Problem 1. (30 marks) FYI, the symmetry group for the square is known as the Dihedral group D_4.

(a) Demonstrate each of the seven rigid motion symmetries of the square (if you include the stay put transformation, you get eight total rigid motion symmetries).

(b) Construct a partial Cayley table for the rigid motion symmetries of the square, filling in the portion that shows:

- all combinations of rotations,
- combinations that lead to the identity element, and
- at least one entry that deals with a combination of two reflections.

Note that the table is not too large, so you could go ahead and fill in the entire Cayley table.

Problem 2. (10 marks) (based on Text Chapter 19 #57) For positive integers a and n, the expression $a \mod n$ means the remainder when a is divided by n. Thus, $23 \mod 4 = 3$ because $23 = 5 \times 4 + 3$. See Chapter 17 Section 4 for further details about modular arithmetic.

Consider the collection of elements $\{0, 1, 2, 3\}$ and the operation $\circ$ on them defined by $a \circ b = (a + b) \mod 4$.

(a) Construct the complete Cayley table for this situation.

(b) Compare it with the Cayley table for rotations only (including the stay put element) that you found in Problem 1.

Problem 3. (20 marks) Construct your own Escher style tiling of the plane. You might want to refer to http://library.thinkquest.org/16661/escher/tessellations.1.html for some ideas.

You solution should clearly explain the mathematical transformations that were performed on an underlying grid to create the Escher style tiling.

Very simplistic Escher style tilings that are well explained will result in a grade of 14/20 on this question. To get a higher grade, construct a more complex tiling involving rotations, multiple modifications, and shade your tiling so it looks like something.