The test will have five sections:

- Matching
- True or False
- Multiple Choice
- Short Answer
- Long Answer

Section 1. Matching Match term or quantity in left column to the one description that best applies from the numbered columns.

regular polygon ___1___

quadrilateral ___8___

rhombus ___10___

tesselation ___2___

isometry ___3___

monohedral tiling ___15___

nonperiodic tiling ___4___

regular tiling ___7___

semiregular tiling ___5___

edge-to-edge tiling ___14___

1. a polygon whose sides and angles are all equal.
2. a tiling.
3. a rigid motion.
4. a tiling in which there is no repetition of the pattern by translation.
5. a tiling that uses a mix of regular polygons with different number of sides but in which are vertex types are alike–the same polygons in the same order at each vertex.
6. a triangle with all sides the same length.
7. an edge-to-edge tiling that uses only one kind of regular polygon.
8. a polygon with four sides.
9. for any two points in the polygon (including the boundary), all the points on the line segment connecting the two points lie inside the polygon (including the boundary).
10. a parallelogram whose sides are all equal–four equal sides and equal opposite equal angles.
11. A pattern that exhibits similarity at ever finer scales.
12. a rectangle with sides that have ratio of the golden ratio.
13. a polygon with n sides.
14. all the tiles are polygons and for every tile, each edge coincides with the entire edge of the bordering tile.
15. a tiling with only one size and shape of tile (the tile is allowed to "turn over" or appear in mirror image form).

Section 2. True or False Circle True (T) or False (F):

(1) The tiling about a point which is labelled 3,6,3,6 would be a regular tiling .... T $\boxed{\text{F}}$

(2) The numbers 21 and 34 are consecutive Fibonacci numbers. The next Fibonacci number in the sequence is 55 .... $\boxed{\text{T}}$ F

(3) A square has translational rigid motion symmetry .... T $\boxed{\text{F}}$

(4) A strip pattern has translational rigid motion symmetry .... $\boxed{\text{T}}$ F

(5) Commutativity ($a \circ b = b \circ a$) is one of the properties that is required of a group .... T $\boxed{\text{F}}$

(6) All strip patterns have glide reflection symmetry .... T $\boxed{\text{F}}$

(7) $81 \mod 3 = 27$ .... T $\boxed{\text{F}}$

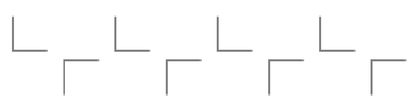
(8) The integers with addition do not form a group since there are no inverses .... T $\boxed{\text{F}}$

(9) A *monohedral tiling* is a tiling that uses two different shapes of tiles .... T $\boxed{\text{F}}$

(10) A *regular tiling* is a tiling that uses two different regular polygons for the tiles .... T $\boxed{\text{F}}$

Section 3. Multiple Choice Circle the most appropriate answer:

(1) Assume the following pattern continues in both directions. Which isometries preserve the pattern?



A) horizontal translation only
B) horizontal translation and horizontal reflection only
$\boxed{\text{C)}}$ horizontal translation and glide reflection only
D) horizontal translation, glide reflection, and vertical reflection only

(2) Assume the following patterns continue in both directions. Which of the patterns has a reflection symmetry?
I: AAAAAAAAAAA II: ZZZZZZZZZZZ

$\boxed{\text{A)}}$ I only B) II only C) Both I and II D) neither

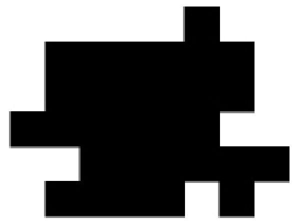
(3) Which of the following triangles can tile the plane?
I: equilateral triangle II: scalene triangle

A) I only B) II only $\boxed{\text{C)}}$ Both I and II D) neither

(4) The interior angle of a decagon (10-gon) is which of the following?

A) 36^o $\boxed{\text{B)}}$ 144^o C) 180^o D) 324^o E) 18^o

(5) Can the tile below be used to tile the plane?



A) No
$\boxed{\text{B)}}$ Yes, with translations only
C) Yes, but with translations and half-turns only
D) Yes, but reflections must be included

Use the following Cayley table to answer questions (6)-(9). The vertical column on the right is the action which is done first. The Cayley table represents a group.:

$\circ$	b	c	d	a
b	a		c	b
c	d	b	a	c
d	c	a	b	d
a	b	c	d	a

(6) The missing entry from the table $c \circ b$ must be:
A) a B) b C) c $\boxed{\text{D)}}$ d

(7) The identity operator I is:
$\boxed{\text{A)}}$ a B) b C) c D) d

(8) The inverse of c (another way of writing the inverse of c would be c^{-1}) is:
A) a B) b C) c $\boxed{\text{D)}}$ d

(9) The quantity $b \circ b \circ c$ is:
A) a B) b $\boxed{\text{C)}}$ c D) d

Section 4. Short Answer

Given the following Cayley Table, where $a \circ b$ means b is done first and read from the left hand column and a is done second and read from the top row:

$\circ$	e	a	b	c	d	f
e	e	a	b	c	d	f
a	a	e	d	f	b	c
b	b	f	e	d	c	a
c	c	d	f	e	a	b
d	d	c	a	b	f	e
f	f	b	c	a	e	d

(1) The identity element is: e

(2) The inverse of a is: a

(3) $(a \circ b) \circ d =$ e

(4) $a \circ a =$ e

(5) $f^{-1} =$ d

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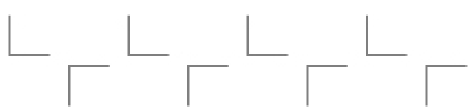
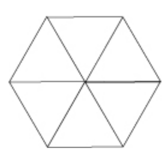
(6) Draw a strip pattern which possesses glide reflection symmetry but not horizontal reflection symmetry.

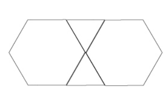
Answers may vary.



(7) Draw an example of the tiling about a point which would be labelled 3,3,3,3,3,3.



(8) Draw an example of the tiling about a point which would be labelled 3,6,3,6.



Section 4. Long Answer

(1) If F_i is the ith Fibonacci number show the ratio F_{i+1}/F_i approaches the number $(1+\sqrt{5})/2$ as i gets large.

Let $x = F_{i+1}/F_i$ when i is large. Then

$$\begin{aligned}
x &= \frac{F_{i+1}}{F_i} \\
x &= \frac{F_i + F_{i-1}}{F_i} \\
x &= 1 + \frac{F_{i-1}}{F_i} \\
x &= 1 + \frac{1}{x} \text{ since } \frac{F_{i-1}}{F_i} = \frac{1}{x} \text{ if } i \text{ is large} \\
x^2 &= x + 1 \\
x^2 - x - 1 &= 0 \\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ Quadratic formula} \\
x &= \frac{1 \pm \sqrt{1+4}}{2} \\
x &= \frac{1+\sqrt{5}}{2}
\end{aligned}$$

We keep only the positive square root since the numbers are all positive.

(2) List all the rigid motion symmetries of the symmetric group of the strip pattern CCCCCCC.

- horizontal translation
- glide reflection
- horizontal reflection

(3) A semiregular tiling has two squares and three regular p-gons at each vertex. What number must p be?

Two squares at the vertex give 180 degrees. This leaves 180 degrees to be split equally between the three regular polygons. The interior angle of an equilateral triangle is 60 degrees. So $p = 3$. The situation looks like the following: