**Exponential Decay**

Exponential Decay is geometric growth with a negative rate of growth.

If $i > 0$, then

growth: $A = P(1+i)^k$

decay: $A = P(1-i)^k$

This decrease in the amount models inflation, where the value of the dollar goes down geometrically, or depreciation, where the value of an item decreases.

You can think of the value of a dollar as depreciating over time much like the value of an item depreciates over time (cars are a prime example of an item whose value decreases over time).

The actual price of an item at any time is said to be in current dollars. To compare prices of items from different times (which will take into account inflation), we use constant dollars, which are dollars from a particular year.

**Example** Suppose you bought a car in early 2002 for \$10,000. If its value (in current dollars) depreciates steadily at 12% per year, what will be its value (in current dollars) in early 2005?

The car was bought in 2002 and we want to know something about the value of the car in 2005, which is 3 years later.

The value of the car depreciates at 12% per year, so $i = 0.12/1 = 0.12$. Since the value of the car is decreasing, we use the formula for $V$ (value):

$$\begin{aligned} V &= P(1-i)^k \\ &= \$10000(1-0.12)^3 \\ &= \$10000(0.88)^3 \\ &= \$10000(0.88)(0.88)(0.88) \\ &= \$6814.72 \end{aligned}$$

Notice that rounding numbers early can significantly change your answer!!!

If you say $(0.88)^3 = 0.68$ instead of $(0.88)^3 = 0.681472$ you get a final value of \$6800.00, which is wrong due to rounding errors.

**The Consumer Price Index**

If inflation stayed constant over the years, we could use the above ideas to compare the cost of an item in an earlier year with the cost of the item today. However, inflation is not a constant! It varies over time.

The Consumer Price Index (CPI) allows us to compare the cost of items in different years.

The CPI represents costs of a basket of goods (food, housing, transportation, etc). This cost is measured each year for the same set of goods. The cost will vary over time, and also over region.

There must be some base number against which all the other numbers are compared, so the CPI for the years 1982–1984 is set to 100 (this is arbitrarily chosen by the Bureau of Labor, they could have chosen something else).

If you want to relate the cost of two items in different years you use the relation:

$$\frac{\text{Cost in Year } A}{\text{Cost in Year } B} = \frac{\text{CPI in Year } A}{\text{CPI in Year } B}$$

**Example** What is the value of a dollar from 1970 in 1987 dollars?

$$\frac{\text{Cost in 1970}}{\text{Cost in 1987}} = \frac{\text{CPI in 1970}}{\text{CPI in 1987}}$$

$$\frac{\text{Value of \$1 in 1970}}{\text{Value of \$1 in 1987}} = \frac{38.8}{113.6}$$

$$\text{Value of \$1 in 1970} = 0.341549\text{Value of \$1 in 1987}$$

$$\text{Value of \$1 in 1970} = \text{ \$0.341549 in 1987}$$

So the value of a 1970 dollar would have been 34 cents in 1987.

**Example** A house cost $39,000 in 1967. How much would the house be in 1999 dollars?

$$\frac{\text{Cost of house in 1999}}{\text{Cost of house in 1967}} = \frac{\text{CPI in 1999}}{\text{CPI in 1967}}$$

$$\frac{\text{Cost of house in 1999}}{\text{Cost of house in 1967}} = \frac{166.6}{33.4} = 4.98802$$

$$\text{Cost of house in 1999} = 4.98802 \times \text{Cost of house in 1967} = 4.98802 \times \$39000 = \$194\,533.93$$