in any of the open bins.

Obviously, the priority list has an influence on the packing. If you first reorder the priority list from biggest to smallest, so you pack big items first and small items last, you have decreasing versions of the three algorithms above.

**I will not ask you to memorize the bin packing algorithms. If you need them on a test I will provide them.**

In the real world, these algorithms are implemented on a computer. One may not know all the items to be packed when one starts packing (think of files on a computer's hard disks), so ordering the lists (which seems like a great idea) might not be possible.

**Example** Say you have 12 objects of varying size to put in bins, that hold  $20 \text{ cm}^3$ . The objects have sizes (in  $\text{cm}^3$ ): 9, 8, 8, 7, 3, 7, 5, 4, 2, 5, 2, 17. This is the priority list for the order in which we try to pack the items.



### Vertex Colouring

**Example** The Managers of a zoo are planning to open a small satellite branch. The animals (A through J) are to be in enclosures in which compatible animals are displayed together. The table below has an X when two animals are not compatible.

	A	B	C	D	E	F	G	H	I	J
A			X					X	X	X
B			X	X				X		
C	X	X		X				X	X	X
D		X	X					X		
E						X	X			X
F					X					X
G					X				X	X
H	X	X	X	X					X	X
I	X		X				X	X		X
J	X		X		X	X	X	X	X	

1. Draw a graph that represents the information in the table.
2. Determine a valid vertex colouring of the graph.
3. What is the minimum number of enclosures needed to avoid housing incompatible animals in the same enclosure?
4. Is it possible to enclose the animals in such a way that each enclosure contains the same number of animals?