Concepts: Literal Equations, Modeling.

Literal Equations have many unspecified variables, but you solve them using the same algebraic techniques. You can just can't simplify as much since you are working with variables instead of numbers. A nice example of a literal equation used in chemistry is the *Combined Gas Law*, which states that for a gas under two different sets of conditions (labeled by the subscript 1 or 2), it is true that

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}.$$

Example An ideal gas in state 1 has $P_1 = 3$ Pa, $V_1 = 20$ cm^3, and $T_1 = 40$ K. This gas is then adjusted so the pressure is $P_2 = 4$ Pa and the volume is $V_2 = 50$ cm^3. What is the temperature of the gas in state 2, using the Combined Gas Law?

$$\begin{aligned}
\frac{P_1V_1}{T_1} &= \frac{P_2V_2}{T_2} \text{ write the equation you will start with} \\
\frac{(3\text{Pa})(20\text{cm}^3)}{(40\text{K})} &= \frac{(4\text{Pa})(50\text{cm}^3)}{T_2} \text{ substitute in the values} \\
T_2 &= \frac{(4\text{Pa})(50\text{cm}^3)}{(3\text{Pa})(20\text{cm}^3)}(40\text{K}) \text{ solve for } T_2 \\
T_2 &= \frac{(4\cancel{\text{Pa}})(50\cancel{\text{cm}^3})}{(3\cancel{\text{Pa}})(20\cancel{\text{cm}^3})}(40\text{K}) \text{ solve for } T_2\text{, cancel units} \\
T_2 &= \frac{(4)(50)}{(3)(20)}(40)\text{K} = \frac{400}{3}\text{K} \sim 133\text{K}
\end{aligned}$$

I split the canceling of units into it's own step, but it need not be. Show as much detail as you need to get the simplification done correctly.

Example Solve the Combined Gas Law for T_2.

$$\begin{aligned}
\frac{P_1V_1}{T_1} \times T_2 &= \frac{P_2V_2}{\cancel{T_2}} \times \cancel{T_2} \\
\frac{\cancel{T_1}}{\cancel{P_1V_1}} \times \frac{\cancel{P_1V_1}}{\cancel{T_1}} \times T_2 &= \frac{T_1}{P_1V_1} \times P_2V_2 \\
T_2 &= \frac{T_1P_2V_2}{P_1V_1}
\end{aligned}$$

Example An ideal gas in state 1 has $P_1 = 2$ Pa, $V_1 = 20$ cm^3, and $T_1 = 12$ K. This gas is then adjusted so the temperature is $T_2 = 80$ K and the volume is $V_2 = 10$ cm^3. What is the pressure of the gas in state 2, using the Combined Gas Law?

$$
\begin{aligned}
\frac{P_1V_1}{T_1} &= \frac{P_2V_2}{T_2} \\
\frac{(2\text{Pa})(20\text{cm}^3)}{(12\text{K})} &= \frac{(P_2)(10\text{cm}^3)}{(80\text{K})} \\
P_2 &= \frac{(2\text{Pa})(20\text{cm}^3)(80\text{K})}{(12\text{K})(10\text{cm}^3)} \\
P_2 &= \frac{3200}{120}\text{Pa} = \frac{80}{3}\text{Pa} \sim 27\text{Pa}
\end{aligned}
$$

Example Solve the van der Waals equation (used to model fluid compression in chemistry) $\left(p + \frac{n^2a}{V^2}\right)(v - nb) = nRT$ for p.

$$
\begin{aligned}
\left(p + \frac{n^2a}{V^2}\right)(v - nb) &= nRT \\
\left(p + \frac{n^2a}{V^2}\right)\cancel{(v - nb)} \times \frac{1}{\cancel{(v - nb)}} &= nRT \times \frac{1}{(v - nb)} \quad \text{Division Principle} \\
p + \frac{n^2a}{V^2} &= \frac{nRT}{(v - nb)} \quad \text{Simplify} \\
p + \cancel{\frac{n^2a}{V^2}} - \cancel{\frac{n^2a}{V^2}} &= \frac{nRT}{(v - nb)} - \frac{n^2a}{V^2} \quad \text{Addition Principle (adding a negative quantity)} \\
p &= \frac{nRT}{(v - nb)} - \frac{n^2a}{V^2} \quad \text{Simplify}
\end{aligned}
$$

Note: Remember, other paths to the final solution are possible.

Modeling

Often you need to construct the model (a literal equation) which then must be solved. When these are coming from your discipline (biology, chemistry, economics, etc.) you can bring to bear your knowledge of the subject matter to help create the model. If the application does not rely on the knowledge of a specific subject, these are often called *word problems* since the solution is about translating the information in the problem into mathematical expressions using your knowledge of basic mathematics. The problem solving strategy for word problems is to gather as many facts as you can and manipulate the facts until a solution presents itself. It is often helpful to organize your thoughts using headings and diagrams. I will use some of these techniques to organize my thoughts in my solutions. It is especially important to define any variables as you write your solution.

Example Six less than five times a number is the same as seven times the number. What is the number?

- **Understand the problem**. We are looking for a number, let's call it x.
- **Write an equation.**

  Six less than five times a number: $5x - 6$
  Seven times the same number: $7x$
  These things are the same: $5x - 6 = 7x$.
- **Solve and state the answer.**

  $$\begin{aligned} 5x - 6 &= 7x \\ -2x &= 6 \\ x &= -3 \end{aligned}$$

  The number is -3.
- **Check.**

  $$5(-3) - 6 \stackrel{?}{=} 7(-3) \Rightarrow -21 \stackrel{?}{=} -21 \checkmark$$

Example Brad is a waiter, and he gets paid \$5.75 per hour, and he can keep his tips. He knows his tips average \$8.80 per table. If he worked an eight-hour shift and took home \$169.20, how many tables did he serve?

Since this is more complicated, let's use headings to organize the information and understand the problem. Start filling information in where ever it seems appropriate. Don't worry about putting something in the "wrong" column, your goal is just to collect information from the problem until you figure out what you need to do to solve it.

Gather Facts	Assign Variables	Basic Formula or Equation	Key Points
Brad is paid \$5.75 per hour. His tips average \$8.80 per table. He worked 8 hours. He took home \$169.20.	We need to know the number of tables Brad waited on, so let that be x.	The amount of money he earns from tips is \$$8.80x$. The amount of money he earned in salary in 8 hours is \$$5.75 \times 8 =$ \$46. This money must add up to \$169.20: $8.80x + 46 = 169.20$.	The number of tables should probably be an integer, and positive.

Now, we can solve the equation:

$$\begin{aligned} 8.80x + 46 &= 169.20 \\ 8.80x &= 123.20 \\ x &= 14 \end{aligned}$$

Brad waited on 14 tables.

Check by working backwards. If he waited on 14 tables he would have earned $\$5.75 \times 8 + 14 \times \$8.80 = \$169.20\checkmark$.