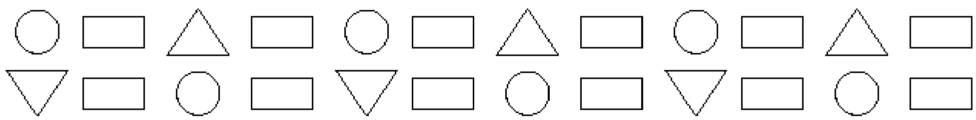
Symmetry Groups of Strip Patterns

For strip patterns, we have different elements of the symmetry group. Let's examine these through a simplified example.

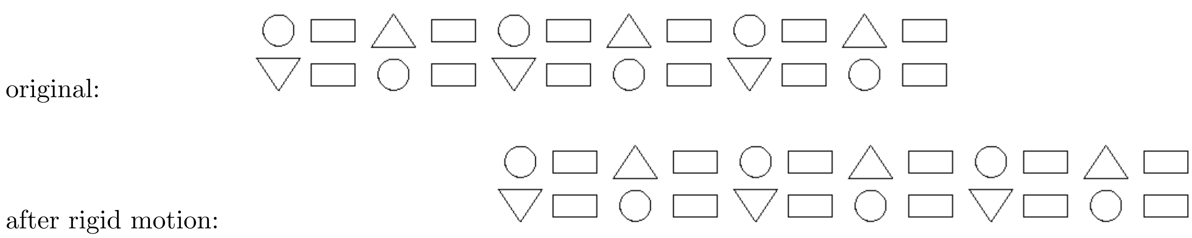


Remember, we think of this pattern extending indefinitely to the left and right.

Tracking the elements of the symmetry group, like we did for the equilateral triangle where we found $G = \{I, a, b, c, d, e\}$, is possible, but a fairly complicated task, so we won't do it. Instead, we will just be finding all the rigid motion symmetries for the strip pattern.

Horizontal Translation Symmetry

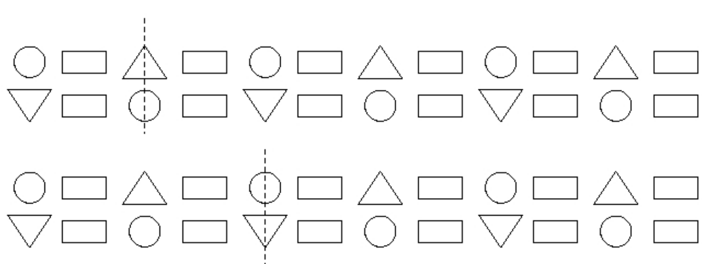
All strip patterns have horizontal translation symmetry. In this case, a translation to the right of 4 would preserve the pattern, and this is the smallest translation that preserves the pattern.



Vertical Reflection Symmetry

Can our strip pattern be reflected across a (particular) vertical line and still remain the same?

We see that our example pattern has this symmetry, if we reflect about a vertical line that bisects any of the circles.

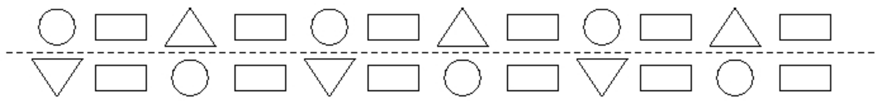


Horizontal Reflection Symmetry

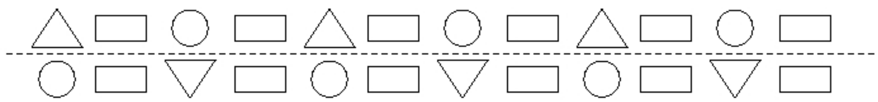
Can our strip pattern be reflected across a (particular) horizontal line and still remain the same?

The only viable horizontal line is the one through the middle of the pattern. Our pattern does not look the same when this reflection is performed, so our pattern does not possess this symmetry.

Before:



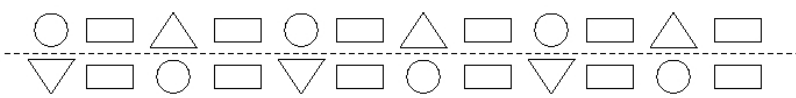
After:



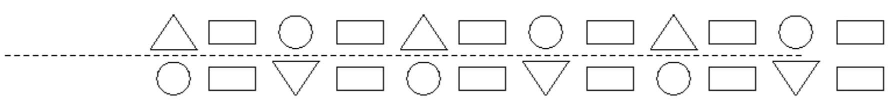
Glide Reflection Symmetry

Can our strip pattern be reflected across a horizontal line through the middle of the pattern, translated some amount, and still remain the same (recall these two rigid motions make up glide translation)?

Before:



After:

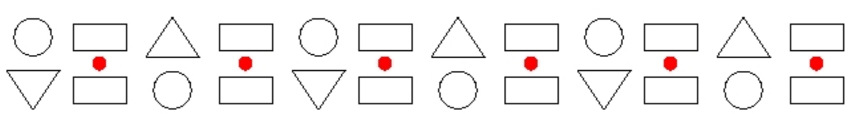


Yes–we see that a reflection about the horizontal line in the middle of the pattern followed by a translation of 2 units to the right would leave the pattern unchanged. Our pattern has glide reflection!

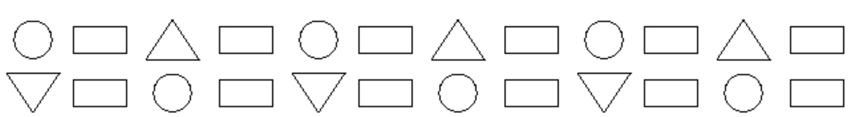
Rotational Symmetry

Can we rotate our strip pattern by 180 degrees and get the same pattern back?

Before:



After:



If we rotate 180 degrees about any of the red dots in the above diagram, we get the pattern back. So our strip pattern has rotational symmetry.

We have now figured out all the symmetries of our strip pattern.

The important thing was actually to identify the symmetries the strip pattern had. Writing the actual group can wait for an abstract algebra class!

Classifying a Strip Pattern Using Crystallographers' Notation

All strip symmetries can be classified into one of seven types. There are only seven types since there are only seven possible ways we can combine the symmetries we listed above.

This leads to the crystallographers' notation for classifying a strip pattern, which is described on page 699-700 of the text. There is a handy flowchart to help you figure out the crystallographers' notation for a given strip pattern.

Here are the seven types, the crystallographer classification for each, and the symmetries they possess.

- $p111$
  - translational
- $p1a1$
  - translational
  - glide reflection
- $p1m1$
  - translational
  - horizontal reflection
- $pm11$
  - translational
  - vertical reflection
- $p112$
  - translational
  - rotation by 180 degrees (or half turn)
- $pma2$
  - translational
  - rotation by 180 degrees (or half turn)
  - vertical reflection
  - glide reflection
- $pmm2$
  - translational
  - rotation by 180 degrees (or half turn)
  - vertical reflection
  - horizontal reflection

The pattern we looked at had vertical reflection symmetry, glide reflection symmetry, and a half turn symmetry (rotation by 180 degrees).

Following the flowchart on page 700 of the text, our strip pattern is type $pma2$.

Note: a pattern with horizontal reflection symmetry automatically has glide reflection symmetry (but not vice-versa!) so we could add glide reflection to the above list for any type with horizontal reflection symmetry.