

The test will have five sections:

- Matching
- True or False
- Multiple Choice
- Short Answer
- Long Answer

**Section 1. Matching** Match term or quantity in left column to the one description that best applies from the numbered columns.

- |                                  |  |
|----------------------------------|--|
| <b>regular polygon</b> _____     | 1. a polygon whose sides and angles are all equal.   |
| <b>quadrilateral</b> _____       | 2. a tiling.   |
| <b>rhombus</b> _____             | 3. a rigid motion.   |
| <b>tesselation</b> _____         | 4. a tiling in which there is no repetition of the pattern by translation.   |
| <b>isometry</b> _____            | 5. a tiling that uses a mix of regular polygons with different number of sides but in which are vertex types are alike—the same polygons in the same order at each vertex.   |
| <b>monohedral tiling</b> _____   | 6. a triangle with all sides the same length.  |
| <b>nonperiodic tiling</b> _____  | 7. an edge-to-edge tiling that uses only one kind of regular polygon.  |
| <b>regular tiling</b> _____      | 8. a polygon with four sides.  |
| <b>semiregular tiling</b> _____  | 9. for any two points in the polygon (including the boundary), all the points on the line segment connecting the two points lie inside the polygon (including the boundary). |
| <b>edge-to-edge tiling</b> _____ | 10. a parallelogram whose sides are all equal—four equal sides and equal opposite equal angles.  |
|                                  | 11. A pattern that exhibits similarity at ever finer scales.   |
|                                  | 12. a rectangle with sides that have ratio of the golden ratio.  |
|                                  | 13. a polygon with $n$ sides.  |
|                                  | 14. all the tiles are polygons and for every tile, each edge coincides with the entire edge of the bordering tile.   |
|                                  | 15. a tiling with only one size and shape of tile (the tile is allowed to "turn over" or appear in mirror image form).   |

**Section 2. True or False** Circle True (T) or False (F):

- (1) The tiling about a point which is labelled 3,6,3,6 would be a regular tiling ..... T F
  - (2) The numbers 21 and 34 are consecutive Fibonacci numbers. The next Fibonacci number in the sequence is 55 ..... T F
  - (3) A square has translational rigid motion symmetry ..... T F
  - (4) A strip pattern has translational rigid motion symmetry ..... T F
  - (5) Commutativity ( $a \circ b = b \circ a$ ) is one of the properties that is required of a group ..... T F
  - (6) All strip patterns have glide reflection symmetry ..... T F
  - (7)  $81 \pmod 3 = 27$  ..... T F
  - (8) The integers with addition do not form a group since there are no inverses ..... T F
  - (9) A *monohedral tiling* is a tiling that uses two different shapes of tiles ..... T F
  - (10) A *regular tiling* is a tiling that uses two different regular polygons for the tiles ..... T F
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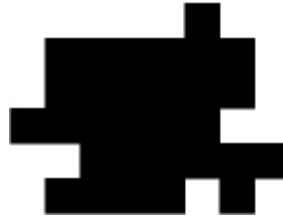
**Section 3. Multiple Choice** Circle the most appropriate answer:

- (1) Assume the following pattern continues in both directions. Which isometries preserve the pattern?



- A) horizontal translation only
  - B) horizontal translation and horizontal reflection only
  - C) horizontal translation and glide reflection only
  - D) horizontal translation, glide reflection, and vertical reflection only
- (2) Assume the following patterns continue in both directions. Which of the patterns has a reflection symmetry?  
I: AAAAAAAAAA      II: ZZZZZZZZZZZZ
- A) I only      B) II only      C) Both I and II      D) neither
- (3) Which of the following triangles can tile the plane?  
I: equilateral triangle      II: scalene triangle
- A) I only      B) II only      C) Both I and II      D) neither
- (4) The interior angle of a decagon (10-gon) is which of the following?
- A)  $36^\circ$       B)  $144^\circ$       C)  $180^\circ$       D)  $324^\circ$       E)  $18^\circ$

(5) Can the tile below be used to tile the plane?



- A) No  
 B) Yes, with translations only  
 C) Yes, but with translations and half-turns only  
 D) Yes, but reflections must be included

Use the following Cayley table to answer questions (6)-(9). The vertical column on the right is the action which is done first. The Cayley table represents a group.:

$\circ$	$b$	$c$	$d$	$a$
$b$	$a$	$c$	$b$	$d$
$c$	$d$	$b$	$a$	$c$
$d$	$c$	$a$	$b$	$d$
$a$	$b$	$c$	$d$	$a$

- (6) The missing entry from the table  $c \circ b$  must be:  
 A)  $a$       B)  $b$       C)  $c$       D)  $d$
- (7) The identity operator  $I$  is:  
 A)  $a$       B)  $b$       C)  $c$       D)  $d$
- (8) The inverse of  $c$  (another way of writing the inverse of  $c$  would be  $c^{-1}$ ) is:  
 A)  $a$       B)  $b$       C)  $c$       D)  $d$
- (9) The quantity  $b \circ b \circ c$  is:  
 A)  $a$       B)  $b$       C)  $c$       D)  $d$
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**Section 4. Short Answer**

Given the following Cayley Table, where  $a \circ b$  means  $b$  is done first and read from the left hand column and  $a$  is done second and read from the top row:

$\circ$	$e$	$a$	$b$	$c$	$d$	$f$
$e$	$e$	$a$	$b$	$c$	$d$	$f$
$a$	$a$	$e$	$d$	$f$	$b$	$c$
$b$	$b$	$f$	$e$	$d$	$c$	$a$
$c$	$c$	$d$	$f$	$e$	$a$	$b$
$d$	$d$	$c$	$a$	$b$	$f$	$e$
$f$	$f$	$b$	$c$	$a$	$e$	$d$

(1) The identity element is: \_\_\_\_\_

(2) The inverse of  $a$  is: \_\_\_\_\_

(3)  $(a \circ b) \circ d =$  \_\_\_\_\_

(4)  $a \circ a =$  \_\_\_\_\_

(5)  $f^{-1} =$  \_\_\_\_\_

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(6) Draw a strip pattern which possesses glide reflection symmetry but not horizontal reflection symmetry.

(7) Draw an example of the tiling about a point which would be labelled 3,3,3,3,3,3.

(8) Draw an example of the tiling about a point which would be labelled 3,6,3,6.

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**Section 5. Long Answer**

(1) If  $F_i$  is the  $i$ th Fibonacci number show the ratio  $F_{i+1}/F_i$  approaches the number  $(1 + \sqrt{5})/2$  as  $i$  gets large.

(2) List all the rigid motion symmetries of the symmetric group of the strip pattern CCCCCC.

(3) A semiregular tiling has two squares and three regular  $p$ -gons at each vertex. What number must  $p$  be?