Mathematical Foundations of Linear Programming

To understand how linear programming works, we need to know how to:

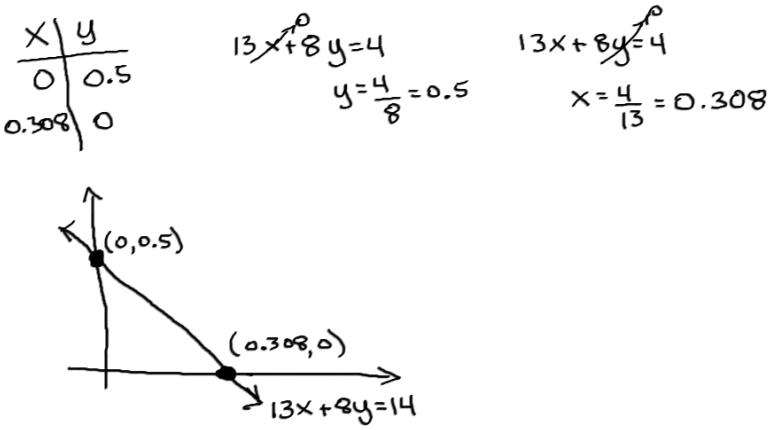
- locate an ordered pair (x, y)
- sketch linear relationships, for example $2x + 4y = 20$
- sketch linear inequalities, for example $2x + 4y \leq 20$
- solve for intersections of lines, for example $2x + 4y = 20$ and $4x + 2y = 16$

Properties of linear relationships:

Horizontal lines have the form $y = b$ and have slope $m = 0$.
Vertical lines have the form $x = a$ and have infinite slope.
Parallel lines have the same slope.

If we want to sketch the line $ax + dy = c$, we can find two ordered pairs that satisfy the equation and then draw a straight line through them. The easy way to do this is evaluate at $x = 0$, then $y = 0$.

Example Sketch $13x + 8y = 4$.



When you are graphing an equation or inequality and not using graph paper (which we often do), here are some important things to do:

- Label your axes, x axis to the right (in the direction of increasing x) and y axis to the top (in the direction of increasing y).
- Include arrows on the ends of your line if the line continues forever.
- include the equation of the line somewhere on the graph beside the line.
- Explicitly label the points you used to create the line. I prefer not to use ticks on the axes, but you can use ticks if you want–but be neat!
- Any annotations you make on the graph (maybe a triangle that shows the slope between two points on the line) should be large and neatly labeled so it is easy to read.
- Make the entire graph large enough to easily read, and redraw it if necessary.

Sketching linear inequalities

1. Replace the inequality symbol by an equality symbol. Graph the line.
   The line is solid if you had $\leq$ or $\geq$ and dashed if you had $<$ or $>$. The solid line means the line itself is part of the set of points that satisfies the inequality.
2. Test a point on one side of the line (the origin $(0, 0)$ is a good point to use as the test point if the line doesn't pass through it). If the inequality is true for the test point, shade the the side of the line that includes the test point. If the inequality is not true, shade the side of the inequality that does not contain the test point.

This procedure works since after Step 1 you have split the plane up into two regions: all the points on one side of the line satisfy the original inequality, and all the points on the other side of the line do not satisfy the inequality. Step 2 is just figuring out which side you want.

Finding Points of Intersections: Solving Linear Systems of Equations

To find where two lines intersect, there are two techniques. You can use either one. Both techniques involve getting down to one equation in one unknown, which you can solve.

The method of substitution involves the following steps:

1. solve one of the equations for one of the unknown variables,
2. substitute the equation from step (1) into the other equation to produce a single equation in a single unknown variable,
3. solve this equation for the unknown variable,
4. substitute into the equation from step (1) to get the second unknown variable.

The method of elimination involves the following steps:

1. rewrite one of the equations so that a coefficient of one of the variables is the opposite (different sign) from the other equation,
2. add the two equations, which will eliminate one of the variables,
3. solve the resulting equation for the unknown variable,
4. use one of the original equations to solve for the other unknown variable.

I like the first method, as it always works for linear systems.

Mixture Problems

Example A clothing manufacturer has 600 yds of cloth available to make shirts and decorated vests. Each shirt requires 3 yds of material and yields a profit of \$5. Each vest requires 2 yds of material and yields a profit of \$2. The manufacturer wants to guarantee that at least 100 shirts and 30 vests are produced, to keep the product line diversified. How many of each garment should be made to maximize profit?

Example A developer has 100 acres that is being divided into 1 acre lots, each with a new construction home. He is going to have two basic types of houses on the lots (with small variations in each type), a modest or a deluxe house. The building costs are \$20,000 on average for the modest house, \$40,000 for the deluxe. The profits are projected to be \$35,000 on average for the modest house, \$50,000 for the deluxe. The developer has \$2.6 million available for the project. How many of each type of house should the developer build to maximize profits?