NonPeriodic Tilings

A periodic tiling is one that has horizontal and vertical translation symmetry.

A fundamental region is a block (which may consist of more than one tile or even parts of tiles) which when translated horizontally and vertically recreates the tiling.

A nonperiodic tiling is one for which there is no translational symmetry.

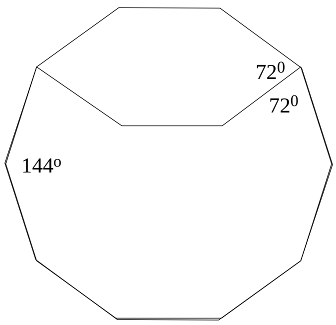
Historically, mathematicians thought that if you could construct a nonperiodic tiling from a set of tiles (one or more tile shapes), then you could also construct a periodic tiling from the set as well.

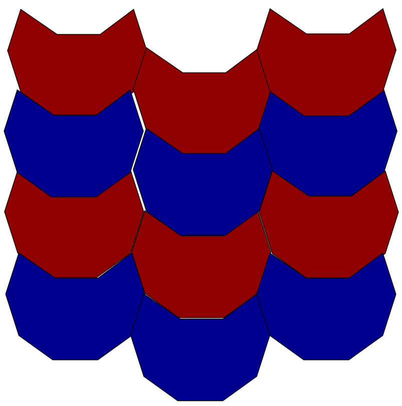
This seems to be true if the set of tiles is a single tile.

However, there exist sets of tiles which tile the plane nonperiodically yet do not tile the plane periodically. The first such sets of tiles were huge, involving over 20,000 tiles! We shall see that now there is an example of such a set which contains only 2 tiles.

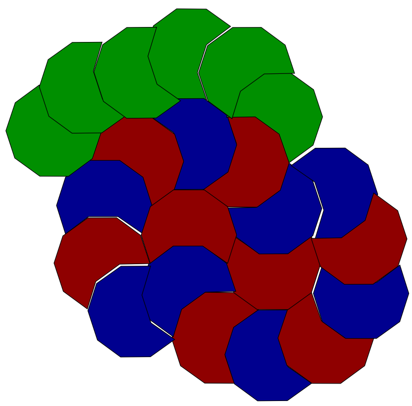
Single Tile NonPeriodic Tiling

Here is an example of a single tile which can tile the plane periodically and nonperiodically. It is based on a 10-gon, with part of the 10-gon removed. The interior angle in a 10-gon is 144^o, and the side with the bite is modified so the interior angle is 72^o. This is important if we are going to cover a vertex, since the interior angles must sum to 360^o around any vertex.





Periodic Tiling

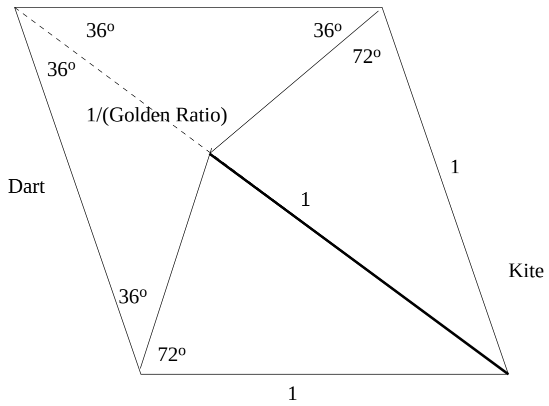


Nonperiodic Tiling

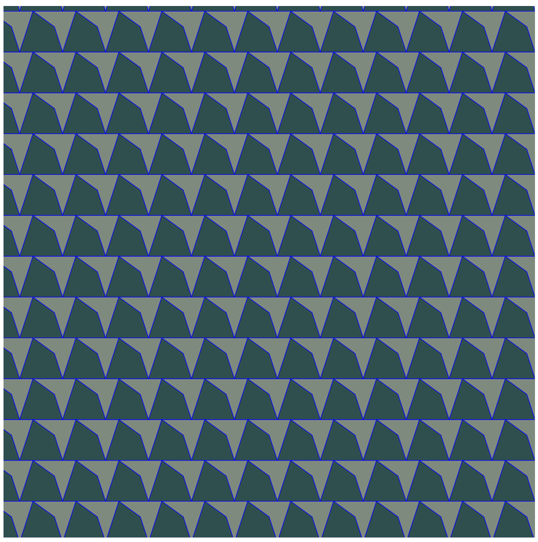
Kites and Darts

There is a set of two tiles which tile the plane periodically. They were found by Roger Penrose and are called *Penrose tiles* or *darts* and *kites*.

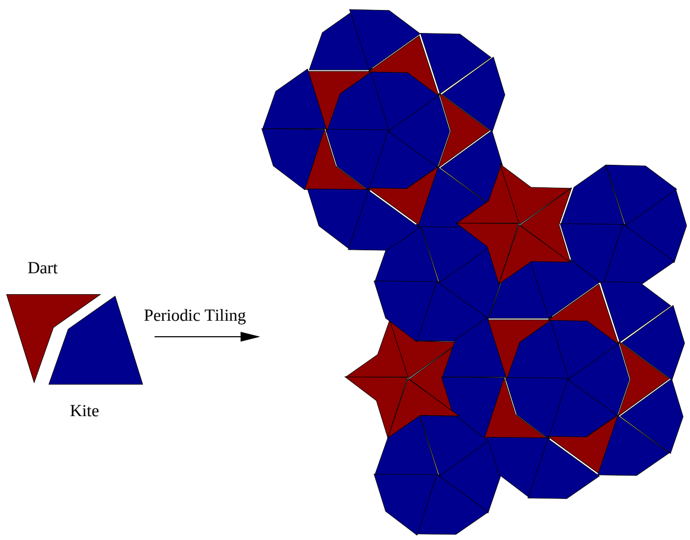
The tiles are constructed from a rhombus (quadrilateral with four equal sides and equal opposite interior angles).



The golden ratio is our old friend $\phi = \dfrac{1+\sqrt{5}}{2}$! Notice the appearance of the angle 72º again.



Periodic Tiling

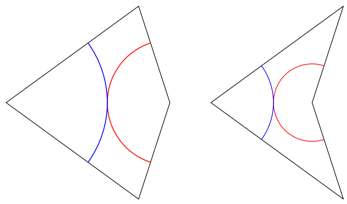


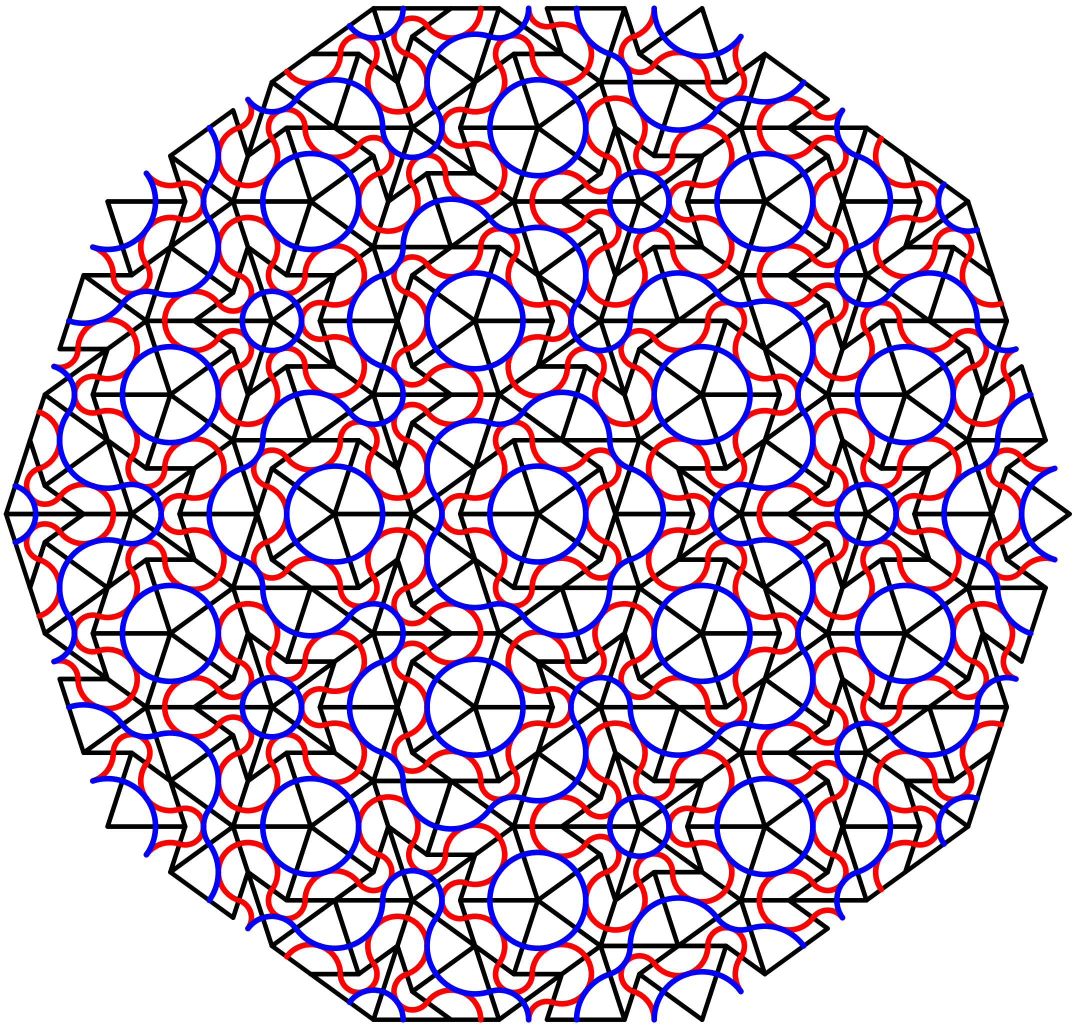
Another Periodic Tiling

Further modification must be made to the kites and darts if we are not to get a periodic tiling with them.

Kites and Darts: The Modification that Leads to Only Periodic Tilings

The kites and darts are modified by adding coloured lines to them, and the rule for tiling is that the tiles must be connected in a manner that makes the colours line up. This results in a nonperiodic tiling, and there is no way to create a periodic tiling under these rules.





References

The figures for the nonperiodic penrose tiling were found at:
http://mathworld.wolfram.com/notebooks/Tilings/PenroseTiles.nb.

The other figures were created using Xfig (requires *nix platform):
http://www.xfig.org/.